HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 12 HSC Assessment 2 Term 1 2023

STUDENT NUMBER: _____

General Instructions

- Reading Time – **5** minutes
- Working Time – **75** minutes
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators may be used
- Show all necessary working
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

Total marks –43

Section I Pages 2 – 3

8 marks

Attempt Questions 1 – 8

Answer on the Objective Response Answer Sheet provided

Section II Pages 4 – 7

35 marks

Attempt Questions 9 – 12. Start each question on a new page. Write your student number on page

Question	***1-8***	***9***	***10***	***11***	***12***	***Total***
Total	*/8*	*/9*	*/9*	*/9*	*/8*	*/43*

This assessment task constitutes 30% of the Higher School Certificate Course School Assessment

Section I

8 marks
Allow about 10 minutes for this section
Use the Multiple Choice answer sheet for Questions 1 – 8

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1. How many ways 4-digit numbers can be formed using 0 to 9 without repetition?

   (A) 4536

   (B) 5040

   (C) 9000

   (D) 10000

2. Given that $y = x^2 - x^3$ and $\frac{dx}{dt} = 3$, find $\frac{dy}{dt}$.

   (A) $3x^2 - x^3$

   (B) $6x - 9x^2$

   (C) $3(x^2 - x^3)$

   (D) $2x - 3x^2$

3. Which expression is equivalent to $n! + (n-1)! + (n-2)!$?

   (A) $(n-2)!$

   (B) $(n-1)!(n^2-1)$

   (C) $n^2(n-2)!$

   (D) $(n-1)!(n-2)$

4. Let $A = \{1, 2, 3, 4, 5\}$. How many subsets of A contain **at least** 3 digits?

   (A) 10

   (B) 15

   (C) 16

   (D) 32

5. Which represents the general term of the expansion of $2x+x^2{}^5$?

(A) ${}^5C_r \times 2^{5-r} \times x^r$

(B) ${}^5C_r \times 2^r \times x^{5+r}$

(C) ${}^rC_5 \times 2^{r-5} \times x^{3r-5}$

(D) ${}^5C_r \times 2^{5-r} \times x^{5+r}$

6. In how many ways can 7 different charms be arranged on circlular bracelet?

(A) 720

(B) 5040

(C) 2520

(D) 360

7. Given $y=1000+300e^{0.2t}$, what is $\frac{dy}{dt}$?

(A) $60e^t$

(B) $0.2\left(y-1000\right)$

(C) $0.2y$

(D) $0.2\left(y+1000\right)$

8. How many students are needed in a school to guarantee that there are at least two students who have the same first two initials?

(A) 676

(B) 677

(C) 672

(D) 680

End of Section I

Section II
35 marks
Attempt Questions 9 - 12
Allow about 60 minutes for this section

Begin each question on a new answering booklet, indicating the question number.
Extra writing paper is available.

Question 9 (9 marks) **Start a new answering booklet.** **marks**

(a) Find the number of arrangements of SAMUEL if the vowels and consonants alternate. **1**

(b) A committee of 5 is to be formed from a group of 4 women and 5 men. How many committees can be formed by more men than women? **1**

(c) Show that ${}^nC_r + {}^nC_{r-1} = \dfrac{(n+1)!}{r!(n-r+1)!}$ **2**

(d) A pool is initially full. It's drained so that at time ***t*** minutes the volume of water, ***V***, in kL, is given by

$$V = 50\left(1 - \frac{t}{40}\right)^2 \quad \text{for} \quad 0 \le t \le 40 .$$

(i) How much water was initially in the pool? **1**

(ii) Show that it takes 20 minutes for the tank to be one-quarter full. **2**

(iii) At what rate was the water draining out the pool when it was one-quarter full? **2**

Question 10 (9 marks)

(a)

(i) Express $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$, **2**

where $R > 0$ and $0 \le \alpha \le \frac{\pi}{2}$.

(ii) Hence, or otherwise, solve $\cos x - \sqrt{3}\sin x = 1$, for $0 \le x \le 2\pi$. **2**

(b) A virus outbreak during a period of time followed an exponential increase model

$$N = Ae^{kt}$$

in which ***N*** is the number of people, ***t*** is the number of days, ***A*** and ***k*** are positive constants.

(i) Initially, 20 people were infected. At the end of 4 weeks, **3**
12000 people were infected. Find the value of ***k***.
Answer correct to 2 decimal places.

