BAULKHAM HILLS HS PERSEVERE

2024 HSC Assessment Task 1

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 1 hour
- Write using non erasable black pen.
- Calculators approved by NESA may be used.
- Answer all questions in the booklet provided.
- In Questions 6-8, show relevant mathematical reasoning and/or calculations.

Total Marks: 31

Section I – 5 marks (pages 2-3)

- Attempt Questions 1-5
- Allow about 7 minutes for this section

Section II - 26 marks (pages 4-6)

- Attempt Questions 6 - 8
- Allow about 53 minutes for this section
- Marks may be deducted for careless or badly arranged work

Section I

5 marks
Attempt Questions 1 – 5
Allow about 7 minutes for this section

Use the multiple-choice answer page in the writing booklet for Questions 1-5.

---

1 Given the points $A(-3,-5)$ and $B(0,7)$, which of the following represents the displacement vector $\overrightarrow{BA}$?

(A)

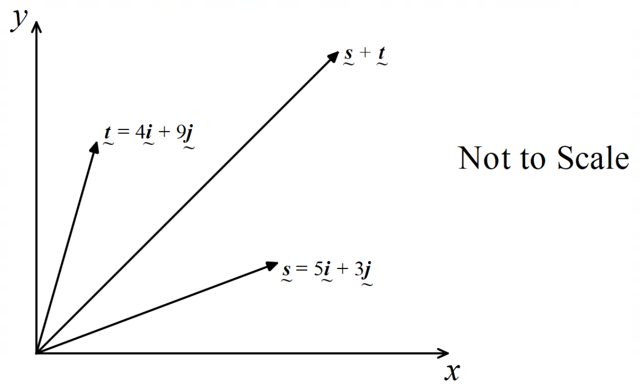
(B)

(C) $3\underset{\sim}{i} - 2\underset{\sim}{j}$

(D) $-3\underset{\sim}{i} + 2\underset{\sim}{j}$

---

2 The diagram below shows the vectors $\underset{\sim}{s}, \underset{\sim}{t}$ and $\underset{\sim}{s} + \underset{\sim}{t}$.



What is the value of $|\underset{\sim}{s} + \underset{\sim}{t}|$?

(A) 12
(B) 15
(C) 16
(D) 20

---

3 Given $\underset{\sim}{a}$ is a vector such that $\underset{\sim}{a} = x\underset{\sim}{i} + y\underset{\sim}{j}$ where $\boldsymbol{x}$ and $\boldsymbol{y}$ are real scalars.
$\underset{\sim}{b}$ is a vector on the line $x = 0$, which of the following is the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$?

(A) $x\underset{\sim}{i}$

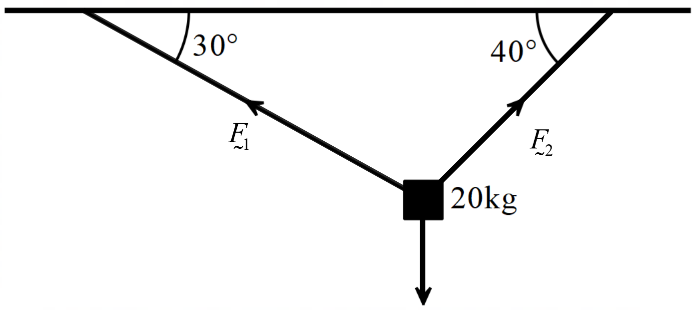
(B) $x\underset{\sim}{j}$

(C) $y\underset{\sim}{i}$

(D) $y\underset{\sim}{j}$

---

4 A particle of mass 20kg is suspended by two strings attached to two points in the same horizontal plane. The two strings make angles of 30° and 40° to the horizontal, as shown below.
The tension in the strings are $\underset{\sim}{F_1}$ and $\underset{\sim}{F_2}$, measured in newtons. (You may assume $g = 9.8 m/s^2$)



Which of the following is closest to the magnitude of the tension of the force in each string in newtons.

(A) $|\underset{\sim}{F_1}| = 160$ N and $|\underset{\sim}{F_2}| = 181$ N

(B) $|\underset{\sim}{F_1}| = 181$ N and $|\underset{\sim}{F_2}| = 160$ N

(C) $|\underset{\sim}{F_1}| = 84$ N and $|\underset{\sim}{F_2}| = 112$ N

(D) $|\underset{\sim}{F_1}| = 112$ N and $|\underset{\sim}{F_2}| = 84$ N

---

5 Mathematical induction is used to prove that $n(2n-1)(2n+1)$ is divisible by 3 for all integers $n \geq 1$. Which of the following is the inductive statement that needs to be proved?

