

HORNSBY GIRLS HIGH SCHOOL



Mathematics Extension 1

HSC Assessment Task 1 2023/2024

STUDENT NUMBER: _____ **Student Name** _____

Teacher Name (circle one): Mr Payne, Mrs Sztajer, Ms Murray, Ms Guan

General Instructions

- Reading Time – 5 minutes
- Working Time – 60 minutes
- Write using black or blue pen
Black pen is preferred
- NESA approved calculators may be used
- A NESA reference sheet is provided
- Show all necessary working
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

Total marks – 45

Section I Pages 2 – 3

5 marks

Attempt Questions 1 – 5

Answer on the multiple choice answer Sheet provided

Section II Pages 4 – 6

40 marks

Attempt Questions 6 – 9. Start each question in a new writing booklet. Write your student number on each writing booklet

At the end of the assessment order your solutions, starting with multiple choice answer sheet, then

Questions 6-9 and place your question paper on top

Question	1-5	6	7	8	9	Total
Total	/5	/11	/9	/10	/10	/45

This assessment task constitutes 20% of the Higher School Certificate Course School Assessment

Section I

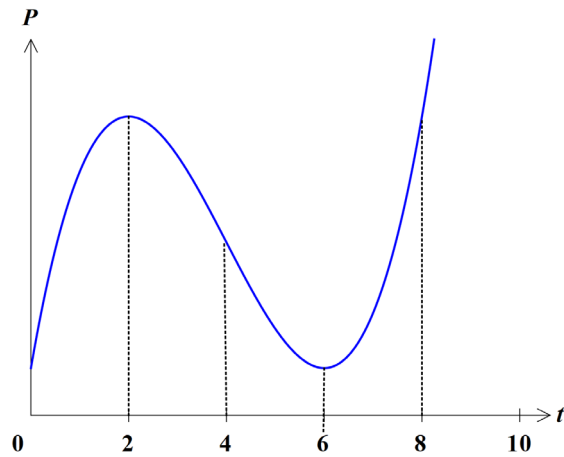
5 marks

Attempt Questions 1 – 5

Allow about 10 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 5

1. The graph shows the share price, P , of a company t months after January 1.



When is the share price increasing at an increasing rate?

- (A) $0 < t < 2$
- (B) $2 < t < 4$
- (C) $4 < t < 6$
- (D) $6 < t < 8$
2. How many possible outcomes are there when arranging the letters of the word ARRANGEMENT?
- (A) $11!$
- (B) $\frac{11!}{2! 2! 2!}$
- (C) $\frac{11!}{2! 2! 2! 2!}$
- (D) $\frac{11!}{4!}$

3. If ${}^{2n}P_n = 4 \times {}^{2n-1}P_{n-1}$, what is the value of n ?
- (A) 1
- (B) 2
- (C) 4
- (D) 8
4. A radioactive substance decays with a half life of 2 hours so that $\frac{dM}{dt} = -kM$. The initial mass is $M_0 = 100$ grams. What is the value of the rate of decay, k ?
- (A) - 0.5
- (B) -0.35
- (C) 0.35
- (D) 0.5
5. A spherical balloon of volume $V \text{ cm}^3$ and radius $r \text{ cm}$ is being inflated at the rate of $20 \text{ cm}^3/\text{s}$. What is the rate of change of the radius with respect to time?
- (A) $\frac{5}{\pi r^2} \text{ cm/s}$
- (B) $\frac{\pi r^2}{5} \text{ cm/s}$
- (C) $5\pi r^2 \text{ cm/s}$
- (D) $80\pi r^2 \text{ cm/s}$

End of Section I

Section II

40 marks

Attempt Questions 6 – 9

Allow about 50 minutes for this section

Begin each question in a new writing booklet, indicating the question number.
Extra writing booklets are available.

Question 6 (11 marks) Start a new writing booklet.

Marks

- (a) A bag contains 12 cards, each with a different number from 1 to 12.

Find, **with justification**, how many cards must be selected without replacement to ensure that at least one pair of cards have numbers that add up to 13.

2

- (b) By considering the coefficients of x^4 on both sides of $(1+x)^8 = (1+x)^4(1+x)^4$,

show that ${}^8C_4 = ({}^4C_0)^2 + ({}^4C_1)^2 + ({}^4C_2)^2 + ({}^4C_3)^2 + ({}^4C_4)^2$.

2

- (c) A container of water, heated to 100°C , is placed in a cool-room where the temperature is maintained at 4°C . After t minutes, the rate of change of the temperature, W° , of the water is given by $\frac{dW}{dt} = -k(W - 4)$, where k is a constant.

