HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 12 HSC Assessment 2 Term 1 2024

STUDENT NUMBER: ________________________ **Student Name**________________________________

Circle your teachers name: Ms Guan, Ms Murray Mr Payne, Ms Sztajer

General Instructions

- Reading Time – **5** minutes
- Working Time – **75** minutes
- Write using black or blue pen
  Black pen is preferred
- NESA-approved calculators may be used
- Show all necessary working
- Marks may be deducted for untidy and poorly arranged work
- Do not use correction fluid or tape
- Do not remove this paper from the examination

Total marks –46

Section I Pages 2 – 4

6 marks

Attempt Questions 1 – 6

Answer on the Objective Response Answer Sheet provided

Section II Pages 5 – 8

38 marks

Attempt Questions 7 – 10. Start each question on a new page. Write your student number on page

Question	***1-6***	***7***	***8***	***9***	***10***	***Total***
Total	*/6*	*/9*	*/10*	*/9*	*/10*	*/44*

This assessment task constitutes 30% of the Higher School Certificate Course School Assessment

Section I

6 marks
Allow about 10 minutes for this section
Use the Multiple Choice answer sheet for Questions 1 – 6

1. Which one of the following vectors is parallel to $\overrightarrow{OE} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$?

   (A) $4\underset{\sim}{i} + 6\underset{\sim}{j}$

   (B) $-4\underset{\sim}{i} + 6\underset{\sim}{j}$

   (C) $6\underset{\sim}{i} + 9\underset{\sim}{j}$

   (D) $-6\underset{\sim}{i} - 9\underset{\sim}{j}$

2. What is the value of $\sin 2x$ given that $\sin x = \frac{5}{13}$ and $\frac{\pi}{2} \le x \le \pi$?

   (A) $\frac{-120}{169}$

   (B) $\frac{-60}{169}$

   (C) $\frac{60}{169}$

   (D) $\frac{120}{169}$

3. Given that $\tan\theta = \frac{1}{3}$, what is the exact value of $\tan\left(\theta + \frac{\pi}{3}\right)$?

   (A) $\frac{\sqrt{3}+3}{3\sqrt{3}-1}$

   (B) $\frac{\sqrt{3}-3}{3\sqrt{3}+1}$

   (C) $\frac{1+3\sqrt{3}}{3-\sqrt{3}}$

   (D) $\frac{1-3\sqrt{3}}{3+\sqrt{3}}$

4. Mathemaical induction is used to prove $1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for all integers n.

Which of the following is a correct expression for part of the proof ?

(A) $\text{LHS} = \frac{(k+1)(k+2)(2k+3)}{6} + k^2$

(B) $\text{LHS} = \frac{(k+1)(k+2)(2k+3)}{6} + (k+1)^2$

(C) $\text{LHS} = \frac{k(k+1)(2k+1)}{6} + k^2$

(D) $\text{LHS} = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

5. Consider the vectors $\underset{\sim}{a} = \begin{pmatrix} m-3 \\ 5 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 2m \\ -4 \end{pmatrix}$.

What are the possible values of m so that $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are perpendicular ?

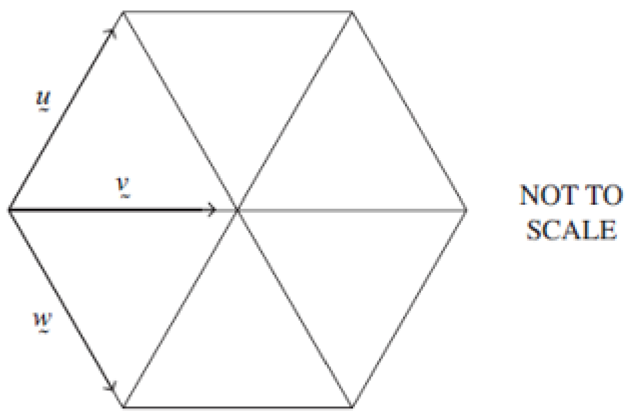
(A) $5, -2$

(B) $-5, 2$

(C) $7, 2\frac{1}{2}$

(D) $8, 2$

6. Six equilateral triangles form a hexagon with side lengths of 4 cm. The vectors $\underset{\sim}{u}$, $\underset{\sim}{v}$ and $\underset{\sim}{w}$ are shown in the diagram.