(A) $3k(2k-1)(2k+1)$ is divisible by 3.

(B) $(k+1)(2k)(2k+2)$ is divisible by 3.

(C) $(k+1)(2k+1)(2k+2)$ is divisible by 3.

(D) $(k+1)(2k+1)(2k+3)$ is divisible by 3.

Section II **Marks**

26 marks
Attempt Questions 6-8
Allow about 53 minutes for this section

Answer each question in the appropriate section of the writing booklet.
Extra writing paper is available.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (10 marks) Use the Question 6 section of the writing booklet.

a) Let $\underset{\sim}{a} = 3\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j}$.

(i) Calculate the magnitude of $\underset{\sim}{a}$. **1**

(ii) Find $\hat{\underset{\sim}{a}}$. **1**

(iii) Find the vector projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$? **1**

(iv) Write a possible vector that is perpendicular to $\underset{\sim}{b}$. **1**

b) Given the vectors $\overrightarrow{OA} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \overrightarrow{OB} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\overrightarrow{OC} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$, **3**
calculate the size of $\angle \boldsymbol{ABC}$ correct to the nearest degree.

c) Use mathematical induction to prove that $\mathbf{23^n - 1}$ is divisible by 11 for all positive integers $\boldsymbol{n}$. **3**

End of Question 6

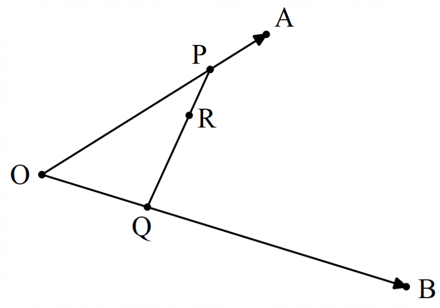
Question 7 (8 marks) Use the Question 7 section of the writing booklet.

a) Three forces act on an object of mass 2 kg. These forces, measured in newtons, are represented by the vectors **2**

$$\begin{bmatrix} 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ -10 \end{bmatrix} \text{ and } \begin{bmatrix} -2 \\ -20 \end{bmatrix}.$$

Calculate the magnitude of the acceleration of the object.

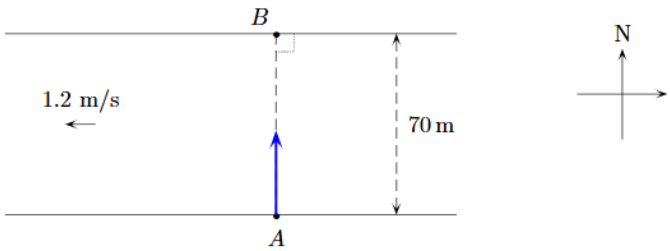
b) In the diagram, $\overrightarrow{OP} = (1-\lambda)\overrightarrow{OA}$, $\overrightarrow{OQ} = \lambda\overrightarrow{OB}$ and $\overrightarrow{PR} = \lambda\overrightarrow{PQ}$, for some real number λ where $0 \le \lambda \le 1$.



(i) Show that $\overrightarrow{OR} = (1-\lambda)^2\overrightarrow{OA} + \lambda^2\overrightarrow{OB}$ where $0 \le \lambda \le 1$. **1**

(ii) Find the Cartesian equation of the curve described by $\overrightarrow{OR}$ when $\overrightarrow{OA} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\overrightarrow{OB} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$. **2**

c) The banks of the river are parallel and 70m apart. Point B is directly opposite point A on the other side of the river, as shown below.



In still water, a yacht can travel at a speed of 1.8 m/s.
The river is flowing downstream at 1.2 m/s, and there is a wind that blows at a constant speed of 0.5 m/s in the north-westerly direction.

(i) Draw a vector diagram labelling all forces. **1**

(ii) On which bearing should the yacht sail in order for it to sail a direct path from point A to point B? Give your answer correct to the nearest degree. **2**

End of Question 7

Question 8 (8 marks) Use the Question 8 section of the writing booklet.

