SYDNEY BOYS HIGH SCHOOL

NESA Number:

Name:

2022

YEAR 12
TERM 1
HSC TASK 2

Maths Class:
12MX

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- Marks may **NOT** be awarded for messy or badly arranged work
- **In Questions 11 – 13, show ALL relevant mathematical reasoning and/or calculations**

Total Marks: 62

Section I – 9 marks (pages 2 – 6)

- Attempt Questions 1 – 9
- Allow about 15 minutes for this section

Section II – 53 marks (pages 7 – 13)

- Attempt Questions 11 – 13
- Allow about 1 hour 15 minutes for this section

Examiner:

RW

NOT TO BE REMOVED FROM EXAMINATION ROOM

Section I

9 marks

Attempt all questions.
Allow about 15 minutes for this section.
Use the multiple choice answer sheet for Questions 1 – 9.

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1 The polynomial $P(x) = 3x^3 - 2x + 1$ has zeroes α, β and γ.

Which of the following is equal to $\alpha\beta + \beta\gamma + \gamma\alpha$?

A. $\frac{-2}{3}$

B. $\frac{-1}{3}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

2 The remainder when a polynomial $P(x)$ is divided by $x^3 + 1$ is $x^2 + 2x + 3$.

Which of the following is the remainder when $P(x)$ is divided by $x + 1$?

A. 0

B. 2

C. 4

D. 6

3 The polynomial $P(x) = x^3 - 3x^2 + 5x - 7$ has zeroes α, β and γ.

Which of the following polynomials has zeroes $\frac{1}{\alpha}$, $\frac{1}{\beta}$ and $\frac{1}{\gamma}$?

A. $P(x) = x^3 + 3x^2 + 5x + 7$

B. $P(x) = x^3 - 3x^2 - 5x - 7$

C. $P(x) = 7x^3 - 5x^2 - 3x + 1$

D. $P(x) = 7x^3 - 5x^2 + 3x - 1$

4 A is driving directly west at 40 km/h on Anzac Parade.

B is driving directly east at 60 km/h on Anzac Parade.

Which of the following is the velocity of A relative to B?

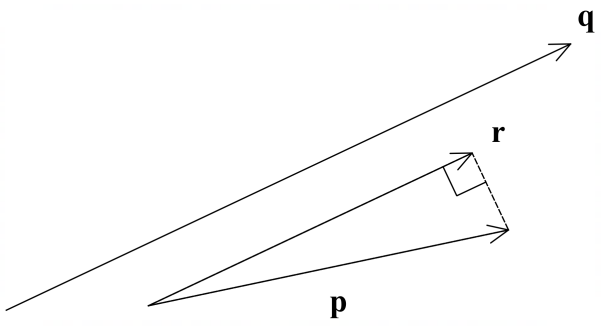
A. 20 km/h directly east.

B. 20 km/h directly west.

C. 100 km/h directly east.

D. 100 km/h directly west.

5 Given the diagram below, which of the following is represented by the vector **r**?



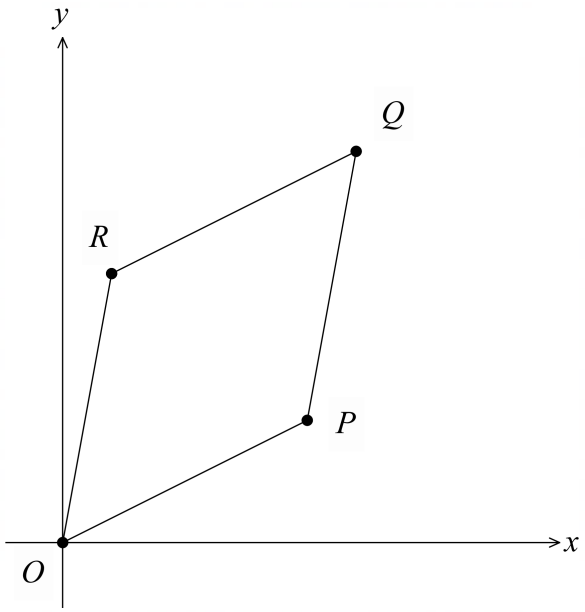
A. The vector projection of **p** onto **q**.