(ii) On which day will the number ***N*** of infections first exceed one million? **2**

Question 11 (9 marks)

(a) Evaluate the coefficient of x^2 in the expansion of $(5+x)(3-2x)^6$. **3**

(b) By expressing $\sin x$ and $\cos x$ in terms of t, where $t = \tan\frac{x}{2}$, solve **3**

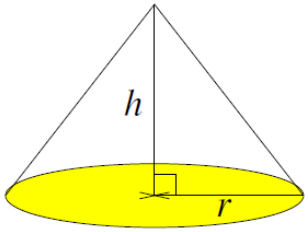
$3\sin x + \cos x = -1$ for $0 \le x \le 2\pi$. Answer correct to 1 decimal places.

(c) Solve the equation $\sin 5x + \sin x = \sin 3x$ for $0 \le x \le \pi$. **3**

Question 12 (8 marks)

(a) Arrange A, B, C, D and E in a circle. How many ways can the circle be arranged so that C and D do not sit together. **1**

(b) Sand pouring from a high conveyor belt is forming a cone with base diameter three times the height. Let ***h*** metres be the height of the pile, and ***r*** metres be its base radius, ***t*** minutes after pouring begins.



(i) Show that $V = \frac{2\pi r^3}{9}$ and $\frac{dV}{dr} = \frac{2\pi r^2}{3}$. **2**

(ii) Given the sand is being poured at a constant rate of $3\ \text{m}^3/\text{min}$, find the rate at which the radius is increasing when the radius is 6 metres. Answer in exact value. **2**

(iii) At what rate must the sand be poured onto the pile if it is required that the base area increases at $12\ \text{m}^2/\text{min}$ when the base radius is 6 metres? **3**

End of Examination

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Hornsby Girls High School
Year 12 Mathematics Extension 1 HSC Assessment 2 2023
Solutions

Solutions	Marker's Comments
Question 1 **A** $9\times9\times8\times7=4536$	50 out of 75 students chose answer B which did not account for the first digit not being a zero
Question 2 **B** $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ $=\left(2x-3x^2\right)\times3$ $=6x-9x^2$	Ax3 Cx2 Dx1
Question 3 **C** $(n-2)!\left(n(n-1)+(n-1)+1\right)$ $=(n-2)!\left(n^2-n+n-1+1\right)$ $=(n-2)!n^2$	Ax2 Bx6 Dx6
Question 4 **C** ${}^5C_3+{}^5C_4+{}^5C_5=16$	Ax4 Bx6
Question 5 **D** ${}^5C_r(2x)^{5-r}(x^2)^r$ $={}^5C_r\times2^{5-r}x^{5-r+2r}$ $={}^5C_r\times2^{5-r}x^{5+r}$	Ax12 Bx2
Question 6 **D** $\frac{6!}{2}=\frac{720}{2}=360$	Ax7 Bx1 Cx6
Question 7 **B** $\frac{dy}{dt}=0.2\times300e^{0.2t}$ $=0.2(y-1000)$	Ax4 Cx1 Dx2
Question 8 **B** $26\times26+1=677$	Ax4 Bx1 Dx2

Solutions	Marker's Comments
Question 9 (a) Vowels: A, U, E Consonants: S, M, L The arrangement can be either VCVCVC or CVCVCV $\therefore 3! \times 3! \times 2 = 72$	There were quite a few students who didn't multiply by 2 here. Aw 1 for correct answer or **correct numerical expression**. (CNE)
(b) 5 men + 0 women: ${}^5C_5$ 4 men + 1 women: ${}^5C_4 \times {}^4C_1$ 3 men + 2 women: ${}^5C_3 \times {}^4C_2$ $\therefore {}^5C_5 + {}^5C_4 \times {}^4C_1 + {}^5C_3 \times {}^4C_2 = 81$	There were quite a few students who didn't include 5 men and 0 women as a possibility. Aw 1 for correct answer or **correct numerical expression**. (CNE)
(c) $${}^nC_r + {}^nC_{r-1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$ $$= \frac{n!(n-r+1)+n!r}{r!(n-r+1)!}$$ $$= \frac{n!(n-r+1+r)}{r!(n-r+1)!}$$ $$= \frac{n!(n+1)}{r!(n-r+1)!}$$ $$= \frac{(n+1)!}{r!(n-r+1)!}$$	There were many who took a lot of work to get the LCD and some who struggled as the simply multiplied the factorials together w/o realising that: r!=rx(r-1)! and (n-r+1)!=(n-r+1)x(n-r)! This skill needs to be practised before your Trial and HSC exam. It is very important that you understand and can do this properly. Aw 2 for correct show procedure Aw 1 for finding LCD.
(d) (i) $$t = 0, V = 50\left(1 - \frac{0}{40}\right)^2$$ $$= 50kL$$	Well done by nearly all students Aw 1 for correct answer or **correct numerical expression**. (CNE)
(ii) $$50 \times \frac{1}{4} = 50\left(1 - \frac{t}{40}\right)^2$$ $$\frac{1}{4} = \left(1 - \frac{t}{40}\right)^2$$ $$\pm\frac{1}{2} = 1 - \frac{t}{40}$$ $$\frac{t}{40} = \frac{1}{2}, \frac{3}{2}$$ $$t = 20, 60$$ $$\therefore t = 20 \quad (0 \le t \le 40)$$	This question posed a few issues depending on method. The easy way was to substitute t=20 and show the volume was 12.5kL **and** this was ¼ of the original volume. Aw 2 for correct answer and relating to original volume. Aw 1 for 12.5kL w/o showing/relating this is ¼ of original. The other way is the more conventional method however there where a lot of students who disregarded the – without a correct reason due to the domain given. This should not be done to the final step. However, this was allowed, **this time**. It may not in the future. Aw 2 for this method with or w/o the +/-