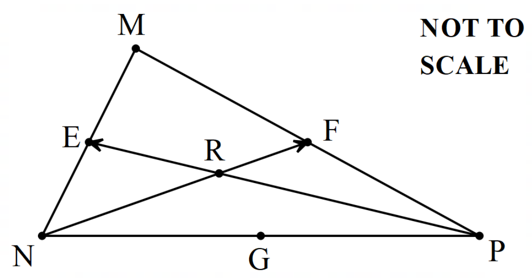
a) (i) By considering the sum of the terms of an arithmetic series, show that **1**

$$(1+2+\ldots+n)^2 = \frac{1}{4}n^2(n+1)^2$$

(ii) By using the Principle of Mathematical induction, prove that **3**

$1^3+2^3+\ldots+n^3=(1+2+\ldots+n)^2$, for all integers where $n \geq 1$

b) In the triangle *MNP* shown in the diagram, the points *E, F, G* are the midpoints of *MN, MP* and *NP* respectively. **4**

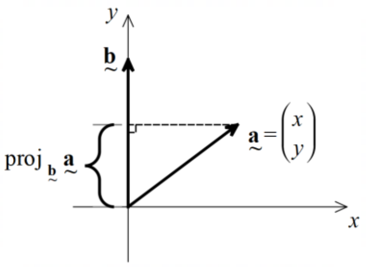
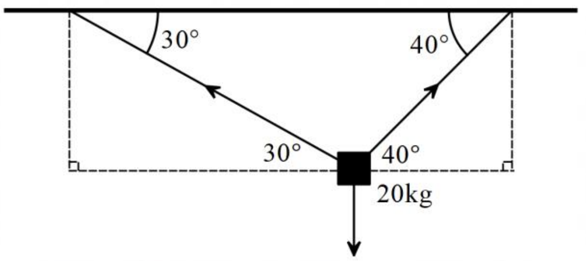


Let $\overrightarrow{MN}=\underset{\sim}{n}$, $\overrightarrow{MP}=\underset{\sim}{p}$, $\overrightarrow{NR}=\lambda\overrightarrow{NF}$, and $\overrightarrow{PR}=\mu\overrightarrow{PE}$, where λ and μ are real numbers between 0 and 1.

By finding two expressions for $\overrightarrow{MR}$, show that the medians $\overrightarrow{PE}$, $\overrightarrow{NF}$ and $\overrightarrow{MG}$ are concurrent and that R divides each median in the ratio of 2:1.

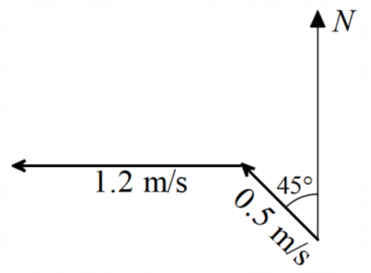
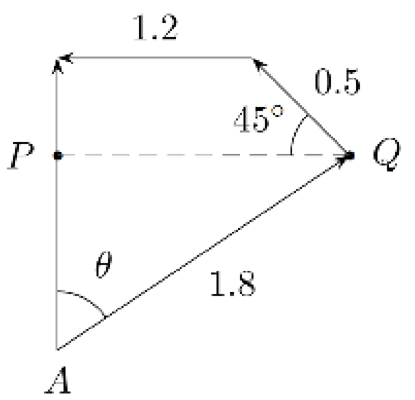
End of Exam