B. The vector projection of **q** onto **p**.

C. The vector projection of **p** perpendicular to **q**.

D. The vector projection of **q** perpendicular to **p**.

6 Given that $OPQR$ is a rhombus in the diagram below, which of the following is **false**?



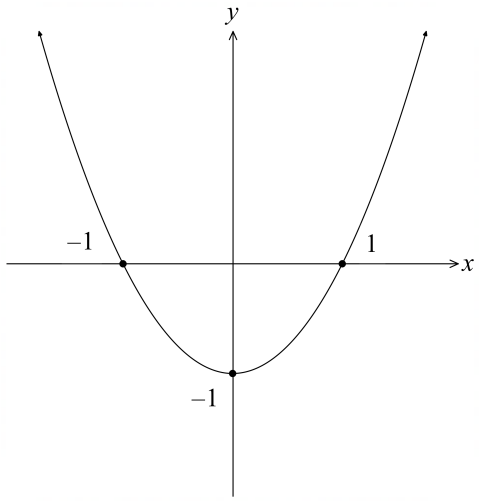
A. $\overrightarrow{OP} = \frac{1}{2}\left(\overrightarrow{OQ} - \overrightarrow{PR}\right)$

B. $\overrightarrow{OP} \cdot \overrightarrow{QR} = \left|\overrightarrow{OP}\right|\left|\overrightarrow{QR}\right|$

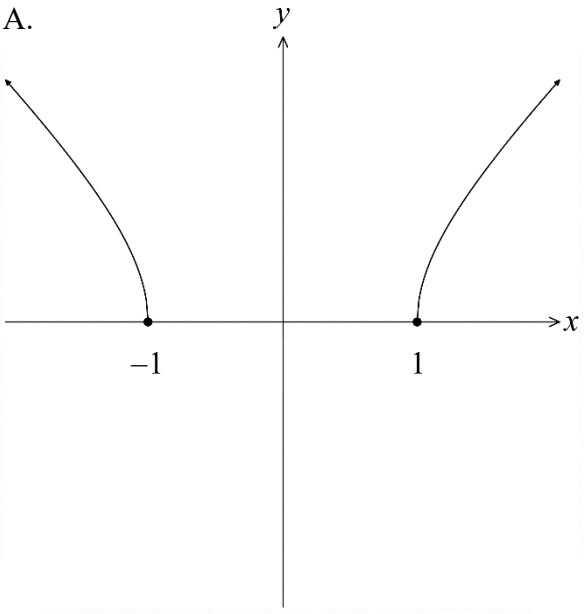
C. $\overrightarrow{OQ} \cdot \overrightarrow{PR} = 0$

D. $\overrightarrow{OR} - \overrightarrow{PQ} = \overrightarrow{OP} - \overrightarrow{RQ}$

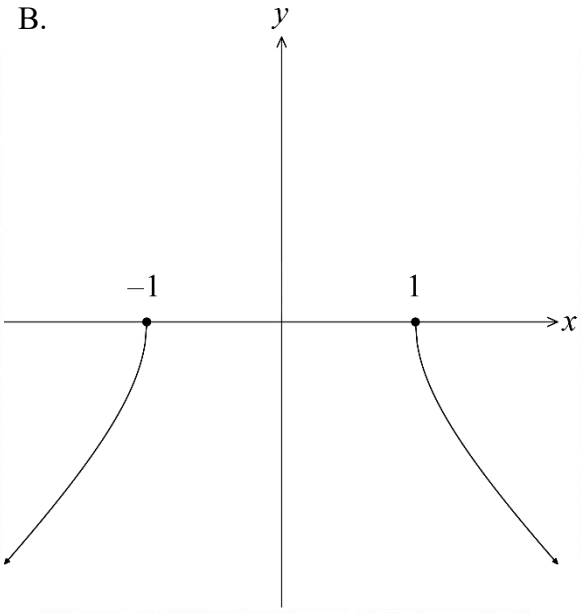
7 Given the graph of $y = f(x)$ below, which of the following is the graph of $y = -\sqrt{f(x)}$?