Solutions	Marker's Comments
Question 9 (d) (iii) $\frac{dV}{dt} = 50 \times 2\left(1-\frac{t}{40}\right)\left(-\frac{1}{40}\right)$ $= \frac{-100}{40}\left(1-\frac{t}{40}\right)$ $= \frac{-5}{2}\left(1-\frac{t}{40}\right)$ $t = 20, \quad \frac{dV}{dt} = \frac{-5}{2}\left(1-\frac{20}{40}\right)$ $= -1.25$ kL/min This means the volume of the pool is decreasing @ 1.25kL/min	This question was generally done well by most students. A few need to brush up on the skill of differentiating using the chain rule. Aw 2 for correct answer of -1.25kL/min. Bare in mind that you may need to interpret what the minus sign means properly in further assessments. i.e. This means the volume of the pool is **decreasing** @ 1.25kL/min, **not** decreasing at -1.25kL/min. Aw 2 for correct answer Aw 1 for correct working of the chain rule.
Question 10 (a) (i) $\cos x - \sqrt{3}\sin x = R\cos(x+\alpha)$ $= R\cos x\cos\alpha - R\sin x\sin\alpha$ $R = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2$ $\tan\alpha = \sqrt{3}, \quad \alpha = \frac{\pi}{3}$ $\therefore \cos x - \sqrt{3}\sin x = 2\cos\left(x+\frac{\pi}{3}\right)$	Done well by most students
(ii) $\cos x - \sqrt{3}\sin x = 1 \quad (0 \le x \le 2\pi)$ $2\cos\left(x+\frac{\pi}{3}\right) = 1$ $\cos\left(x+\frac{\pi}{3}\right) = \frac{1}{2}$ $x+\frac{\pi}{3} = \frac{\pi}{3}, 2\pi-\frac{\pi}{3}, 2\pi+\frac{\pi}{3} \quad \left(\frac{\pi}{3} \le x+\frac{\pi}{3} \le 2\pi+\frac{\pi}{3}\right)$ $\therefore x = 0, \frac{4\pi}{3}, 2\pi$	Many students did not consider the domain $\frac{\pi}{3} \le x+\frac{\pi}{3} \le 2\pi+\frac{\pi}{3}$ and therefore missed the solution $x = 2\pi$

