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Name:

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Teacher:.....



Pymble Ladies' College

MATHEMATICS EXTENSION 1

HSC ASSESSMENT TASK 3

Term 2 2024

General Instructions

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using black or blue pen
- Erasable pens are not to be used
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks 33

SECTION 1 – 5 marks

- Attempt Questions 1-5

SECTION II – 28 marks

- Attempt Questions 7-9
- Answer each question in the appropriate space in the Answer Booklet.

Mark	/33
Rank	/
Highest Mark	/33

Section I

Multiple Choice.

Use the multiple-choice answer sheet for Questions 1-5.

Each question is worth 1 mark in this section

1 Which of the following is equivalent to $\int \frac{3}{4+9x^2} dx$?

(A) $\frac{1}{2} \tan^{-1} \left(\frac{2x}{3} \right) + C$

(B) $\frac{1}{2} \tan^{-1} \left(\frac{3x}{2} \right) + C$

(C) $2 \tan^{-1} \left(\frac{2x}{3} \right) + C$

(D) $2 \tan^{-1} \left(\frac{3x}{2} \right) + C$

2 The slope field for a differential equation is sketched below.

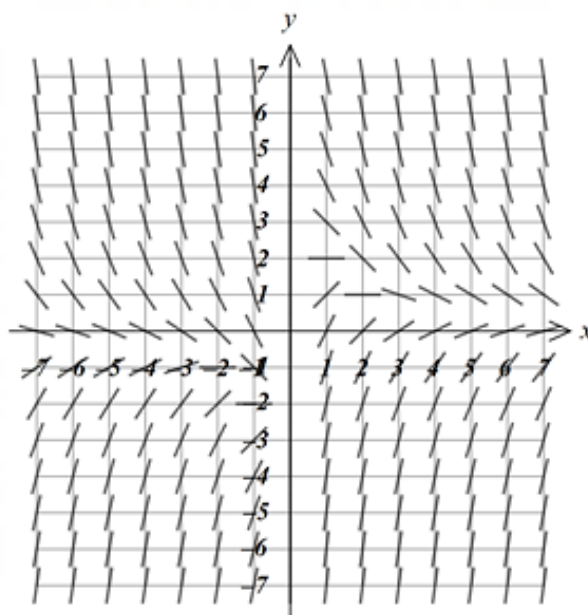
Which of the following is the correct differential equation for this slope field?

(A) $\frac{dy}{dx} = \frac{2}{x} - y$

(B) $\frac{dy}{dx} = \frac{2}{x} + y$

(C) $\frac{dy}{dx} = \frac{2}{y} - x$

(D) $\frac{dy}{dx} = \frac{2}{y} + x$



- 3 The equation $y = e^{kx}$ satisfies the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0$.
What are the possible values of k ?
- (A) $k = 3$ or $k = -4$
(B) $k = -1$ or $k = 12$
(C) $k = -3$ or $k = 4$
(D) $k = 1$ or $k = -12$
- 4 The curve $y = \sqrt{9 - (x-1)^2}$ is rotated about the x -axis to form a solid of revolution.
Which of the following integrals would give the volume of the solid?
- (A) $V = \pi \int_{-3}^3 (9 - (x-1)^2) dx$
(B) $V = \pi \int_{-3}^3 (9 - (x-1)^2)^2 dx$
(C) $V = \pi \int_{-2}^4 (9 - (x-1)^2) dx$
(D) $V = \pi \int_{-2}^4 (9 - (x-1)^2)^2 dx$
- 5 If $(g'(x))^2 = g(x)$ for all real x and $g(0) = 0$, $g(4) = 4$ and $g(1) = a$, which of the following gives the value of a ?
- (A) $a = 4$
(B) $a = 1$
(C) $a = \frac{1}{2}$
(D) $a = \frac{1}{4}$

End of Section I

Section II

(28 marks)

Answer these questions in the Answer Book provided.

Extra writing space is available at the end of each question.

Your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 7 (7 marks)

(a) Find $\int \cos 5x \sin x \, dx$. 2

(b) Find $\int \frac{1}{2} \cos^2 \left(\frac{x}{4} \right) dx$. 2

(c) Find the particular solution to $\frac{dy}{dx} = \frac{x+1}{y}$ given $y(0) = -2$. 3

Question 8 (10 marks)**Marks**

(a) Use the substitution $u = 1 - 2x$ to find $\int_{-4}^0 x\sqrt{1-2x} \, dx$. **3**

(b) Find the value of $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sin x - \cos x)^2 \, dx$. **3**

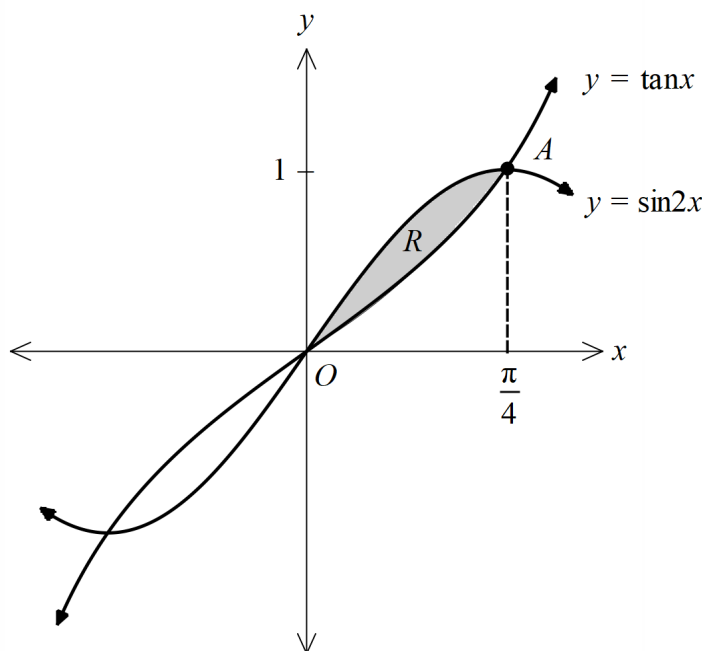
(c) (i) Differentiate $x \cos^{-1} x - \sqrt{1-x^2}$. **2**

(ii) Hence, evaluate $\int_{-1}^1 3 \cos^{-1} x \, dx$. **2**

- (a) The shaded region R is bounded by the curve $y = \sin 2x$ and $y = \tan x$ as shown in the diagram.

4

A has coordinates $\left(\frac{\pi}{4}, 1\right)$.



Show that the volume of the solid of revolution formed when R is rotated about the x -axis is $\frac{\pi}{8}(3\pi - 8)$ units³.

Question 9 continues on page 6

- (b) A team of ecologists released 500 fish into a marine park with a maximum carrying capacity of 10 000. It was found that the number tripled during the first year. The fish population, N , in the marine park after t years is modelled by the logistic equation:

$$\frac{dN}{dt} = kN(10\,000 - N) \quad \text{where } k \text{ is a constant.}$$

- (i) Given $\frac{10\,000}{N(10\,000 - N)} = \frac{1}{N} + \frac{1}{10\,000 - N}$, solve the differential equation 3

to show that the fish population after t years is $N = \frac{10\,000}{1 + 19e^{-10\,000kt}}$.

- (ii) Show that $k = \frac{1}{10\,000} \ln\left(\frac{57}{17}\right)$. 2

- (iii) Correct to the nearest month, how long will it take for the fish population in the marine park to reach 7000 fish? 2

End of Assessment