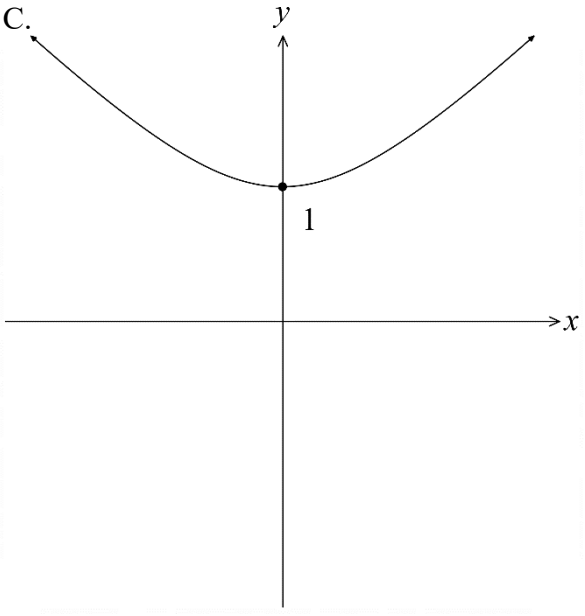
A.



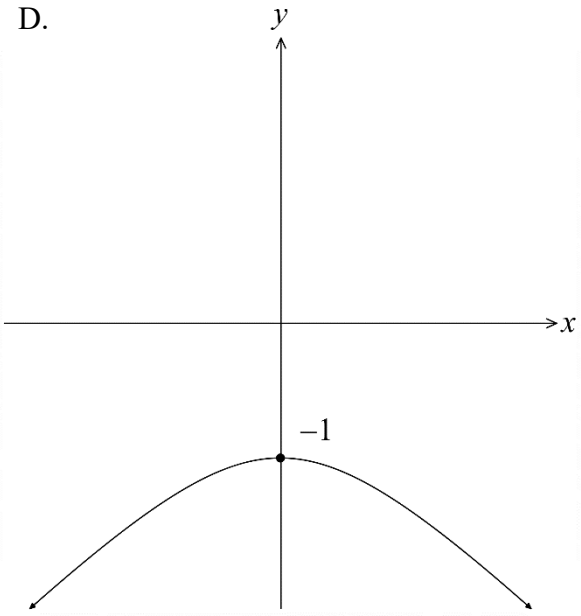
B.



C.



D.



8 Which of the following is the derivative of $x^2y + xy^2$?

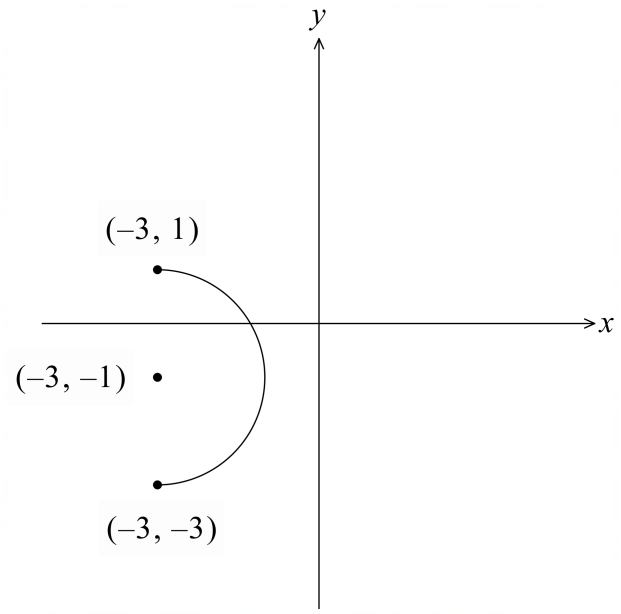
A. $2xy\dfrac{dy}{dx} + y^2\dfrac{dy}{dx}$

