# HORNSBY GIRLS HIGH SCHOOL

# Mathematics Extension 1

Year 12
Assessment 2 2022

**STUDENT NUMBER:** ____________________

**General Instructions**

- Reading Time – **5** minutes
- Working Time – **75** minutes
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators may be used
- A table of standard integrals is provided
- Show all necessary working
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

**Total marks – 47**

**Section I** Pages 2 – 3

5 marks

Attempt Questions 1 – 5

Answer on the Objective Response Answer Sheet provided

**Section II** Pages 4 – 6

42 marks

Attempt Questions 6 – 8. Start each question on a new page. Write your student number on page

At the end of the assessment:

- Order your solutions, starting with Multiple Choice answer sheet, then Questions 6-8
- Place your question paper on top
- Staple through

| ***Question*** | ***1-5*** | ***6*** | ***7*** | ***8*** | ***Total*** |
|---|---|---|---|---|---|
| ***Total*** | */5* | */14* | */14* | */14* | */47* |

*This assessment task constitutes 30% of the Higher School Certificate Course School Assessment*

# Section I

**5 marks**
**Attempt Questions 1 – 5**
**Allow about 10 minutes for this section**

Use the Multiple Choice answer sheet for Questions 1 – 5

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1. How many arrangements can be made using all the letters of the word ISOSCELES?

   (A) 30 240

   (B) 362 880

   (C) 60 480

   (D) 90 720

2. In the expansion of $(x-2)^4(3-4x)$, the coefficient of $x^3$ is:

   (A) 96

   (B) $-96$

   (C) $-120$

   (D) 120

3. A book club accepts members from the age of 30 to 60 including 30 and 60 years old. How many members would the book club need to guarantee that at least two members are the same age?

   (A) 30

   (B) 31

   (C) 32

   (D) 33

4. Which expression is equivalent to $\cos x + 2\sin x$ ?

(A) $5\sin\ x - 0.46$

(B) $\sqrt{5}\cos\ x - 1.1$

(C) $\sqrt{5}\sin\ x + 1.1$

(D) $5\cos\ x - 1.1$

5. Expanding $(1-x)^n$ then differentiating with respect to ***x***, we will then have:

(A) $1 + x + x^2 + x^3 + \dots$

(B) $\binom{n}{1} + 2\binom{n}{2}x + 3\binom{n}{3}x^2 + 4\binom{n}{4}x^3 + \dots$

(C) $1 - x + x^2 - x^3 + \dots$

(D) $-\binom{n}{1} + 2\binom{n}{2}x - 3\binom{n}{3}x^2 + 4\binom{n}{4}x^3 + \dots$

**End of Section I**

## Section II

**42 marks**
**Attempt Questions 6 – 8**
**Allow about 60 minutes for this section**

Begin each question on a new answering booklet, indicating the question number.
Extra writing paper is available.

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**Question 6** (14 marks) **Start a new answering booklet.** **marks**

(a) Solve ${}^{n}C_2 + {}^{n+1}C_2 = 8$ for the value of $n$. **2**

(b) In the expansion of $\left(\dfrac{1}{x^2} - 2x^2\right)^{10}$, find the term independent of $x$. **2**

(c) Seven people are seated at a round table. What is the probability that two people, Jenny and Lindsay, do not sit next to one another? **2**

(d) Differentiate $f(x) = \ln(x \sin x), \quad (\sin x \neq 0)$ . **2**

(e) Given that $2\sin x \cos x \sec(2x) = 1$ , solve for $x$ where $0 \le x \le \pi$ . **2**

(f) Given $f(x) = \sin^3(2x)$ ,

(i) show that $f'(x) = 6\cos(2x) - 6\cos^3(2x)$ . **2**

(ii) Find the gradient of the normal to $f(x)$ at $x = \dfrac{2\pi}{3}$. **2**

**Question 7** (14 marks) **Start a new answering booklet.**

(a) Consider set $S = \{1, 3, 5, 7, 9\}$,

(i) How many numbers greater than 5000 can be formed from the digits in $S$ if no digit is repeated? **2**

(ii) How many subsets of $S$ contains at least three digits? **1**

(b)

(i) If $t = \tan x$, show that $\dfrac{1+\sin 2x}{\cos 2x} = \dfrac{1+t}{1-t}$. **2**