Which of the following is the value of $\underset{\sim}{u} \cdot (\underset{\sim}{u} + \underset{\sim}{v} + \underset{\sim}{w})$?

(A) 8

(B) 16

(C) 32

(D) 48

End of Section I

Section II

38 marks
Attempt Questions 7 - 10
Allow about 60 minutes for this section

Begin each question on a new answering booklet, indicating the question number.
Extra writing paper is available.

Question 7 (9 marks) **Start a new writing booklet.** **Marks**

(a) Prove $\dfrac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}=\tan\theta$. **2**

(b) Let $\underset{\sim}{i}$ and $\underset{\sim}{j}$ be unit vectors in the directions of east and north respectively. Jo is rowing in the ocean and her velocity $\underset{\sim}{v}_1$ m/s is given by the vector $\underset{\sim}{v}_1=-\underset{\sim}{i}+2\underset{\sim}{j}$. The oceans current has a velocity of $\underset{\sim}{v}_2$ m/s where $\underset{\sim}{v}_2=-\dfrac{\underset{\sim}{i}}{2}-\dfrac{\underset{\sim}{j}}{2}$.

(i) Find the resultant velocity $\underset{\sim}{v}$. **1**

(ii) What is the direction of the resultant velocity to the nearest degree? **1**

(iii) What is the magnitude of the the resultant velocity, correct to 1 decimal place? **1**

(c)

(i) Prove by mathematical induction

$1^3+2^3+3^3+...+n^3=\dfrac{1}{4}n^2(n+1)^2$ for all positive integers n. **3**

(ii) Hence evaluate $2^3+4^3+6^3+...+20^3$. **1**

Question 8 (10 marks) **Start a new writing booklet.** **Marks**

(a) Use sums to products of trigonometric ratios to solve the equation
$\cos x + \cos 2x + \cos 3x = 0,\ 0 \le x \le 2\pi.$ **3**

(b) An object of mass 20 kg is suspended in equilibrium by two strings attached to two points in the same horizontal plane. If the two strings make angles of 30^o and 45^o respectively to the horizontal, **use component form** to find the magnitude of the tension force in each string, in newtons, correct to one decimal place.
Use $g = 9.8$ m/s^2. **3**

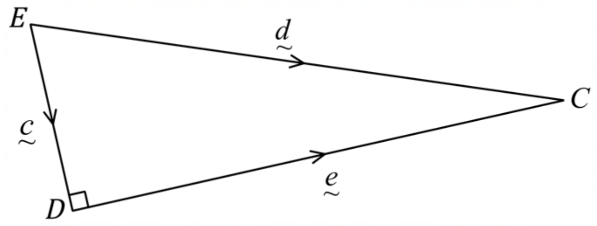
(c) (i) Express $\sqrt{3}\sin x - \cos x$ in the form $R\sin(x-\alpha)$, **2**

where $R > 0$ and $0 \le \alpha \le \dfrac{\pi}{2}$.

(ii) Hence, solve $\sqrt{3}\sin x - \cos x = \sqrt{2}$, for $0 \le x \le 2\pi$. **2**

Question 9 (9 marks) **Start a new writing booklet.** **Marks**

(a) ΔDEC has a right angle at D.



Use vector methods to prove that $|\underset{\sim}{d}|^2 = |\underset{\sim}{c}|^2 + |\underset{\sim}{e}|^2$.