Marking Guideline
Year 12 - Extension 1 - Task 1 2024

Qn	Suggested solutions	Marking Guidelines
1	$\overrightarrow{BA}=\begin{bmatrix}-3\\-5\end{bmatrix}-\begin{bmatrix}0\\7\end{bmatrix}=\begin{bmatrix}-3\\-12\end{bmatrix}$	**B**
2	$\lvert \underset{\sim}{s}+\underset{\sim}{t}\rvert = \lvert 4\underset{\sim}{i}+9\underset{\sim}{j}+5\underset{\sim}{i}+3\underset{\sim}{j}\rvert$ $=\lvert 9\underset{\sim}{i}+12\underset{\sim}{j}\rvert$ $=\sqrt{9^2+12^2}$ $=15$	**B**
3		**D**
4	 Vector sum in the $\underset{\sim}{i}$ direction: $\lvert \underset{\sim}{F_2}\rvert\cos 40^\circ - \lvert \underset{\sim}{F_1}\rvert\cos 30^\circ = 0$ $\lvert \underset{\sim}{F_2}\rvert = \dfrac{\cos 30^\circ}{\cos 40^\circ}\lvert \underset{\sim}{F_1}\rvert - (1)$ Vector sum in the $\underset{\sim}{j}$ direction: $\lvert \underset{\sim}{F_1}\rvert\sin 30^\circ - \lvert \underset{\sim}{F_2}\rvert\sin 40^\circ - 20g = 0 - (2)$ Sub (1) → (2) $\lvert \underset{\sim}{F_1}\rvert\sin 30^\circ - \dfrac{\cos 30^\circ}{\cos 40^\circ}\lvert \underset{\sim}{F_1}\rvert\times\sin 40^\circ - 20\times 9.8 = 0$ $\lvert \underset{\sim}{F_1}\rvert = 159.8\,\text{N}$ Sub into (1) $\lvert \underset{\sim}{F_2}\rvert = 1.1305\times 159.8$ $\lvert \underset{\sim}{F_2}\rvert = 180.6\,\text{N}$ $\therefore \lvert \underset{\sim}{F_1}\rvert = 160\,\text{N}$ and $\lvert \underset{\sim}{F_2}\rvert = 181\,\text{N}$	**A**
5	$(k+1)\big(2[k+1]-1\big)\big(2[k+1]+1\big)=(k+1)(2k+1)(2k+3)$	**D**

6a(i)	$\lvert \underset{\sim}{a} \rvert = \sqrt{3^2+1^2} = \sqrt{10}$	**1 mark** • Correct solution
6a(ii)	$\hat{\underset{\sim}{a}} = \dfrac{\underset{\sim}{a}}{\lvert \underset{\sim}{a} \rvert}$ $\hat{\underset{\sim}{a}} = \dfrac{3\underset{\sim}{i}+1\underset{\sim}{j}}{\sqrt{3^2+1^2}}$ $\hat{\underset{\sim}{a}} = \dfrac{3}{\sqrt{10}}\underset{\sim}{i} + \dfrac{1}{\sqrt{10}}\underset{\sim}{j}$	**1 mark** • Correct solution
6a(iii)	$\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b} = \dfrac{\underset{\sim}{b}\cdot\underset{\sim}{a}}{\underset{\sim}{a}\cdot\underset{\sim}{a}}\underset{\sim}{a}$ $= \dfrac{3\times 2+1\times -1}{3^2+1^2}\begin{bmatrix}3\\1\end{bmatrix}$ $= \begin{bmatrix}1.5\\0.5\end{bmatrix}$	**1 mark** • Correct solution
6a(iv)	$\begin{bmatrix}2\\-1\end{bmatrix}\cdot\begin{bmatrix}x_1\\y_1\end{bmatrix} = 0$ $2x_1 - y_1 = 0$ $2x_1 = y_1$ must be in the form $\begin{bmatrix}a\\2a\end{bmatrix}$, where a is a real number eg. $\begin{bmatrix}1\\2\end{bmatrix}$	**1 mark** • Correct solution
6b	$\overrightarrow{BA} = \begin{bmatrix}-2\\2\end{bmatrix}$ $\overrightarrow{BC} = \begin{bmatrix}2\\1\end{bmatrix}$ $\cos\angle ABC = \dfrac{\overrightarrow{BA}\cdot\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$ $\cos\angle ABC = \dfrac{2\times -2+2\times 1}{\sqrt{(-2)^2+2^2}\times\sqrt{2^2+1^2}}$ $\cos\angle ABC = \dfrac{-2}{\sqrt{8}\times\sqrt{5}}$ $\angle ABC = 108.43...^\circ$ $\angle ABC = 108^\circ$ (nearest degree)	**3 marks** • Correct solution. **2 marks** • Calculates the dot product of $\overrightarrow{BA}$ and $\overrightarrow{BC}$. **1 mark** • Finds $\overrightarrow{BA}$ or $\overrightarrow{BC}$.