(ii) Hence, find and check all the possible solutions of $\dfrac{1+\sin 2x}{\cos 2x} = 2$ for $0 \le x \le 2\pi$. **3**

(c)

(i) Show that $2\sin x + 2\cos\left(x + \dfrac{\pi}{6}\right) = \sqrt{3}\cos x + \sin x$. **2**

(ii) Hence, write $2\sin x + 2\cos\left(x + \dfrac{\pi}{6}\right)$ in the form $R\cos(x-\alpha)$. **2**

where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

(iii) Hence, or otherwise, solve $2\sin x + 2\cos\left(x + \dfrac{\pi}{6}\right) = 1$ for $0 < x < 2\pi$. **2**

**Question 8** (14 marks) **Start a new page.**

(a) Mary, John and five other people form a queue. In how many ways can the seven people line up if John is always somewhere behind Mary? **2**

(b) The concentration $C$ of carbon dioxide $CO_2$ in an experiment is modelled by function

$C(t) = 100\cos\left(\frac{\pi t}{2} - \frac{\pi}{4}\right) + 100$, where $t$ is the number of hours after mid night.

(i) Find the first time at which the concentration of $CO_2$ reaches its maximum and minimum values. **2**

(ii) Differentiate $C(t)$. **2**

(iii) Solve $C'(t) = 0$ where $0 \le t \le 4.5$. **2**

(iv) Show that the curve of $C(t)$ intersects the $y$-axis at $50 \ \sqrt{2}+1$ . **1**

(v) Sketch the graph of $C(t)$ where $0 \le t \le 4.5$. Clearly show its range, y-intercept, and label the coordinates of its maximum and minimum values. **3**

(c) Given $(1+x)^{11} = (1+x)^3(1+x)^8$ , show that **2**

$$\binom{11}{5} = \binom{8}{5} + \binom{3}{1}\binom{8}{4} + \binom{3}{2}\binom{8}{3} + \binom{8}{2}$$

**End of paper**

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# HORNSBY GIRLS HIGH SCHOOL

# Mathematics Extension 1
# Year 12
# Assessment 2 2022

## MULTIPLE CHOICE ANSWER SHEET

**Student Number**:________________________

Questions 1 – 5

Completely fill the response oval representing the most correct answer.

| | | | | |
|---|---|---|---|---|
| 1. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 2. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 3. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 4. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 5. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |

# Hornsby Girls High School
# Year 12 Mathematics Extension 1 HSC Assessment2 2022
# Solutions

| Solutions | Marker's Comments |
|---|---|
| **Question 1** **A**<br>$\frac{9!}{3!2!}=30240$ | Generally well done. |
| **Question 2** **C**<br>$\binom{4}{3}x^3(-2)\times 3+\binom{4}{2}x^2(-2)^2(-4x)\times 3$<br>$=-24x^3-96x^3$<br>$=-120x^3$ | Many students forgot to add the second part which also gives $x^3$. |
| **Question 3** **C**<br>There are 31 ages from age 30 to 60.<br>31+1=32 by pigeonhole principle we need 32 members to guarantee that at least two members are the same age. | Generally well done. |
| **Question 4** **B**<br>$r=\sqrt{1^2+2^2}$<br>$=\sqrt{5}$<br>$\alpha=\tan^{-1}\left(\frac{2}{1}\right)$<br>$\approx 1.107$ | Generally well done. |
| **Question 5** **D**<br>$(1-x)^n=\binom{n}{0}1^n(-x)^0+\binom{n}{1}1^{n-1}(-x)^1+\binom{n}{2}1^{n-2}(-x)^2+\binom{n}{3}1^{n-3}(-x)^3+......$<br>$=1-\binom{n}{1}x+\binom{n}{2}x^2+\binom{n}{3}x^3+......$<br>$\frac{d}{dx}(1-x)^n=0-\binom{n}{1}+2\binom{n}{2}x-3\binom{n}{3}x^2+......$ | Generally well done. |

