Mathematics Extension 1

HSC Marking Feedback 2022

Question 11

Part (a) (i)

Students should:

- know that when a vector is multiplied by a scalar the result is a vector
- be able to add vectors in two dimensions.

In better responses, students were able to:

- find the scalar multiple of a vector before adding it to the other vector
- work separately with the components of each vector.

Areas for students to improve include:

- using the correct notation for vectors
- avoiding arithmetic errors.

Part (a) (ii)

Students should:

- understand that the dot product of two vectors is a scalar
- select and use the most appropriate form of the equation for dot product of vectors from the Reference Sheet.

In better responses, students were able to:

- show full working before arriving at the numerical answer.

Areas for students to improve include:

- using the most appropriate form of the equation for the dot product of vectors from the Reference Sheet
- understanding that the dot product of two vectors is a scalar
- completing a variety of examples of the application of the dot product, with emphasis on the fact that the result is a number
- avoiding arithmetic errors when adding and subtracting numbers.

Part (b)

Students should:

- realise that integration by substitution is used to simplify the integration of a more complex expression
- be able to arrive at an integrand involving only the substituted variable
- change the limits of the integral to match the boundaries of the substituted variable.

In better responses, students were able to:

- clearly set out the substitution and find dx in terms of du
- transform the integrand and its limits in terms of the substituted variable
- find the correct primitive and apply the limits correctly.

Areas for students to improve include:

- understanding how the derivative of the substituted variable is used to facilitate the formation of the transformed integral
- ensuring that the correct limits are applied to the transformed integrand
- evaluating integrals involving variables with negative fractional indices.

Part (c)

Students should:

- be able to use the binomial theorem to expand the given binomial expression raised to a given power
- know how to use the general term formula to find the required terms in the expansion of a binomial.

In better responses, students were able to:

- correctly expand the given binomial to include only the terms required
- use the general term formula to correctly identify the coefficients of the required terms.

Areas for students to improve include:

- expanding binomials in which one of the terms is negative
- using the general term formula to find individual terms without expanding the binomial entirely
- using index laws with fractions (both positive and negative) raised to odd and even powers.

Part (d)

Students should:

- understand that the dot product of 2 perpendicular vectors is zero or that the product of their gradients is equal to –1
- be able to apply the dot product.

In better responses, students were able to:

- state that the dot product of 2 perpendicular vectors is zero

- apply the dot product rule to arrive at the correct quadratic equation
- correctly factorise the quadratic to arrive at the required values of a.

Areas for students to improve include:

- expanding bracketed terms
- factorising and solving quadratic equations.

Part (e)

Students should:

- understand that the sum or difference of a sine or cosine function with the same frequency can be written as a single sine or cosine function with the same frequency
- be able to find the amplitude and horizontal translation of the resulting single trigonometric function.

In better responses, students were able to:

- use the coefficients of the given functions to find the amplitude of the resulting trigonometric function
- use the coefficients of the given functions to find the auxiliary angle and the relevant quadrant.

Areas for students to improve include:

- using the Reference Sheet to expand compound angle functions such as $R\sin(x+\alpha)$
- equating coefficients to find the correct values of R and α for different types of auxiliary angle problems, observing restrictions on the quadrant of the argument.

Part (f)

Students should:

- be able to solve an inequality with a variable in the denominator.

In better responses, students were able to:

- multiply both sides of the inequality by the square of the denominator, solve the quadratic inequality and provide the correct solution
- find the critical points and test the inequality in each region to obtain the correct solution.

Areas for students to improve include:

- understanding and using different methods for solving inequalities with a variable in the denominator, such as multiplying by the square of the denominator, finding critical values and testing regions, considering options involving the polarity of the denominator and sketching curves to find the solutions
- not using the method of multiplying both sides by the denominator when solving an inequality
- understanding that the solution cannot include the value of the variable that makes the denominator zero
- showing clear working

- writing the solution of a quadratic inequation correctly.

Question 12

Part (a)

Students should:

- be able to draw all 3 slopes at the given points clearly
- read the question carefully.

In better responses, students were able to:

- draw the 3 slopes correctly
- demonstrate that the slope at P was steeper than that at Q.

Areas for students to improve include:

- showing working for their substitution into the required derivative
- drawing relative slopes correctly with respect to their steepness.

Part (b)

Students should:

- be able to apply the Pigeonhole Principle.