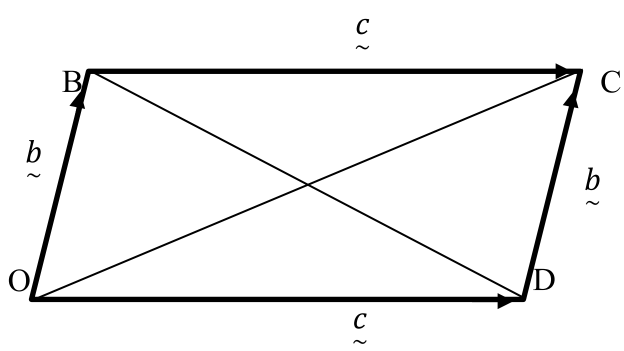
That is, prove Pythagoras' Theorem. **3**

(b) By expressing $\sin x$ and $\cos x$ in terms of $\boldsymbol{t}$, where $t = \tan\dfrac{x}{2}$, solve **3**

$3\sin x - 2\cos x = 2$ for $0^o \le x \le 360^o$. Answer correct to the nearest minute.

(c) Consider the parallelogram *OBCD* where $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{OD} = \underset{\sim}{c}$.
Show that the diagonals of a parallelogram meet at right angles if and only if it is a rhombus.

3



Question 10 (10 marks) **Start a new writing booklet.** **Marks**

(a) Consider the vectors $\underset{\sim}{a} = 3\underset{\sim}{i} - 4\underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + \underset{\sim}{j}$.

(i) Find the acute angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ to the nearest degree. **1**

(ii) Find the unit vector in the direction of $\underset{\sim}{a}$. **1**

(iii) Find the <u>scalar</u> projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$. **1**

(iv) Find the <u>vector</u> projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$. **1**

(v) Find the vector projection of $\underset{\sim}{b}$ perpendicular to the direction of $\underset{\sim}{a}$. **1**

(b) (i) Prove the trigonometric identity $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. **2**

(ii) For the equation $8a^3 - 6a = 1$, show that $\cos 3\theta = \frac{1}{2}$ by letting $a = \cos\theta$. **1**

(iii) Hence, find the solutions to the equation $8a^3 - 6a = 1$ in exact values. **2**

End of Examination

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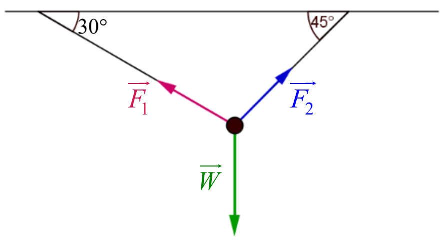
Hornsby Girls High School
Year 12 Mathematics Extension 1 HSC Assessment 2 2024
Solutions

Solutions	Marker's Comments
1. **B** $-4\underset{\sim}{i}+6\underset{\sim}{j}=-2\left(2\underset{\sim}{i}-3\underset{\sim}{j}\right)$ $=-2\overrightarrow{OE}$ parallel but in opposite directions.	
2. **A** $\sin x=\frac{5}{13}, \cos x=\frac{-12}{13}$ x is obtuse $\sin 2x=2\sin x\cos x$ $=2\times\frac{5}{13}\times\frac{-12}{13}$ $=\frac{-120}{169}$	
3. **C** $\tan\left(\theta+\frac{\pi}{3}\right)=\frac{\tan\theta+\tan\frac{\pi}{3}}{1-\tan\theta\tan\frac{\pi}{3}}$ $=\frac{\frac{1}{3}+\sqrt{3}}{1-\frac{1}{3}\times\sqrt{3}}$ $=\frac{1+3\sqrt{3}}{3-\sqrt{3}}$	
4. **D** $1^2+2^2+3^2+......+k^2+\left(k+1\right)^2$ $=\frac{k(k+1)(2k+1)}{6}+\left(k+1\right)^2$	