B. $2x\dfrac{dy}{dx} + 2y\dfrac{dy}{dx}$

C. $x^2\dfrac{dy}{dx} + 2xy + 2xy\dfrac{dy}{dx} + y^2$

D. $x^2y\dfrac{dy}{dx} + 2xy + xy^2\dfrac{dy}{dx} + y^2$

9 Which of the following pair of parametric equations represents the semicircle shown below?



A. $\begin{cases} x = 2\cos t + 3 \\ y = 2\sin t + 1 \end{cases}$ for $\dfrac{-\pi}{2} \le t \le \dfrac{\pi}{2}$

B. $\begin{cases} x = 2\sin t + 3 \\ y = 2\cos t + 1 \end{cases}$ for $\dfrac{-\pi}{2} \le t \le \dfrac{\pi}{2}$

C. $\begin{cases} x = 2\cos t - 3 \\ y = 2\sin t - 1 \end{cases}$ for $\dfrac{-\pi}{2} \le t \le \dfrac{\pi}{2}$

D. $\begin{cases} x = 2\sin t - 3 \\ y = 2\cos t - 1 \end{cases}$ for $\dfrac{-\pi}{2} \le t \le \dfrac{\pi}{2}$

End of Section I

Section II

53 marks

Attempt Questions 11–13
Allow about 1 hour 15 minutes for this section
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (18 marks) Start a NEW Answer Booklet

(a) The polynomial $P(x) = x^3 + px^2 + qx + r$ has zeroes α, β and γ.

Express the following in terms of p, q and r:

(i) $\alpha + \beta + \gamma$ **1**

(ii) $\alpha^2 + \beta^2 + \gamma^2$ **2**

(b) Prove by mathematical induction that for all positive integers n: **3**

$$2 + 5 + 8 + \ldots + (3n - 1) = \frac{n(3n+1)}{2}$$

(c) OAB is a triangle with $\overrightarrow{OA} = 5\underset{\sim}{i}$, $\overrightarrow{OB} = 7\underset{\sim}{j}$ and M is the midpoint of $\overrightarrow{AB}$.

Using vector methods:

(i) Show that $\overrightarrow{OM} = \frac{5}{2}\underset{\sim}{i} + \frac{7}{2}\underset{\sim}{j}$. **2**

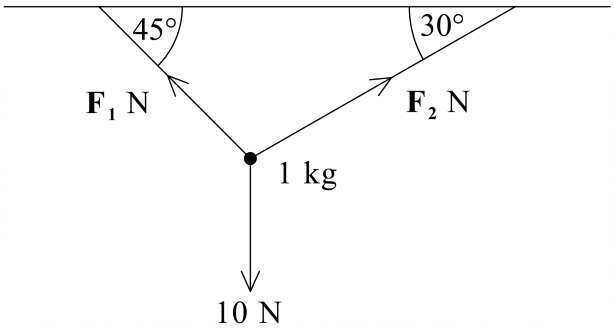
(ii) Prove that $\overrightarrow{OM}$ is **not** perpendicular to $\overrightarrow{AB}$. **1**

Question 11 continues on page 8

(d) The polynomial $P(x) = x^4 + x^3 + cx^2 - x + e$ has zeroes $x = 1$ and $x = 2$. **2**

Find the value of c and e.

(e) In the diagram below, a particle of mass 1 kilogram is suspended in the air by two strings. **2**



The particle exerts a downward force of 10 newtons.

Find the magnitude of the tension forces $\mathbf{F_1}$ and $\mathbf{F_2}$ in newtons

Correct your answer to 2 decimal places.

Question 11 continues on page 9

(f) A parabola is represented by $x = 2at$ and $y = at^2$ for some arbitrary constant $a \neq 0$.