In better responses, students were able to:

- demonstrate the Pigeonhole Principle correctly and provide a correct conclusion.

Areas for students to improve include:

- applying the Pigeonhole Principle in a variety of situations
- indicating a remainder after division and, hence, concluding the need of an extra 'hole' (ceiling function) or stating the minimum number of 'holes' required.

Part (c)

Students should:

- be able to differentiate a given function using the product rule
- be able to substitute the given x – coordinate into the derived equation to find the slope of the tangent
- be able to use the given x – and y – coordinates to find the equation of the tangent.

In better responses, students were able to:

- differentiate the given function, find the slope and the correct equation of the tangent.

Areas for students to improve include:

- using the Reference Sheet to differentiate $\tan^{-1} x$ correctly
- evaluating $\tan^{-1} x$ when $x = 1$
- reading the question carefully.

Part (d) (i)

Students should:

- be able to separate the variables in the differential equation
- be able to integrate to obtain T as a function involving an exponential equation
- be able to use the given conditions to find A and k.

In better responses, students were able to:

- recognise $T_1 = 12$ and substitute early
- separate the variables and integrate to obtain an equation in exponential form
- use the given conditions to find A and k.

Areas for students to improve include:

- separating the variables in a differential equation
- integrating to obtain T as a function involving an exponential equation with $A = e^c$
- using the given conditions to evaluate the relevant equations to find A and k.

Part (d) (ii)

Students should:

- substitute for T in the given exponential equation to arrive at the correct answer.

In better responses, students were able to:

- substitute for T and arrived at the correct time to the nearest minute.

Areas for students to improve include:

- using their calculator to obtain an answer correct to the nearest minute with respect to time.

Part (e)

Students should:

- be able to use the binomial distribution result to obtain the expected score of the game
- be able to find the expected score of one ball and then multiply that to obtain the result of 4 balls
- read the question carefully.

In better responses, students were able to:

- use the binomial distribution result and multiply that by the points either earned or deducted to arrive at the expected score
- find the expected score of one ball.

Areas for students to improve include:

- identifying the correct formula from the Reference Sheet for the binomial distribution.

Part (f)

Students should:

- establish the base case
- write a statement assuming that k is true
- use the assumption statement to prove the inductive step.

In better responses, students were able to:

- prove the base case was true, assume that k was true and prove the inductive step with a conclusion
- substitute and establish the base case without numerical errors
- manipulate the algebra in the inductive step without error.

Areas for students to improve include:

- showing the statement is true for the base case
- providing a proof by mathematical induction including all necessary steps.

Question 13

Part (a)

Students should:

- know that two vectors are perpendicular if their dot product is equal to zero
- use the correct notation for vectors and magnitude of vectors
- complete a proof with logical steps and reasoning as required.

In better responses, students were able to:

- rewrite direction vectors, for example $\overrightarrow{BH} = \underset{\sim}{h} - \underset{\sim}{b}$
- state the property of perpendicular vectors
- provide reasons for their steps (as required)
- recognise $\underset{\sim}{a}.\underset{\sim}{a} = \left|\underset{\sim}{a}\right|^2$ and apply this knowledge to prove the given statement.

Areas for students to improve include:

- knowing how to find and use the dot product of two vectors
- showing all the steps in a proof with reasons (as required)
- understanding and using vector notation.

Part (b)

Students should:

- write an integral expression for the required volume with the correct limits
- use the double angle formula for $\cos 2\theta$ correctly

- be able to integrate trigonometric functions
- substitute the limits of integration into the primitive of the volume
- solve an equation to find the value of the unknown.

In better responses, students were able to:

- provide the correct integral for the volume
- correctly use the double angle formula
- integrate the function and correctly substitute the limits of integration
- solve the resultant equation for k.

Areas for students to improve include:

- knowing and using the volume of solids of revolution formula
- using the Reference Sheet to change $sin^2 kx$ to $\cos 2kx$
- simplifying an integral by moving a constant to the front of the integral sign
- knowing how to integrate trigonometric functions.

Part (c)

Students should:

- understand the conditions on domain and range of functions and their inverses
- use the property $g\big(f(x)\big) = x$
- test the possibility of $f(x)$ having an inverse.