Solutions	Marker's Comments
5. **A** $\underset{\sim}{a}\cdot\underset{\sim}{b}=0$ $2m(m-3)+5(-4)=0$ $2m^2-6m-20=0$ $2\left(m^2-3m-10\right)=0$ $2(m-5)(m+2)=0$ $m=5,-2$	
6. **B** $\underset{\sim}{u}+\underset{\sim}{v}+\underset{\sim}{w}=2\underset{\sim}{v}$ equilateral triangles, angels are 60° each $\lvert\underset{\sim}{u}\rvert=\lvert\underset{\sim}{v}\rvert=\lvert\underset{\sim}{w}\rvert=4$ $\underset{\sim}{u}\cdot(\underset{\sim}{u}+\underset{\sim}{v}+\underset{\sim}{w})=\underset{\sim}{u}\cdot 2\underset{\sim}{v}$ $=\lvert\underset{\sim}{u}\rvert\lvert 2\underset{\sim}{v}\rvert\cos 60°$ $=4\times 2\times 4\times\frac{1}{2}$ $=16$	
7. a) $LHS=\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}$ $=\frac{\sin\theta+2\sin\theta\cos\theta}{1+\cos\theta+2\cos^2\theta-1}$ $=\frac{\sin\theta(1+2\cos\theta)}{\cos\theta(1+2\cos\theta)}$ $=\frac{\sin\theta}{\cos\theta}$ $=\tan\theta$	1 for using two double angle results 1 for factorising and simplifying to SHOW the required result
b) i) $\underset{\sim}{v}=\underset{\sim}{v}_1+\underset{\sim}{v}_2$ $=-\underset{\sim}{i}+2\underset{\sim}{j}+-\frac{\underset{\sim}{i}}{2}-\frac{\underset{\sim}{j}}{2}$ $=-\frac{3}{2}\underset{\sim}{i}+\frac{3}{2}\underset{\sim}{j}$	Done well by most students Be careful not to make transcription errors NOT $\underset{\sim}{v}=\underset{\sim}{v}_1-\underset{\sim}{v}_2$
b) ii) $\tan\theta=\frac{\frac{3}{2}}{\frac{-3}{2}}=-1$ $\theta=135°$ The direction is $N45^0W$ or 315°	Some students did not interpret the angle to find the bearing

Solutions	Marker's Comments
b) iii) $$\lvert \underset{\sim}{v} \rvert = \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{3}{2}\right)^2}$$ $$= \sqrt{\frac{9}{4} + \frac{9}{4}}$$ $$= 2.121320344....$$ $$\approx 2.1 \text{ m/s}$$	
c) i) for $n = 1$, $LHS = 1^3 = 1$, $RHS = \frac{1}{4}(1)^2(1+1)^2$ $= \frac{1}{4} \times 2^2$ $= 1$ $\therefore$ Prove true for $n = 1$. Assume true for $n = k$, i.e. assume $1^3 + 2^3 + 3^3 + ... + k^3 = \frac{1}{4}k^2(k+1)^2$ is true require to prove true for $n = k+1$, i.e. $1^3 + 2^3 + 3^3 + ... + k^3 + (k+1)^3 = \frac{1}{4}(k+1)^2(k+2)^2$ $LHS = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$ from assumption $= \frac{1}{4}(k+1)^2\left(k^2 + 4(k+1)\right)$ $= \frac{1}{4}(k+1)^2\left(k^2 + 4k + 4\right)$ $= \frac{1}{4}(k+1)^2(k+2)^2$ $= RHS$ If true for $n = k$, proven true for $n = k+1$. Since true for $n = 1$, also true for $n = 1+1 = 2$, $n = 1+2 = 3$, and so on for all positive integers n.	It is an essential part of the induction process to indicate that the assumption has been used. Lots of algebraic errors and/or insufficient working to **prove** that LHS=RHS It is **strongly recommended** that students use the conclusion as modelled here, in class and in the NESA topic guidance.
c) ii) $2^3 + 4^3 + 6^3 + ... + 20^3$ $= (2\times1)^3 + (2\times2)^3 + (2\times3)^3 + ... + (2\times10)^3$ $= 2^3\times1^3 + 2^3\times2^3 + 2^3\times3^3 + ...2^3\times10^3$ $= 2^3\times\left(1^3 + 2^3 + 3^3 + ... + 10^3\right)$ $= 2^3\times\left(\frac{1}{4}10^2(10+1)^2\right)$ $= 8\times\frac{1}{4}\times10^2\times11^2$ $= 24200$	Not done well