(b) (i) $t = 0, N = 20$ $20 = Ae^{k(0)}$ $A = 20$ $t = 4 \times 7 = 28, N = 12000$ $12000 = 20e^{k(28)}$ $e^{28k} = 600$ $28k = \ln 600$ $k = \dfrac{\ln 600}{28}$ $= 0.22846...$ ≈ 0.23	There were quite a few careless numerical errors e.g. $\frac{12000}{20} \neq 60$ and some understanding errors involved with working with logs. If students are making errors with converting from exponential form to log form $e^{28k} = 600$ $28k = \ln 600$ they may have greater success by taking log of both sides instead. $e^{28k} = 600$ $\ln e^{28k} = \ln 600$ $28k \ln e = \ln 600$ $28k \times 1 = \ln 600$ $28k = \ln 600$
Solutions	
Question 10 (b) (ii) $20e^{\frac{\ln 600}{28}t} > 1000000$ $e^{\frac{\ln 600}{28}t} > 50000$ $\dfrac{\ln 600}{28}t > \ln 50000$ $t > \dfrac{28 \ln 50000}{\ln 600}$ $t > 47.3592...$ On the 48th day, N will exceed 1 million.	Most students dealt with the equation rather than the inequation to find t=47.35925019 if they used the exact value of k or t=47.04251428 if they used k=0.23 which is fine but then they did not interpret their answer correctly to state the 48th day.
Question 11 (a) $(5+x)(3-2x)^6$ $= (5+x)\,{}^6C_r 3^{6-r}(-2)^r x^r$ $= 5 \times {}^6C_r 3^{6-r}(-2)^r x^r + x \times {}^6C_r 3^{6-r}(-2)^r x^r$ $= 5\,{}^6C_r 3^{6-r}(-2)^r x^r + {}^6C_r 3^{6-r}(-2)^r x^{r+1}$ $r = 2,\ 5\,{}^6C_2 3^{6-2}(-2)^2 x^2 = 24300x^2$ $r = 1,\ {}^6C_1 3^{6-1}(-2)^1 x^{1+1} = -2916x^2$ $24300x^2 - 2916x^2 = 21384x^2$ $\therefore$ the coefficient of $x^2 is 21384$.	Many students were able to write the general form correctly. Some students couldn't multiply $(5+x)$ across and add the correct coefficients. Some students expanded $(3-2x)^6$ and then multiple with $(5+x)$ to find the correct terms, which also gives the same answer. Some students are still not familiar with binomial expansion.

(b) ii) Let $t=\tan\frac{x}{2}$, $\frac{3\times 2t}{1+t^2}+\frac{1-t^2}{1+t^2}=-1$ $6t-1-t^2=-1-t^2$ $6t=-2$ $t=-\frac{1}{3}$ $\tan\frac{x}{2}=-\frac{1}{3},\quad 0\le x\le 2\pi$ $\frac{x}{2}=\pi-\tan^{-1}\frac{1}{3}$ $x=5.6396...$ $x\approx 5.6$ Check $x=\pi$, $3\sin\pi+\cos\pi=0-1=-1$ $\therefore x\approx 5.6$ or $x=\pi$	Many students managed to solve for *t* correctly. Some students didn't find the correct solutions for *x* in the domain. Some students forgot to check $x=\pi$.
(c) $\sin 5x+\sin x=\sin 3x\quad (0\le x\le\pi)$ $2\sin 3x\cos 2x=\sin 3x$ $\sin 3x(2\cos 2x-1)=0$ $\sin 3x=0,\quad \cos 2x=\frac{1}{2}$ $3x=0,\pi,2\pi,3\pi,\quad 2x=\frac{\pi}{3},\frac{5\pi}{3}$ $\therefore x=0,\frac{\pi}{3},\frac{2\pi}{3},\pi,\frac{\pi}{6},\frac{5\pi}{6}$	Many students were not able to use appropriate angle formula to convert the equation into a form to solve for *x*. The product to sum formula is on the formula sheet. Some students didn't find the solutions in the correct domain for $\sin 3x$ and $\cos 2x$.
Question 12 (a) $2\times 3\times 2=12$	About 25% of students did not get the answer because they forgot to switch the two people around.
(b) (i) $2r=3h, h=\frac{2}{3}r$ $V=\frac{1}{3}\pi r^2 h$ $=\frac{1}{3}\pi r^2\left(\frac{2}{3}r\right)$ $=\frac{2}{9}\pi r^3$ $\frac{dV}{dr}=\frac{2}{9}\times 3\pi r^2$ $=\frac{2}{3}\pi r^2$	Some students were unable to get an equation for h in terms of r and therefore were unable to get an equation for V. This part was done well!

(ii) $\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}$ $=\frac{3}{2\pi r^2}\times\frac{dV}{dt}$ sub $\frac{dV}{dt}=3, r=6$ $\frac{dr}{dt}=\frac{3}{2\pi(6)^2}\times 3$ $=\frac{1}{8\pi}$ mm/min	Done well!
(iii) $A=\pi r^2, \frac{dA}{dr}=2\pi r,$ $\frac{dV}{dt}=\frac{dV}{dr}\times\frac{dr}{dA}\times\frac{dA}{dt}$ $=\frac{2}{3}\pi r^2\times\frac{1}{2\pi r}\times\frac{dA}{dt}$ sub $\frac{dA}{dt}=12, r=6$ $\frac{dV}{dt}=\frac{2}{3}\pi(6)^2\times\frac{1}{2\pi(6)}\times 12$ $=24$ mm^3/min	Poorly done! Many students did not get an expression for $\frac{dr}{dt}$ using the conditions for the area of the base.

End of Paper