6c	Test for $n=1$ $\text{LHS} = 23^1 - 1$ $= 11\times 2$ $\therefore$ True for $n=1$ Assume true for $n=k$ ie. $23^k - 1 = 11P$, where P is an integer. Prove true for $n=k+1$ AIM : $23^{k+1} - 1 = 11Q$, where Q is an integer. Proof: LHS = $23^{k+1} - 1$ $= 23^k \times 23 - 1$ $= (11P+1)\times 23 - 1$ if the assumption is true $= 23\times 11P + 22$ $= 11(23P+2)$ $= 11Q$ since $(23P+2)$ is an integer $= \text{RHS}$ $\therefore$ true for $n=k+1$, if it is true for $n=k$. $\therefore$ result is true by Mathematical Induction for all integers $n\geq 1$, since it is true for $n=1$.	**3 marks** • Correct induction. **2 marks** • Uses the assumption to prove the required result. **1 mark** • Correct base case.
7a)	Resultant vector $= \begin{bmatrix}5\\6\end{bmatrix} + \begin{bmatrix}7\\-10\end{bmatrix} + \begin{bmatrix}-2\\-20\end{bmatrix} = \begin{bmatrix}10\\-24\end{bmatrix}$ magnitude $= \sqrt{10^2 + (-24)^2}$ $= 26$ $F = ma$ $26 = 2a$ $a = 13\,\text{ms}^{-2}$	**2 marks** • Correct solution **1 mark** • Find the resultant vector
7b(i)	$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$ $\overrightarrow{OR} = (1-\lambda)\overrightarrow{OA} + \lambda\overrightarrow{PQ} \quad -(1)$ $\overrightarrow{PQ} = \lambda\overrightarrow{OB} - (1-\lambda)\overrightarrow{OA}$ sub $\overrightarrow{PQ}$ into (1) $\overrightarrow{OR} = (1-\lambda)\overrightarrow{OA} + \lambda\left[\lambda\overrightarrow{OB} - (1-\lambda)\overrightarrow{OA}\right]$ $\overrightarrow{OR} = (1-2\lambda+\lambda^2)\overrightarrow{OA} + \lambda^2\overrightarrow{OB}$	**1 marks** • Correct solution

7b(ii)	$\overrightarrow{OR}=(1-\lambda)^2\begin{bmatrix}1\\2\end{bmatrix}+\lambda^2\begin{bmatrix}-1\\3\end{bmatrix}$ $=\begin{bmatrix}1-2\lambda\\2-4\lambda+5\lambda^2\end{bmatrix}$ $x=1-2\lambda$ and $y=2-4\lambda+5\lambda^2$ $\lambda=\frac{1-x}{2}$ sub λ into y $y=2-4\left(\frac{1-x}{2}\right)+5\left(\frac{1-x}{2}\right)^2$ $y=\frac{5x^2}{4}-\frac{x}{2}+\frac{5}{4}$	**2 marks** • Correct solution **1 mark** • Finds the correct parametric equations
7c(i)		**1 mark** • Correct solution
7c(ii)	Forces parallel to the river bank ie. $\left\|\overrightarrow{PQ}\right\|$:  $1.8\sin\theta-0.5\cos 45°-1.2=0$ In ΔAPQ, $\sin\theta=\dfrac{\frac{6}{5}+\frac{1}{2\sqrt{2}}}{1.8}$ $\theta=59.6647..°$ $\therefore$ The bearing that the fish should head to is 060°T (to the nearest degree) $1.8\cos\theta+0.5\sin 45°$	**2 marks** • Correct solution **1 mark** • Finds $\left\|\overrightarrow{PQ}\right\|$

8a(i)	$S_n = \frac{n}{2}(a+l)$ $1+2+...+n = \frac{n}{2}(1+n)$ square both sides $\left(1+2+...+n\right)^2 = \frac{n^2}{4}(1+n)^2$	**1 mark** • Correct solution
8a(ii)	Test for $n=1$ $\text{LHS} = 1^3 = 1$ $\quad\quad$ $\text{RHS} = 1^2 = 1$ $\therefore$ True for $n=1$ Assume true for $n=k$ ie. $1^3+2^3+...+k^3 = \left(1+2+....+k\right)^2$ Prove true for $n=k+1$ $AIM: \ 1^3+2^3+...+k^3+(k+1)^3 = \left(1+2+....+k+[k+1]\right)^2$ Proof: LHS = $1^3+2^3+...+k^3+\left(k+1\right)^3$ $=(1+2+...+k)^2+(k+1)^3$ if the assumption is true $=\frac{k^2}{4}\left(k+1\right)^2+\left(k+1\right)^3$ using part (i) $=\frac{\left(k+1\right)^2}{4}\left[k^2+4(k+1)\right]$ $=\frac{\left(k+1\right)^2}{4}\left[k+2\right]^2$ RHS = $\left(1+2+....+k+[k+1]\right)^2$ $=\left[\frac{(k+1)}{2}(1+k+1)\right]^2$ $=\frac{(k+1)^2}{4}(k+2)^2$ $=\text{LHS}$ $\therefore$ true for $n=k+1,$ if it is true for $n=k.$ $\therefore$ result is true by Mathematical Induction for all integers $n \geq 1,$ since it is true for $n=1.$	**3 marks** • Correct induction. **2 marks** • Uses the assumption to prove the required result. **1 mark** • Correct base case, showing correct substitution. Note: Marks were deducted if proving for $1^3+2^3+...+n^3 = \frac{1}{4}n^2(n+1)^2$ without linking to the question.