Solutions	Marker's Comments
8. a) $\cos x+\cos 2x+\cos 3x=0$ $\cos 3x+\cos x+\cos 2x=0$ $2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)+\cos 2x=0$ $2\cos 2x\cos x+\cos 2x=0$ $\cos 2x(2\cos x+1)=0$ $\cos 2x=0, \quad 2\cos x+1=0$ $\cos 2x=0, \quad 0\le 2x\le 4\pi$ $\qquad$ $\cos x=\frac{1}{2}, \quad 0\le x\le 2\pi$ $2x=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2}$ $\qquad$ $x=\pi-\frac{\pi}{3},\pi+\frac{\pi}{3}$ $\therefore x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4},\frac{2\pi}{3},\frac{4\pi}{3}$	27% of cohort did not use or know the 'Sum to Product' formula as required. Students should make sure they are competent with this trigonometric identity. Generally did well.
b) Required in component form.  $\vec{F}_1=-\lvert\vec{F}_1\rvert\cos 30^\circ \underset{\sim}{i}+\lvert\vec{F}_1\rvert\sin 30^\circ \underset{\sim}{j}$ $\vec{F}_2=\lvert\vec{F}_2\rvert\cos 45^\circ \underset{\sim}{i}+\lvert\vec{F}_2\rvert\sin 45^\circ \underset{\sim}{j}$ $\vec{W}=0\underset{\sim}{i}-mg\underset{\sim}{j}$ Since $\sum F=0$ $\left(-\lvert\vec{F}_1\rvert\cos 30^\circ \underset{\sim}{i}+\lvert\vec{F}_1\rvert\sin 30^\circ \underset{\sim}{j}\right)$ $+\left(\lvert\vec{F}_2\rvert\cos 45^\circ \underset{\sim}{i}+\lvert\vec{F}_2\rvert\sin 45^\circ \underset{\sim}{j}\right)-mg\underset{\sim}{j}=0$ Vectors in $\underset{\sim}{i}$ direction: $-\lvert\vec{F}_1\rvert\cos 30^\circ+\lvert\vec{F}_2\rvert\cos 45^\circ=0$ $-\lvert\vec{F}_1\rvert\times\frac{\sqrt{3}}{2}+\lvert\vec{F}_2\rvert\times\frac{\sqrt{2}}{2}=0$ $\lvert\vec{F}_1\rvert\times\frac{\sqrt{3}}{2}=\lvert\vec{F}_2\rvert\times\frac{\sqrt{2}}{2}$ $\lvert\vec{F}_1\rvert=\lvert\vec{F}_2\rvert\times\frac{\sqrt{2}}{\sqrt{3}} \qquad (1)$	Diagram should be drawn to aid the student and marker for reference to the different vectors and angles used. Many students did not know how to use vector notation in their working. 50% of cohort did not write the forces as vectors and expressed these vectors in component form as required. Students need to pay attention to the required expression. Students must draw the vectors in the diagrams to indicate the directions of the vectors given. Generally, students were able to solve for the magnitude of the two forces.