| Solutions | Marker's Comments |
|---|---|
| **Question 6**<br>(a)<br>$^{n}C_2 + ^{n+1}C_2 = 8$<br>$\frac{n!}{2(n-2)!} + \frac{(n+1)!}{2(n+1-2)!} = 8$<br>$\frac{n(n-1)(n-2)!}{2(n-2)!} + \frac{(n+1)n(n-1)!}{2(n-1)!} = 8$<br>$\frac{n(n-1)}{2} + \frac{(n+1)n}{2} = 8$<br>$n^2 - n + n^2 + n = 16$<br>$2n^2 = 16$<br>$n^2 = 8$<br>$n = \pm\sqrt{8}$<br>$n = \pm 2\sqrt{2}$<br>but n must be a non-negative integer<br>$\therefore$ no solution | 1 for correctly rewriting in factorial notation.<br><br>1 for solving and discarding solutions by recognising that n must be a non-negative integer, in this case n>2.<br>Only 3 students achieved the second mark. |
| (b)<br>$\left(x^{-2} - 2x^2\right)^{10} = {}^{10}C_r \left(x^{-2}\right)^{n-r} \left(-2x^2\right)^r$<br>$= {}^{10}C_r (-2)^r \left(x^{-2}\right)^{n-r} x^{2r}$<br>$= {}^{10}C_r (-2)^r x^{-2(n-r)+2r}$<br>$= {}^{10}C_r (-2)^r x^{-20+4r}$<br>$-20 + 4r = 0$ for constant term<br>$r = 5$<br>$\therefore {}^{10}C_5 (-2)^5 = -8064$ | Done well by most students.<br>A few numerical errors. |
| (c)<br>$\frac{4 \times 5!}{6!} = \frac{2}{3}$<br>or:<br>$\frac{6! - 2!5!}{6!} = \frac{2}{3}$ | Some students found the number of arrangements but did not find the probability. |
| (d) i)<br>$f(x) = \ln(x \sin x)$<br>$= \ln x + \ln(\sin x)$<br>$f'(x) = \frac{1}{x} + \frac{1}{\sin x}\cos x$<br>$= \frac{1}{x} + \cot x$<br>**or**<br>$f'(x) = \frac{1}{x \sin x}(x \cos x + \sin x)$<br>$= \frac{x \cos x + \sin x}{x \sin x}$<br>$= \cot x + \frac{1}{x}$ | Done well by most students. |

| | |
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| (e)<br>$2\sin x\cos x\sec 2x=1, \quad 0\le x\le \pi$<br>$\sin 2x\sec 2x=1, \quad 0\le 2x\le 2\pi$<br>$\dfrac{\sin 2x}{\cos 2x}=1$<br>$\tan 2x=1$<br>$2x=\dfrac{\pi}{4},\dfrac{5\pi}{4}$<br>$x=\dfrac{\pi}{8},\dfrac{5\pi}{8}$ | Poorly done.<br>Some students tried to put it in terms of *sinx* and *cosx* instead of *sin2x* and *cos2x*. That method failed to correctly go anywhere and made the solving process of the incorrect equation easier. |
| **Solutions** | |
| **Question 6**<br>(f) i)<br>$f'(x)=3\sin^2(2x)\cos(2x)\times 2$<br>$=6\sin^2(2x)\cos(2x)$<br>$=6(1-\cos^2(2x))\cos(2x)$<br>$=6\cos(2x)-6\cos^3(2x)$ | Reasonably well done.<br>Some students used<br>$\sin^2(2x)$<br>$=(\sin 2x)^2$<br>$=(2\sin x\cos x)^2$<br>which was not helpful for this question since the result was in terms of *2x* not *x*. |
| (f) ii)<br>$f'\left(\dfrac{2\pi}{3}\right)=6\cos\left(\dfrac{4\pi}{3}\right)-6\cos^2\left(\dfrac{4\pi}{3}\right)$<br>$=6\left(-\dfrac{1}{2}\right)-6\left(-\dfrac{1}{2}\right)^3$<br>$=-3+\dfrac{6}{8}$<br>$=-\dfrac{9}{4}$<br>$m_{normal}=\dfrac{4}{9}$ | Many students made errors with evaluating<br>$-6\cos^2\left(\dfrac{4\pi}{3}\right)$ |
| **Question 7**<br>(a) i)<br>4-digit numbers: $3\times4\times3\times2$<br>5-digit numbers: $5!$<br>$\therefore 3\times4\times3\times2+5!=192$ | Generally well done.<br>Some students forget to add the 5-digit case. |
| (a) ii)<br>$2^5-({}^5C_0+{}^5C_1+{}^5C_2)=16$<br>*or*<br>${}^5C_3+{}^5C_4+{}^5C_5=16$ | Some students didn't add the ${}^5C_5$ case. |

