Ms Davis **Mr Feng**
Mrs Greenberg **Mrs Kerr**
Ms Lau

Name: ……..……..

Teacher:…….

MATHEMATICS EXTENSION 1

HSC ASSESSMENT

TASK 3

Term 2
2021

Time Allowed: 50 minutes

General Instructions

- Start each question in a **new** booklet.
- Write using black or blue pen.
- Erasable pens are not to be used.
- Approved calculators may be used.
- Diagrams are not drawn to scale.
- All necessary working should be shown in every question.
- Marks may be deducted for careless or untidy work.
- A reference sheet is provided.

Mark	**/30**
Rank	**/**
Highest Mark	**/30**

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Section I

Multiple Choice. (Each question in this section is worth 1 mark)

Use Multiple Choice answer sheet for questions 1 - 6.

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1 What is the value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)$?

(A) $\frac{\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{-\pi}{6}$

(D) $\frac{-5\pi}{6}$

2 Which of the following is <u>not</u> equivalent to $\int \frac{-dx}{\sqrt{1-x^2}}$?

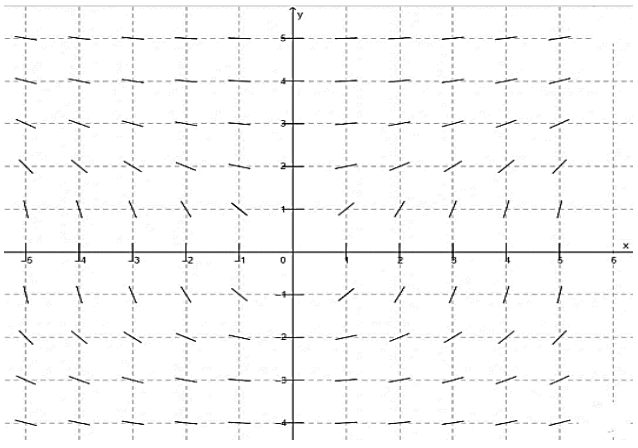
(A) $\sin^{-1}(-x)$

(B) $-\sin^{-1}(x)$

(C) $\cos^{-1}(-x)$

(D) $\cos^{-1}(x)$

3 Which of the given differential equations could produce the slope field shown?



(A) $\frac{dy}{dx}=\frac{x}{y^2}$

(B) $\frac{dy}{dx}=xy$

(C) $\frac{dy}{dx}=\frac{x}{y}$

(D) $\frac{dy}{dx}=x^2y$

4 What is $\int \frac{dx}{25+4x^2}$?

(A) $\tan^{-1}\left(\frac{2x}{5}\right)+C$

(B) $\frac{5}{x}\tan^{-1}\left(\frac{2x}{5}\right)+C$

(C) $\frac{1}{25}\tan^{-1}\left(\frac{2x}{5}\right)+C$

(D) $\frac{1}{10}\tan^{-1}\left(\frac{2x}{5}\right)+C$

5 Which expression is equivalent to $\int_0^{\frac{\pi}{3}} \tan x \sec^2 x dx$ when using the substitution $u = \tan x$?

(A) $\int_0^{\frac{1}{\sqrt{3}}} u\ du$

(B) $\int_0^{\sqrt{3}} u\ du$

(C) $\int_0^{\frac{1}{\sqrt{3}}} u\left(u^2+1\right) du$

(D) $\int_0^{\sqrt{3}} u\left(u^2+1\right) du$

6 Which of the following satisfies the differential equation $2x - y'e^y = 0$?

(A) $y = \ln|x|$

(B) $y = e^x$

(C) $y = \ln\left|x^2+4\right|$

(D) $y = e^{x^2}$

Section II

Answer each Question in a SEPARATE Booklet

Question 7 (8 marks) **Marks**

(a) Given $y = x\sin^{-1} x,$ find $\dfrac{dy}{dx}.$ **2**

(b) The area bounded by the curve $y = \sin x,$ the x-axis and the line $x = \dfrac{\pi}{3}$ is rotated about the x-axis. Find the volume generated. **3**

(c) Evaluate $\displaystyle\int_{-1}^{0} x\sqrt{1+x}\,dx$ by using the substitution $u = 1 + x.$ **3**

Question 8 (8 marks)

Answer in a SEPARATE Booklet **Marks**

(a) Show that $y = \sin x$ satisfies the differential equation $y'' + y = 0$. **2**

(b) Find $\int \frac{\ln 2x}{x}\, dx$ by using the substitution $u = \ln 2x$. **2**