Solutions	Marker's Comments
Vectors in $\underset{\sim}{j}$ direction: $\left\|\vec{F}_1\right\|\sin 30° + \left\|\vec{F}_2\right\|\sin 45° = 20g$ $\left\|\vec{F}_1\right\| \times \frac{1}{2} + \left\|\vec{F}_2\right\| \times \frac{\sqrt{2}}{2} = 20 \times 9.8 \qquad (2)$ substitute equation (1) into (2): $\left\|\vec{F}_2\right\| \times \frac{\sqrt{2}}{\sqrt{3}} \times \frac{1}{2} + \left\|\vec{F}_2\right\| \times \frac{\sqrt{2}}{2} = 20 \times 9.8$ $\therefore \left\|\vec{F}_2\right\| = \frac{20 \times 9.8}{\frac{\sqrt{2}}{\sqrt{3}} \times \frac{1}{2} + \frac{\sqrt{2}}{2}}$ $= 175.7287925....$ $\approx 175.7 \text{ N}$ $\therefore \left\|\vec{F}_1\right\| \approx 175.7 \times \frac{\sqrt{2}}{\sqrt{3}}$ $= 143.4819583....$ $\approx 143.5 \text{ N}$	
c) i) $\sqrt{3}\sin x - \cos x = R\sin(x - \alpha)$ $= R\sin x\cos\alpha - R\cos x\sin\alpha$ $R\sin\alpha = 1, R\cos\alpha = \sqrt{3}$ $\tan\alpha = \frac{1}{\sqrt{3}}, \quad \alpha = \frac{\pi}{6}$ $(R\sin\alpha)^2 + (R\cos\alpha)^2 = 1^2 + \left(\sqrt{3}\right)^2$ $R^2 = 4$ $R = 2 \quad (R > 0)$ $\therefore \sqrt{3}\sin x - \cos x = 2\sin\left(x - \frac{\pi}{6}\right)$	Mostly did well.

Solutions	Marker's Comments
c) ii) $\sqrt{3}\sin x - \cos x = \sqrt{2}$ $2\sin\left(x - \frac{\pi}{6}\right) = \sqrt{2}$ $\sin\left(x - \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2}, \quad -\frac{\pi}{6} \le x - \frac{\pi}{6} \le 2\pi - \frac{\pi}{6}$ $x - \frac{\pi}{6} = \frac{\pi}{4}, \frac{3\pi}{4}$ $x = \frac{\pi}{4} + \frac{\pi}{6}, \frac{3\pi}{4} + \frac{\pi}{6}$ $x = \frac{5\pi}{12}, \frac{11\pi}{12}$	Mostly did well.
9. a) $\underset{\sim}{d} = \underset{\sim}{c} + \underset{\sim}{e}$ $\lvert\underset{\sim}{d}\rvert^2 = \underset{\sim}{d} \cdot \underset{\sim}{d}$ $= (\underset{\sim}{c} + \underset{\sim}{e}) \cdot (\underset{\sim}{c} + \underset{\sim}{e})$ $= \underset{\sim}{c} \cdot \underset{\sim}{c} + \underset{\sim}{e} \cdot \underset{\sim}{c} + \underset{\sim}{c} \cdot \underset{\sim}{e} + \underset{\sim}{e} \cdot \underset{\sim}{e}$ $= \underset{\sim}{c} \cdot \underset{\sim}{c} + 2\underset{\sim}{e} \cdot \underset{\sim}{c} + \underset{\sim}{e} \cdot \underset{\sim}{e}$ $= \underset{\sim}{c} \cdot \underset{\sim}{c} + 2(0) + \underset{\sim}{e} \cdot \underset{\sim}{e} \quad \left(\text{since } \underset{\sim}{e} \perp \underset{\sim}{c}, \underset{\sim}{e} \cdot \underset{\sim}{c} = 0\right)$ $= \underset{\sim}{c} \cdot \underset{\sim}{c} + \underset{\sim}{e} \cdot \underset{\sim}{e}$ $= \lvert\underset{\sim}{c}\rvert^2 + \lvert\underset{\sim}{e}\rvert^2$	Mostly well done. There were some students who tried to use Pythagoras to prove Pythagoras which can't be done. Some students need to watch their setting out as it was sometimes hard to follow and may have gotten benefit of the doubt.