| | |
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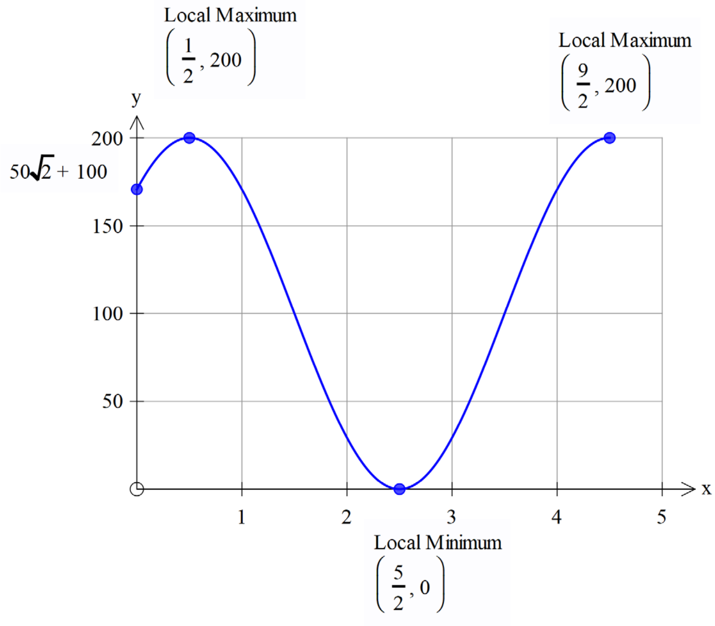
| (b) i)<br>$\dfrac{1+\dfrac{2t}{1+t^2}}{\dfrac{1-t^2}{1+t^2}} = \dfrac{1+t^2+2t}{1-t^2}$<br>$= \dfrac{(1+t)^2}{(1+t)(1-t)}$<br>$= \dfrac{1+t}{1-t}$ | Generally well done.<br>Some students forgot the t formula. It can also be done using Pythagoras. |
| **Question 7**<br><br>(b) ii)<br>$\dfrac{1+t}{1-t} = 2$<br>$1+t = 2-2t$<br>$3t = 1$<br>$t = \dfrac{1}{3}$<br>$\tan x = \dfrac{1}{3}, \quad 0 \le x \le 2\pi$<br>$x \approx 0.322, \pi + 0.322$<br>Check $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$<br>$\dfrac{1+\sin\pi}{\cos\pi} = \dfrac{1}{-1} = -1 \ne 2$<br>$\dfrac{1+\sin 3\pi}{\cos 3\pi} = \dfrac{1}{-1} = -1 \ne 2$<br>$\therefore x \approx 0.322, 3.463$ | Generally well done find the two solutions.<br><br>Many students checked the possible solution for $x=\pi$. Note that for $t=\tan x$, we should check $x=\dfrac{\pi}{2},\dfrac{3\pi}{2}$. |
| (c) i)<br>$LHS = 2\sin x + 2\cos x\cos\dfrac{\pi}{6} - 2\sin x\sin\dfrac{\pi}{6}$<br>$= 2\sin x + 2\cos x\left(\dfrac{\sqrt{3}}{2}\right) - 2\sin x\left(\dfrac{1}{2}\right)$<br>$= 2\sin x + \sqrt{3}\cos x - \sin x$<br>$= \sqrt{3}\cos x + \sin x$ | Generally well done. |
| (c) ii)<br>$r = \sqrt{3+1}$<br>$= 2$<br>$\tan\alpha = \dfrac{1}{\sqrt{3}}$<br>$\alpha = \dfrac{\pi}{6}$<br>$\therefore \sqrt{3}\cos x + \sin x = 2\cos\left(x - \dfrac{\pi}{6}\right)$ | Generally well done.<br>Some students find the wrong $\alpha$. |

| | |
|---|---|
| (c) iii) $2\cos\left(x-\frac{\pi}{6}\right)=1, \qquad 0\le x\le 2\pi$ <br> $\cos\left(x-\frac{\pi}{6}\right)=\frac{1}{2}, \qquad -\frac{\pi}{6}\le x-\frac{\pi}{6}\le 2\pi-\frac{\pi}{6}$ <br> $x-\frac{\pi}{6}=\frac{\pi}{3},\frac{5\pi}{3}$ <br> $x=\frac{\pi}{2},\frac{11\pi}{6}$ | Generally well done. |