(i) By using parametric differentiation, find $\frac{dy}{dx}$. **1**

(ii) The point $P\left(2ap, ap^2\right)$ lies on the parabola. **2**

Show that the equation of the normal to P is $x + py - 2ap - ap^3 = 0$.

(iii) The normal to P intersects the parabola again at $Q\left(2aq, aq^2\right)$. **2**

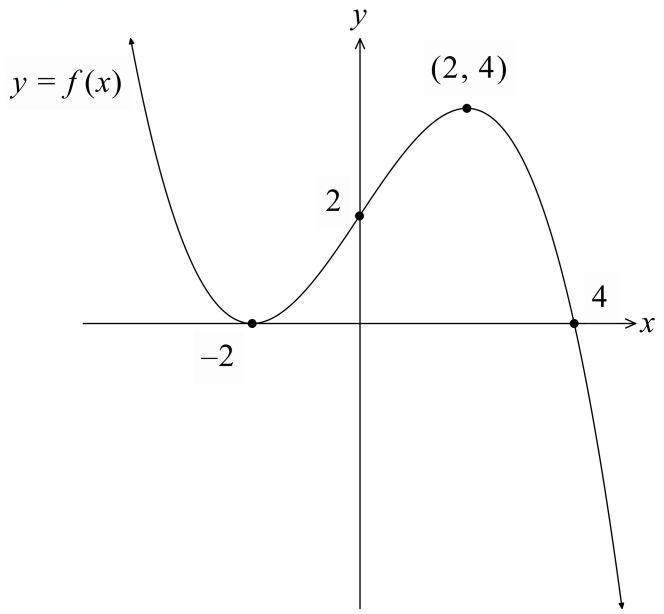
Show that $p^2 + pq + 2 = 0$.

End of Question 11

Question 12 (18 marks) Start a NEW Answer Booklet

(a) The graph of $y = f(x)$ below has a double zero at $x = -2$ and a single zero at $x = 4$.

Also, there is a stationary point at $(2,4)$ and y intercept at $(0,2)$.



(i) Sketch the following transformations of $y = f(x)$ **on the insert sheet provided**.

(α) $y = \dfrac{1}{f(x)}$ **2**

(β) $y = \dfrac{1}{4}\left(f(x)\right)^2$ **2**

(γ) $|y| = f(|x|)$ **2**

(ii) Consider the function $g(x) = 8 + 2x - x^2$. **2**

State the nature of the zeroes of the function $y = f(x)g(x)$.

Question 12 continues on page 11

(b) Suppose $\underset{\sim}{a} = -6\underset{\sim}{i} + 3\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} - 8\underset{\sim}{j}$.

(i) Find $\hat{\underset{\sim}{a}}$. **1**

(ii) Hence or otherwise, find the vector projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$. **2**

(iii) The vector $\underset{\sim}{c}$ is the vector $\underset{\sim}{b}$ reflected about its projection onto $\underset{\sim}{a}$. **2**

Find $\underset{\sim}{c}$.

(c) The polynomial $P(x) = x^3 - 3x^2 - 24x + 80$ has a double zero.

(i) Find the double zero. **2**

(ii) Hence, sketch $y = P(x)$, clearly indicating the nature of the x intercepts. **3**

Your sketch **must** be at least half a page in size.

You do **not** need to locate the point of inflection.

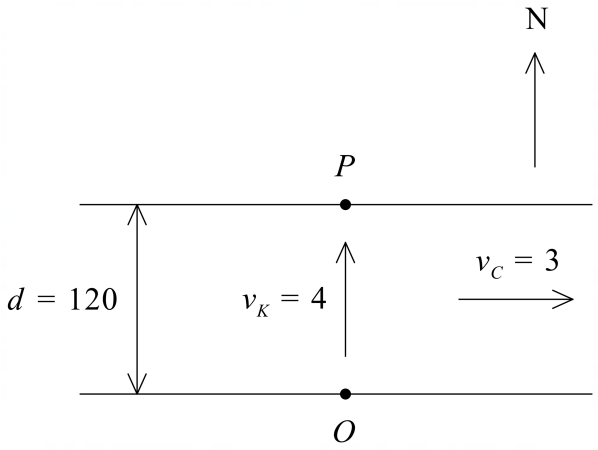
End of Question 12

Question 13 (17 marks) Start a NEW Answer Booklet

(a) In the diagram below, the current of a river flows east at a velocity $v_C = 3$ km/h.