Solutions	Marker's Comments
b) $3\dfrac{2t}{1+t^2}-2\dfrac{1-t^2}{1+t^2}=2$ $6t-2\left(1-t^2\right)=2\left(1+t^2\right)$ $6t-2+2t^2=2+2t^2$ $6t=4$ $t=\dfrac{2}{3}$ $\tan\dfrac{x}{2}=\dfrac{2}{3}$ for $0^o\le\dfrac{x}{2}\le 180^o$ $\dfrac{x}{2}\approx 33°41'$ $x\approx 67°23'$ Test $x=180°$, $LHS=3\sin 180°-2\cos 180°$ $=3(0)-2(-1)$ $=2$ $\therefore x=67°23',\quad 180°$	This question was very well done by nearly everyone. The common errors were either -not simplifying the t formula expression properly. -not testing for 180^o (which is a solution) -not correctly following the domain
c) Since $OBCD$ is a parallelogram, opposite sides are e $\overrightarrow{OB}=\overrightarrow{DC}=\underset{\sim}{b},\quad \overrightarrow{OD}=\overrightarrow{BC}=\underset{\sim}{c}$ $\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{BC}=\underset{\sim}{b}+\underset{\sim}{c}$ $\overrightarrow{DB}=\overrightarrow{DO}+\overrightarrow{OB}=\underset{\sim}{b}-\underset{\sim}{c}$ $\overrightarrow{OC}\bullet\overrightarrow{DB}=\left(\underset{\sim}{b}+\underset{\sim}{c}\right)\bullet\left(\underset{\sim}{b}-\underset{\sim}{c}\right)$ $=\underset{\sim}{b}\bullet\underset{\sim}{b}-\underset{\sim}{c}\bullet\underset{\sim}{b}+\underset{\sim}{b}\bullet\underset{\sim}{c}-\underset{\sim}{c}\bullet\underset{\sim}{c}$ $=\underset{\sim}{b}\bullet\underset{\sim}{b}-\underset{\sim}{c}\bullet\underset{\sim}{c}$ $=\lvert\underset{\sim}{b}\rvert^2-\lvert\underset{\sim}{c}\rvert^2$ If $\overrightarrow{OC}\bullet\overrightarrow{DB}=0$ then the diagonals will be perpendicu $\overrightarrow{OC}\bullet\overrightarrow{DB}=\lvert\underset{\sim}{b}\rvert^2-\lvert\underset{\sim}{c}\rvert^2=0$ if $\lvert\underset{\sim}{b}\rvert=\lvert\underset{\sim}{c}\rvert$ $\therefore\lvert\overrightarrow{OB}\rvert=\lvert\overrightarrow{BC}\rvert=\lvert\overrightarrow{DC}\rvert=\lvert\overrightarrow{OD}\rvert$ $\therefore$ $OBCD$ is a rhombus.	This was generally well done. Most students realised they needed to set the vector product of the diagonals to 0 and as a result this could only be true if adjacent sides have the same modulus. A few students stated $\underset{\sim}{b}=\underset{\sim}{c}$ which is not true. The moduli are equal. i.e. $\lvert\underset{\sim}{b}\rvert=\lvert\underset{\sim}{c}\rvert$ is true.

Solutions	**Marker's Comments**
10. a) i) $\underset{\sim}{a}\cdot\underset{\sim}{b}=\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert\cos\theta$ $\cos\theta=\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert}$ $=\dfrac{3\times2+(-4)\times1}{\sqrt{3^2+4^2}\sqrt{2^2+1^2}}$ $=\dfrac{2}{5\sqrt{5}}$ $\theta=79.69515353....$ $=79°42'$	Well done. Most of the students understand how to find the angle between two vectors.
ii) $\underset{\sim}{\hat{a}}=\dfrac{3\underset{\sim}{i}-4\underset{\sim}{j}}{\sqrt{3^2+4^2}}=\dfrac{1}{5}\left(3\underset{\sim}{i}-4\underset{\sim}{j}\right)$	Well done. Most of the students understand how to find the unit vectors
iii) $\underset{\sim}{b}\cdot\underset{\sim}{\hat{a}}=\left(2\underset{\sim}{i}+\underset{\sim}{j}\right)\cdot\dfrac{1}{5}\left(3\underset{\sim}{i}-4\underset{\sim}{j}\right)$ $=\dfrac{1}{5}(2\times3-4\times1)$ $=\dfrac{2}{5}$ Or: $\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{\lvert\underset{\sim}{a}\rvert}=\dfrac{3\times2-4}{\sqrt{3^2+4^2}}$ $=\dfrac{2}{5}$	Well done. Most of the students understand how to find scalar projection.
iv) $\left(\underset{\sim}{b}\cdot\underset{\sim}{\hat{a}}\right)\underset{\sim}{\hat{a}}=\dfrac{2}{5}\times\dfrac{1}{5}\left(3\underset{\sim}{i}-4\underset{\sim}{j}\right)$ $=\dfrac{2}{25}\left(3\underset{\sim}{i}-4\underset{\sim}{j}\right)$	Generally, well done. Some students were not able to find the vector projection correctly.
v) $\left(\underset{\sim}{b}\cdot\underset{\sim}{\hat{a}}\right)\underset{\sim}{\hat{a}}=\dfrac{2}{5}\times\dfrac{1}{5}\left(3\underset{\sim}{i}-4\underset{\sim}{j}\right)$ $\underset{\sim}{b}-\dfrac{2}{25}\left(3\underset{\sim}{i}-4\underset{\sim}{j}\right)=\left(2\underset{\sim}{i}+\underset{\sim}{j}\right)-\dfrac{2}{25}\left(3\underset{\sim}{i}-4\underset{\sim}{j}\right)$ $=\dfrac{1}{25}\left(50\underset{\sim}{i}+25\underset{\sim}{j}-6\underset{\sim}{i}+8\underset{\sim}{j}\right)$ $=\dfrac{1}{25}\left(44\underset{\sim}{i}+33\underset{\sim}{j}\right)$	Many students were not sure what the vector projection of b perpendicular to the direction of a.