(c) (i) Find the x coordinate of the intersection point of the curves $y = (x+1)^2$ and $y = (x-3)^2$. **1**

(ii) Find the volume of the solid generated when the area bounded by the curves, $y = (x+1)^2$, $y = (x-3)^2$ and the x-axis is rotated about the x-axis. **3**

Question 9 (8 marks)

Answer in a SEPARATE Booklet **Marks**

(a) Find the area bounded by the curve $y = \cos^{-1} x$ and the x and y axes. **3**

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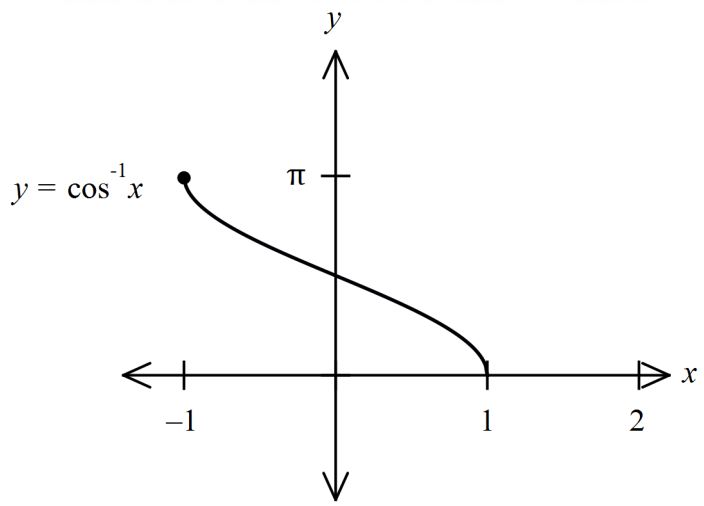


(b) The area A cm^2 is occupied by a bacterial colony. The colony has its growth modelled by the logistic equation $\frac{dA}{dt} = \frac{1}{25}A(50 - A)$ where $t \geq 0$ and t is measured in days. At time $t = 0$, the area occupied by the bacteria colony is $\frac{1}{2}$cm^2.

(i) Show that $\frac{1}{A(50-A)} = \frac{1}{50}\left(\frac{1}{A} + \frac{1}{50-A}\right)$. **1**

(ii) Using the result from above in (i), solve the logistic equation and hence show that $A = \frac{50}{1+99e^{-2t}}$. **3**

(iii) According to this model, what is the limiting area of the bacterial colony? **1**

End of Assessment

1. C 2. C 3. A
4. D 5. B 6. C

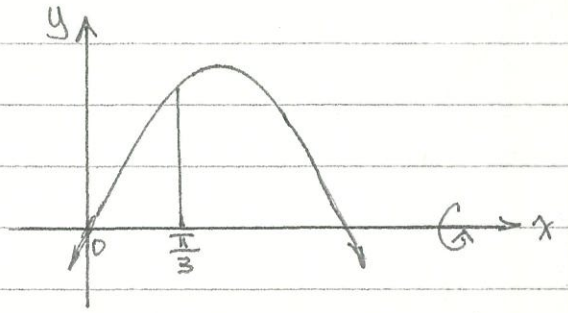
Question 7 (8 marks)

a) $y = x \sin^{-1} x$

$y' = 1 \times \sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \times x$

$= \sin^{-1} x + \frac{x}{\sqrt{1-x^2}}$ (2)

b) Volume



$= \pi \int_0^{\frac{\pi}{3}} \sin^2 x \, dx$

$= \frac{\pi}{2} \int_0^{\frac{\pi}{3}} (1 - \cos 2x) \, dx$

$= \frac{\pi}{2} \left[x - \frac{1}{2} \sin 2x \right]_0^{\frac{\pi}{3}}$

$= \frac{\pi}{2} \left[\frac{\pi}{3} - \frac{1}{2} \sin \frac{2\pi}{3} - 0 + \frac{1}{2} \sin 0 \right]$

$= \frac{\pi}{2} \left[\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right]$ cubic units (3)

c) $\int_{-1}^{0} x \sqrt{1+x} \, dx$; $u = 1 + x$

$= \int_0^1 \sqrt{u} (u - 1) \, du$; $du = dx$

$= \int_0^1 (u^{\frac{3}{2}} - u^{\frac{1}{2}}) \, du$; $x = 0, \ u = 1 + 0 = 1$

$= \left[\frac{2}{5} u^{\frac{5}{2}} - \frac{2}{3} u^{\frac{3}{2}} \right]_0^1$; $x = -1, \ u = 1 + (-1) = 0$

$= \frac{2}{5} - \frac{2}{3} - 0 + 0$

$= \frac{-4}{15}$ (3)

Question 8 (8 marks)

a) $y = \sin x$

$y' = \cos x$ (1).