A kayaker paddles directly north across the river at a velocity $v_K = 4$ km/h, starting from O.

The river is known to be $d = 120$ m wide and point P lies directly north of O.



(i) Find the time it takes for the kayak to reach the other side of the river. **2**

Also find the horizontal displacement of the kayak relative to P.

On the return trip, the kayaker starts from P and wants to finish at O.

(ii) Find the angle relative to O that the kayaker should paddle at to achieve this. **2**

Write your answer as a true bearing and rounded to the nearest degree.

(iii) Find the time taken for the return trip if this is achieved. **1**

Round your answer to the nearest second.

Question 13 continues on page 13

(b) Prove by mathematical induction that $2^{3n}-1$ is divisible by 7 for positive integers $n \geq 1$. **3**

(c) It is known that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

(i) By considering $x = 2\cos\theta$, find the solutions to $x^3 - 3x = \sqrt{3}$. **3**

(ii) Hence, show that $\sec\frac{\pi}{18} + \sec\frac{11\pi}{18} + \sec\frac{13\pi}{18} = -2\sqrt{3}$. **2**

(d) (i) Suppose that α, β and γ are consecutive terms in a geometric sequence. **1**

Show that $\beta^2 = \alpha\gamma$.

(ii) Consider the polynomial equation $x^3 - px^2 + qx - r = 0$ for $p \neq 0$ and $r \neq 0$. **3**

Show that if $r = \frac{q^3}{p^3}$, then $\frac{q}{p}$ is a root of the polynomial.

Hence, show that the roots form a geometric sequence.

End of paper

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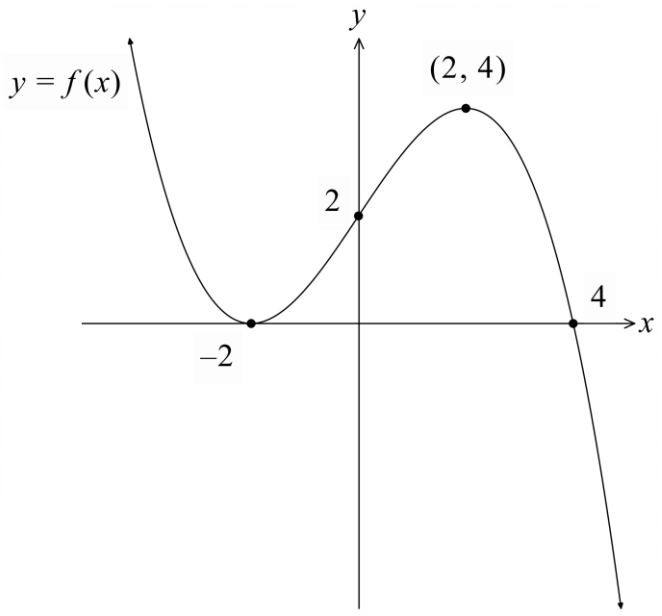
Question 12 (18 marks)