| Solutions | Marker's Comments |
|---|---|
| **Question 8** <br> (a) <br> $6!+5\times5!+4\times5!+3\times5!+2\times5!+5!=2520$ <br> *or* <br> $\frac{7!}{2}=2520$ | For those who realised this was simply half of the possibilities it was an easy 2 mark question. Some did the longer method well but a a lot fell short using this method. |
| (b) i) <br> When $\cos\left(\frac{\pi t}{2}-\frac{\pi}{4}\right)=1,\ C(t)=200$ which is the maximum. <br> $\frac{\pi t}{2}-\frac{\pi}{4}=0$ <br> $\frac{\pi t}{2}=\frac{\pi}{4}$ <br> $t=\frac{1}{2}$ <br> $\therefore$ The maximum occurs 0.5 hour after midnight. <br> When $\cos\left(\frac{\pi t}{2}-\frac{\pi}{4}\right)=-1,\ C(t)=0$ which is the minimum. <br> $\frac{\pi t}{2}-\frac{\pi}{4}=\pi$ <br> $\frac{\pi t}{2}=\frac{5\pi}{4}$ <br> $t=\frac{5}{2}$ <br> $\therefore$ The minimum occurs 2.5 hours after midnight. | For those who realised the cosine function/graph has a max of 1 when the angle is 0 and a min at -1 when the angle is pi in the domain it was straight forward, although a number of students did not find **when** these things happened. |
| (b) ii) <br> $C'(t)=100\left(\frac{\pi}{2}\right)\left(-\sin\left(\frac{\pi t}{2}-\frac{\pi}{4}\right)\right)$ <br> $=-50\pi\sin\left(\frac{\pi t}{2}-\frac{\pi}{4}\right)$ | Mostly well done although a few students need to revise their Chain rule with Trig |

| | |
|---|---|
| (b) iii)<br>$-50\pi\sin\left(\frac{\pi t}{2}-\frac{\pi}{4}\right)=0$<br>$\sin\left(\frac{\pi t}{2}-\frac{\pi}{4}\right)=0, \quad 0\le t\le 4.5$<br>$\frac{\pi t}{2}-\frac{\pi}{4}=0,\pi,2\pi$<br>$\frac{\pi t}{2}=\frac{\pi}{4},\frac{5\pi}{4},\frac{9\pi}{4}$<br>$t=\frac{1}{2},\frac{5}{2},\frac{9}{2}$ | Generally well done, however some students did not connect that some of the solutions should have matched those in b(i) which was fine but would help with the graph. A lot did not find **all** the solutions in the domain given. |

| Solutions | Marker's Comments |
|---|---|
| **Question 8**<br>(b) iv)<br>$t=0, C(0)=100\cos\left(-\frac{\pi}{4}\right)+100$<br>$=100\left(\frac{\sqrt{2}}{2}\right)+100$<br>$=50\sqrt{2}+100$ | There is a typo in the question. It should be $50(\sqrt{2}+2)$. The mark was given for substituting t=0 and working down. |
| (b) v) | A lot of very nice graphs here. But some that need a lot of work. Marks were generally awarded for y intercept from working above, correct shape of a cosine shifted ½ a unit to the right and max/min in correct places according to previous work. |




(c)

$LHS:\binom{11}{5}$ is the coefficient of $x^5$ for $(1+x)^{11}, n=11, r=5$

$$(1+x)^3=\binom{3}{0}+\binom{3}{1}x+\binom{3}{2}x^2+\binom{3}{3}x^3$$

$$(1+x)^8=\binom{8}{0}+\binom{8}{1}x+\binom{8}{2}x^2+\binom{8}{3}x^3+\ldots\ldots+\binom{8}{8}x^8$$

RHS: The coefficient of $x^5$ for $(1+x)^3(1+x)^8\, is:$

$$\binom{3}{1}x\binom{8}{4}x^4+\binom{3}{2}x^2\binom{8}{3}x^3+\binom{3}{3}x^3\binom{8}{2}x^2+\binom{8}{5}x^5$$

$$=\binom{3}{1}\binom{8}{4}+\binom{3}{2}\binom{8}{3}+\binom{8}{2}+\binom{8}{5}$$

With these types of questions, you need to explain with a few words what the LHS and RHS mean and show what is happening. Some did this very well. Some need to refine/learn how to do these questions.

**End of Paper**