<table>
<tr>
<td>8b(iii)</td>
<td>
Here are 4 ways we can obtain the vector $\overrightarrow{MR}$
$\overrightarrow{MR} = \overrightarrow{MN} + \overrightarrow{NR}$
$\overrightarrow{MR} = \underset{\sim}{n} + \lambda\overrightarrow{NF} \quad -(1)$

$\overrightarrow{MR} = \overrightarrow{MP} + \overrightarrow{PR}$
$\overrightarrow{MR} = \underset{\sim}{p} + \mu\overrightarrow{PE} \quad -(2)$

$\overrightarrow{MR} = \overrightarrow{ME} + \overrightarrow{ER}$
$\overrightarrow{MR} = \frac{1}{2}\underset{\sim}{n} - (1-\mu)\overrightarrow{PE} \quad -(3)$

$\overrightarrow{MR} = \overrightarrow{MF} + \overrightarrow{FR}$
$\overrightarrow{MR} = \frac{1}{2}\underset{\sim}{p} - (1-\lambda)\overrightarrow{NF} \quad -(4)$
$\overrightarrow{NF} = -\underset{\sim}{n} + \frac{1}{2}\underset{\sim}{p}$
$\overrightarrow{PE} = -\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{n}$
Sub $\overrightarrow{NF}$ into (1) or (4)
<table>
<tr>
<td>Sub $\overrightarrow{NF}$ into (1):
$\overrightarrow{MR} = \underset{\sim}{n} + \lambda\left(-\underset{\sim}{n} + \frac{1}{2}\underset{\sim}{p}\right)$
$\overrightarrow{MR} = (1-\lambda)\underset{\sim}{n} + \frac{1}{2}\lambda\underset{\sim}{p} \quad -(5)$</td>
<td>Sub $\overrightarrow{NF}$ into (4):
$\overrightarrow{MR} = \frac{1}{2}\underset{\sim}{p} - (1-\lambda)\left(-\underset{\sim}{n} + \frac{1}{2}\underset{\sim}{p}\right)$
$\overrightarrow{MR} = \frac{1}{2}\underset{\sim}{p} + \underset{\sim}{n} - \frac{1}{2}\underset{\sim}{p} - \lambda\underset{\sim}{n} + \frac{1}{2}\lambda\underset{\sim}{p}$
$\overrightarrow{MR} = (1-\lambda)\underset{\sim}{n} + \frac{1}{2}\lambda\underset{\sim}{p}$</td>
</tr>
</table>
Sub $\overrightarrow{PE}$ into (2) or (3)
<table>
<tr>
<td>Sub $\overrightarrow{PE}$ into (2):
$\overrightarrow{MR} = \underset{\sim}{p} + \mu\left(-\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{n}\right)$
$\overrightarrow{MR} = (1-\mu)\underset{\sim}{p} + \frac{1}{2}y\underset{\sim}{n} \quad -(6)$</td>
<td>Sub $\overrightarrow{PE}$ into (3):
$\overrightarrow{MR} = \frac{1}{2}\underset{\sim}{n} - (1-y)\left(-\underset{\sim}{p} + \frac{1}{2}\underset{\sim}{n}\right)$
$\overrightarrow{MR} = \frac{1}{2}\underset{\sim}{n} + \underset{\sim}{p} - \frac{1}{2}\underset{\sim}{n} - y\underset{\sim}{p} + \frac{1}{2}y\underset{\sim}{n}$
$\overrightarrow{MR} = (1-y)\underset{\sim}{p} + \frac{1}{2}y\underset{\sim}{n}$</td>
</tr>
</table>
using (5) and (6) to equate $\underset{\sim}{p}$ and $\underset{\sim}{n}$:
$\frac{1}{2}\lambda = 1-\mu \quad -(7)$
$\frac{1}{2}\mu = 1-\lambda \quad -(8)$
</td>
<td>
4 marks
• Correct solution by showing $\overrightarrow{MR}$ and $\overrightarrow{MG}$ are collinear and proving ratio of 2:1 for all 3 medians.