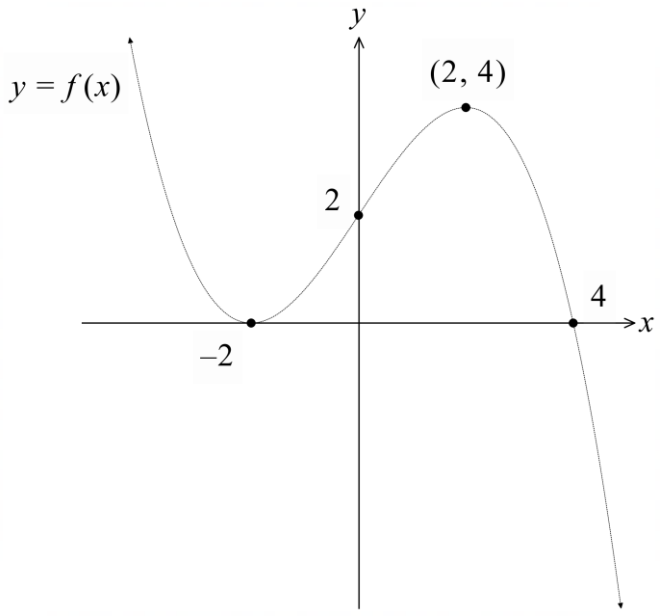
(a) The graph of $y = f(x)$ below has a double root at $x = -2$ and a single root at $x = 4$.

Also, there is a stationary point at $(2, 4)$ and y intercept at $(0, 2)$.



(i) Sketch the following transformations of $y = f(x)$:

(α) $y = \dfrac{1}{f(x)}$ **2**



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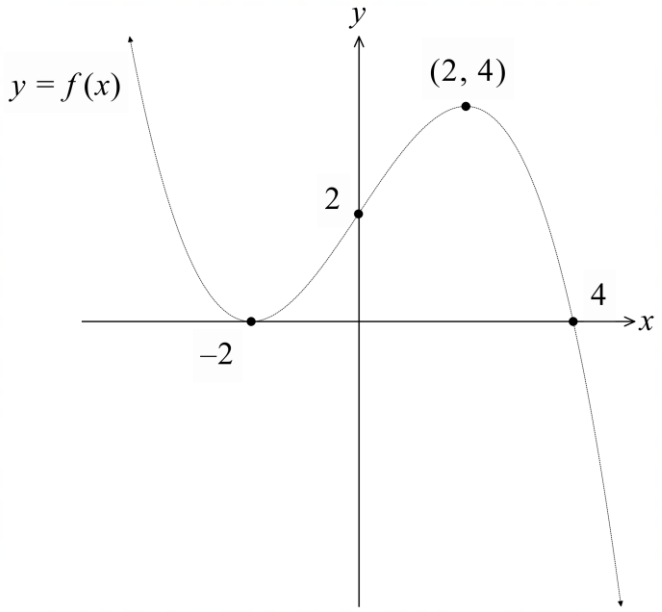
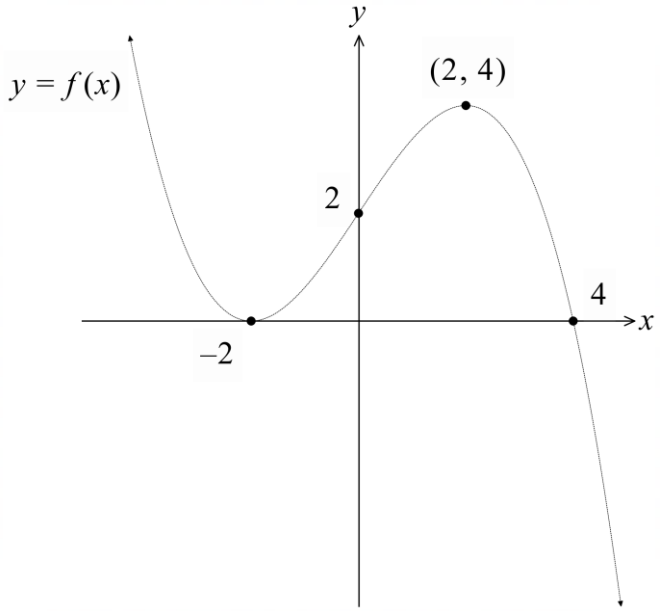
(β) $y = \frac{1}{4}\left(f(x)\right)^2$ **2**



(γ) $|y| = f(|x|)$ **2**



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