<table>
<tr><th>Solutions</th><th>Marker's Comments</th></tr>
<tr><td>

10. b) i)
$LHS = \cos(2\theta + \theta)$
$= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$
$= (2\cos^2\theta - 1)\cos\theta - 2\sin\theta\cos\theta\sin\theta$
$= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$
$= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$
$= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^2\theta$
$= 4\cos^3\theta - 3\cos$
$= RHS$

</td><td>Well done. Students were able to use compound angle formula to prove the identity.</td></tr>
<tr><td>

b) ii)
using part i) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$,
let $a = \cos\theta$
$\cos 3\theta = 4a^3 - 3a$
$2\cos 3\theta = 8a^3 - 6a$
solving $8a^3 - 6a = 1$
$2\cos 3\theta = 1$
$\cos 3\theta = \frac{1}{2}$

</td><td>Well done.</td></tr>
<tr><td>

b) iii)
$\cos 3\theta = \frac{1}{2}$
$3\theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}, 2\pi + \frac{\pi}{3}$
($8a^3 - 6a = 1$ can have a maximum of 3 solutions)
$3\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3} \ldots$
$\theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9} \ldots$
$\therefore a = \cos\frac{\pi}{9}, \cos\frac{5\pi}{9}, \cos\frac{7\pi}{9} \left(\cos\frac{5\pi}{9} = \cos\frac{13\pi}{9}, \cos\frac{7\pi}{9} = \cos\frac{11\pi}{9} \ldots\right)$
alternatively,
$3\theta = 2n\pi + \frac{\pi}{3}$
$\theta = \frac{2n\pi}{3} + \frac{\pi}{9}$
$\theta = \frac{\pi}{9}, \frac{2(1)\pi}{3} + \frac{\pi}{9}, \frac{2(2)\pi}{3} + \frac{\pi}{9}$
($n = 0, 1, 2$ as there are maximum of 3 solutions)
$\theta = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

</td><td>

Many students are able to find the solutions $\frac{\pi}{9}$ and $\frac{5\pi}{9}$, but not able to find $\frac{7\pi}{9}$.
Many students are not able to connect the solutions to the cubic polynomial and realise that there can only be maximum 3 distinct solutions.
Many students didn't realise that $\cos\frac{5\pi}{9} = \cos\frac{13\pi}{9}, \cos\frac{7\pi}{9} = \cos\frac{11\pi}{9} \ldots$ as it's a periodic function. So there are only 3 distinct solutions.

Some students use general solutions incorrectly. And they have to substitute the values of n to give the 3 solutions.

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End of Paper