3 marks
• Solves for $\boldsymbol{\lambda = \frac{2}{3}}$ and $\boldsymbol{\mu = \frac{2}{3}}$

2 marks
• Finds expression of $\overrightarrow{MR}$ in terms of $\underset{\sim}{n}, \underset{\sim}{p}, \lambda$ and μ only
ie.
$\overrightarrow{MR} = (1-\mu)\underset{\sim}{p} + \frac{\mu}{2}\underset{\sim}{n}$
$\overrightarrow{MR} = (1-\lambda)\underset{\sim}{n} + \frac{\lambda}{2}\underset{\sim}{p}$

1 mark
• Finds one correct expression of $\overrightarrow{MR}$ using $\underset{\sim}{n}$ and $\underset{\sim}{p}$ listed below:

$\overrightarrow{MR} = \underset{\sim}{n} + \lambda\overrightarrow{NF}$
$\overrightarrow{MR} = \underset{\sim}{p} + \mu\overrightarrow{PE}$
$\overrightarrow{MR} = \frac{1}{2}\underset{\sim}{n} - (1-\mu)\overrightarrow{PE}$
$\overrightarrow{MR} = \frac{1}{2}\underset{\sim}{p} - (1-\lambda)\overrightarrow{NF}$
</td>
</tr>
</table>

Sub (7) into (8)

$$\frac{1}{2}\mu = 1 - 2(1-\mu)$$

$$\mu = \frac{2}{3}$$

Sub y into (7)

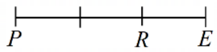
$$\lambda = 2\left(1 - \frac{2}{3}\right) = \frac{2}{3}$$

$$\therefore \lambda = \mu = \frac{2}{3}$$

$$\overrightarrow{NR} = \frac{2}{3}\overrightarrow{NF}$$

$$\therefore \left|\overrightarrow{NR}\right| : \left|\overrightarrow{NF}\right| = 2:1$$

$$\overrightarrow{PR} = \frac{2}{3}\overrightarrow{PE}$$



$$\left|\overrightarrow{PR}\right| : \left|\overrightarrow{RE}\right| = 2:1$$

Use (1), (2), (3) or (4) to obtain

$$\overrightarrow{MR} = \frac{1}{3}\left(\underset{\sim}{p} + \underset{\sim}{n}\right)$$

$$\overrightarrow{MG} = \overrightarrow{MN} + \overrightarrow{NG}$$

$$\overrightarrow{MG} = \underset{\sim}{n} + \overrightarrow{NG}$$

we know

$$\overrightarrow{NG} = \frac{1}{2}\overrightarrow{NP}$$

$$\overrightarrow{NG} = \frac{1}{2}(\underset{\sim}{p} - \underset{\sim}{n})$$

$$\overrightarrow{MG} = \underset{\sim}{n} + \frac{1}{2}(\underset{\sim}{p} - \underset{\sim}{n})$$

$$\overrightarrow{MG} = \frac{1}{2}(\underset{\sim}{p} + \underset{\sim}{n})$$

$$= \frac{3}{2}\overrightarrow{MR}$$

Since $\overrightarrow{MG}$ is a scalar multiple of $\overrightarrow{MR}$
and intersect at the same point M,
then M, R, G are collinear.

	$\left\|\overrightarrow{MR}\right\| : \left\|\overrightarrow{RG}\right\|$ $= \left\|\overrightarrow{MR}\right\| : \left\|\overrightarrow{MG} - \overrightarrow{MR}\right\|$ $= \frac{1}{3}\left\|\underset{\sim}{n} + \underset{\sim}{p}\right\| : \frac{1}{2}\left\|\underset{\sim}{n} + \underset{\sim}{p}\right\| - \frac{1}{3}\left\|\underset{\sim}{n} + \underset{\sim}{p}\right\|$ $= \frac{1}{3}\left\|\underset{\sim}{n} + \underset{\sim}{p}\right\| : \frac{1}{6}\left\|\underset{\sim}{n} + \underset{\sim}{p}\right\|$ $= \frac{1}{3} : \frac{1}{6}$ $= 2:1$ $\overrightarrow{PE}, \overrightarrow{NF}$ and $\overrightarrow{MG}$ is concurrent at M, in the ratio 2:1	