- (i) Verify, by substitution, that the function $W = Ae^{-kt} + 4$, where A is a constant, satisfies the rate of change.

2

- (ii) After 15 minutes, the temperature falls to 52°C . Find, to the nearest degree, the temperature of the water after **a further** 5 minutes.

3

- (iii) Find, to the nearest minute, the amount of time that the water needs to be in the cool-room before its temperature drops below 10°C .

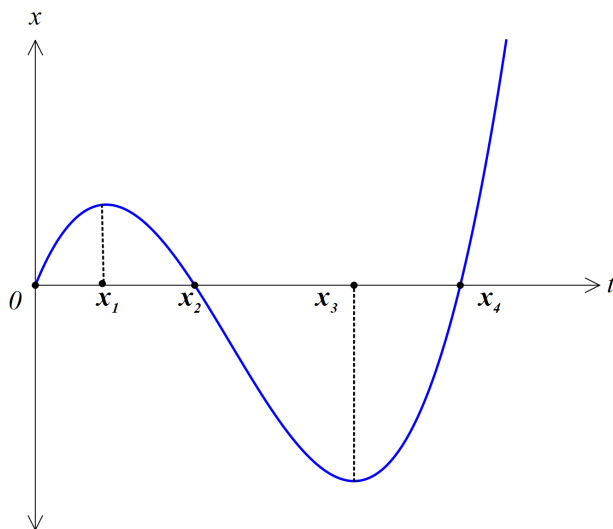
2

Question 7 (9 marks) Start a new writing booklet.

- (a) Find the term of $\left(\frac{3}{x^2} - \frac{x}{2}\right)^6$ that is independent of x .

3

- (b) The graph below represents the displacement (x m/s) of a particle moving in a straight line after t seconds.



- (i) At what times is the particle at rest? **1**
- (ii) Draw a possible graph to represent the velocity (v m/s) of the particle after t seconds. **2**

- (c) A 5m ladder is leaning against a wall so that the foot of the ladder is x metres from the base of the wall and the top of the ladder reaches a height of h metres up the wall. The foot of the ladder is slipping away from the base of the wall so that $\frac{dx}{dt} = 0.1$ m/s.

Find $\frac{dh}{dt}$ when $x = 3$.

3

Question 8 (10 marks) Start a new writing booklet.

- (a) The seven letters of the word RACECAR are arranged at random in a straight line. What is the probability that the sequence is the same from right to left as it is from left to right? 2
- (b) Ten people arrive at a restaurant.
- (i) Find how many ways they can be assigned to
- (α) a large table for 6 people and a smaller table for 4 people. 1
- (β) two indistinguishable tables for 5 people each. 1
- (ii) Find how many ways the ten people can be seated at a large table for 6 and a smaller table for 4 if both tables are circular. 1
- (c) Given $(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k$, differentiate and then show that $\sum_{k=1}^n k \binom{n}{k} = n \times 2^{n-1}$ 2
- (d) Use mathematical induction to prove that $15^n + 2^{3n} - 2$ is a multiple of 7 3
- for all positive integers n .

Question 9 (10 marks) Start a new writing booklet.

- (a) A group of 6 people have tickets to the theatre, sitting in a row. How many ways can they be arranged if Mr A and Mr B do not sit next to one another? **2**
- (b) A particle is moving in a straight line so that its displacement, x metres, from a fixed point O on the line at time, t seconds, is given by $x = 5t - t^2 - t^3$.
- (i) Find the times when the particle is at rest. **2**
- (ii) Describe the motion of the particle when $t = 3$ with reference to its displacement, which way it is moving and justify whether it is speeding up or slowing down. **2**
- (c) (i) For positive integers n and r , with $r < n$, show that ${}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$. **2**
- (iii) Hence, prove that $\sum_{r=0}^n {}^{n+r}C_r = {}^{2n+1}C_n$ **2**

End of paper

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Mathematics Extension 1

HSC Task 1 2023/2024

STUDENT NUMBER: _____

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
 A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☒ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ☒ B ☒ ^{correct} C ☐ D ☐

- | | | | | |
|----|-------------------------|-------------------------|-------------------------|-------------------------|
| 1. | A ☐ | B ☐ | C ☐ | D ☐ |
| 2. | A ☐ | B ☐ | C ☐ | D ☐ |
| 3. | A ☐ | B ☐ | C ☐ | D ☐ |
| 4. | A ☐ | B ☐ | C ☐ | D ☐ |
| 5. | A ☐ | B ☐ | C ☐ | D ☐ |