$y'' = -\sin x$

$LHS = y'' + y$

$= -\sin x + \sin x$

$= 0$ (2)

$= RHS$

b) $\int \frac{\ln 2x}{x}\, dx$; $u = \ln 2x$ (1).

$du = \frac{1}{x}\, dx$

$= \int u\, du$

$= \frac{1}{2} u^2 + C$

$= \frac{1}{2} (\ln 2x)^2 + C$ (2)

c) i) $(x+1)^2 = (x-3)^2$

$x^2 + 2x + 1 = x^2 - 6x + 9$

$x = 1$ (1)

ii) Volume

$= \pi \int_{-1}^{1} (x+1)^4\, dx$ (1).

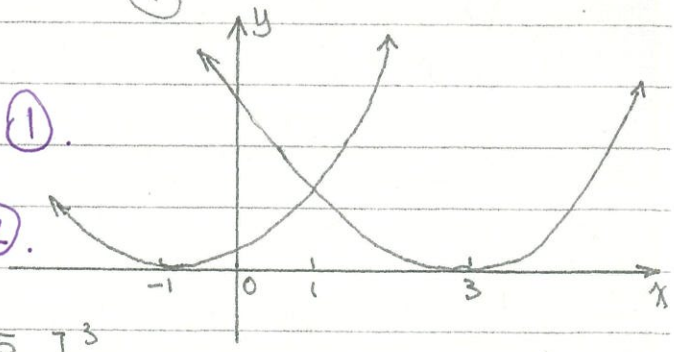
$+ \pi \int_{1}^{3} (x-3)^4\, dx$ (2).

$= \pi \left[\frac{1}{5}(x+1)^5\right]_{-1}^{1}$

$+ \pi \left[\frac{1}{5}(x-3)^5\right]_{1}^{3}$

$= \pi \left[\frac{1}{5}(2)^5 - 0 + 0 - \frac{1}{5}(-2)^5\right]$

$= \frac{64\pi}{5}$ cubic units (3)



Question 9 (8 marks)

a) $y = \cos^{-1} x$

$\Rightarrow x = \cos y$

Area $= \int_0^{\frac{\pi}{2}} \cos y \, dy$

$= \left[\sin y \right]_0^{\frac{\pi}{2}}$

$= \sin \frac{\pi}{2} - \sin 0$

$= 1 - 0$

$= 1$ square unit

(3) — Correct

(2) — Correct integral, correct limits, correct integration

(1) — correct integral

$\int_0^{\pi/2} \cos x \, dx \rightarrow (1)$

(3)

b) i) $RHS = \frac{1}{50}\left(\frac{1}{A} + \frac{1}{50-A}\right)$

$= \frac{1}{50} \times \frac{50-A+A}{A(50-A)}$

$= \frac{1}{50} \times \frac{50}{A(50-A)}$

$= \frac{1}{A(50-A)}$

$= LHS$

(1) r/w

ii) $\frac{dA}{dt} = \frac{1}{25} A(50-A)$

$\int \frac{dA}{A(50-A)} = \int \frac{dt}{25}$

$\frac{1}{50} \int \left(\frac{1}{A} + \frac{1}{50-A}\right) dA = \int \frac{dt}{25}$

$\ln A - \ln(50-A) = 2t + C$

$\ln\left(\frac{50-A}{A} \times k\right) = -2t$

When $t = 0$, $A = \frac{1}{2}$;

$\ln\left(\frac{50-0.5}{0.5} k\right) = 0$

$99k = 1$

$k = \frac{1}{99}$

$\ln\left(\frac{50-A}{A} \times \frac{1}{99}\right) = -2t$

(3) — All correct.

(2) — Correctly integrated with initial condition

(1) — Just the set up

$$\frac{50-A}{A} \times \frac{1}{99} = e^{-2t}$$

$$\frac{50-A}{A} = 99e^{-2t}$$

$$50 - A = 99Ae^{-2t}$$

$$50 = A + 99Ae^{-2t}$$

$$50 = A(1 + 99e^{-2t})$$

$$A = \frac{50}{1+99e^{-2t}} \quad (3)$$

iii) As $t \to \infty$,

$e^{-2t} \to 0$, $1 + 99e^{-2t} \to 1$

So $A \to 50$

∴ Limiting area $= 50\ cm^2$. (1)