In better responses, students were able to:

- use graphs as a form of explanation connecting functions and their inverses
- find one value of x for which $g\big(f(x)\big) \neq x$
- recognise $f(x)$ as failing the horizontal line test or not being a one-to-one function, hence no inverse in the given domain of $f(x)$
- show the domain and range of $f(x)$ does not match the range and domain of $g(x)$ respectively, hence no inverse in the given domain of $f(x)$.

Areas for students to improve include:

- reading the question carefully and extracting the appropriate information
- providing clear logical explanations
- understanding the tests required for an inverse to exist in a given domain.

Part (d)

Students should:

- write an expression for a monic cubic polynomial and find its derivative
- state the relationships between roots and the coefficients of a polynomial of degree 3.

In better responses, students were able to:

- find an expression for the monic cubic polynomial $P(x)$ and its derivative $P'(x)$
- expand $(\alpha + \beta + \gamma)^2$ to obtain an expression for $\alpha^2 + \beta^2 + \gamma^2$
- obtain an expression for $P'(\alpha)$ + $P'(\beta)$ + $P'(\gamma)$
- use these expressions to find the required value.

Areas for students to improve include:

- reading the question carefully to determine what equations or expressions are required
- building the answer step by step from the provided statements
- stating known facts about the polynomial.

Part (e) (i)

Students should:

- understand the parameters associated with binomial and normal distributions
- use the Reference Sheet formulae to find the mean, standard deviation and z - score
- use either the normal distribution graph or the z - score table to find the probability.

In better responses, students were able to:

- find the mean and standard deviation of the sample
- calculate the required z - score
- use the table of z - scores provided or use the normal distribution graph on the Reference Sheet to find the required probability.

Areas for students to improve include:

- familiarising themselves with the information provided on the Reference Sheet
- determining the values to be substituted into the z - score formula
- understanding the significance of the score obtained from the z - score table
- knowing how to read values from the z - score table.

Part (e) (ii)

Students should:

- understand normal and binomial distributions
- understand the significance of sample size.

In better responses, students were able to:

- relate the size of the sample to the validity of the method used.

Areas for students to improve include:

- understanding the importance of sample size
- providing specific reasons rather than general comments.

Question 14

Part (a)

Students should:

- use integration to solve a differential equation.

In better responses, students were able to:

- separate the variables correctly
- find the primitives of each integrand
- evaluate the constant of integration
- determine whether the solution involved the positive or negative case of $|y|$
- simplify the resulting expression.

Areas for students to improve include:

- developing the algebraic skills necessary to successfully separate variables and simplify expressions
- correctly integrating $\frac{1}{y}$ to obtain $ln|y|$
- understanding how to determine the solution from an expression involving $|y|$ and justifying the use of the positive or negative case.

Part (b)

Students should:

- use their knowledge of vectors to prove the required result.

In better responses, students were able to:

- draw a large, clearly labelled diagram which displayed each of the vectors mentioned in the question
- use the concept of the perpendicular distance from a point to a line or Pythagoras' Theorem to explain clearly why $|\vec{u} - \lambda_0 \vec{v}| \leq |\vec{u} - \lambda \vec{v}|$.

Areas for students to improve include:

- drawing large, clearly labelled diagrams
- understanding where the projection lies on the diagram
- understanding what $\vec{u} - \lambda \vec{v}$ and $|\vec{u} - \lambda \vec{v}|$ represent geometrically
- providing clear explanations.

Part (c)

Students should:

- use knowledge of projectile motion to solve a problem.

In better responses, students were able to:

- demonstrate an orderly progression through the stages of the problem

- find the time of flight of the projectile
- find the horizontal distance travelled in this time by both the projectile and the target
- equate the distances to find an expression for the location of the point of impact
- determine the maximum range of the projectile
- use calculus to show that the point of impact was within 37% of the maximum range.

Areas for students to improve include:

- planning their approach to this type of question
- developing the algebraic skills to simplify the different trigonometric expressions
- understanding which of the given equations represented horizontal and which represented vertical motion
- understanding how to find the time of flight
- recognising that the maximum range for the point of impact was not necessarily the maximum range of the projectile
- recognising that calculus was needed to demonstrate progress to the supplied solution.

Part (d)

Students should:

- use a normal distribution to approximate a binomial distribution.

In better responses, students were able to:

- recognise the values needed and use them to find the relevant expressions for $E(x)$ and $Var(x)$
- find the relevant z - score to represent the probability
- put these into the formula for z – score to generate a quadratic equation
- solve the quadratic equation to determine the number of tickets which should be sold.

Areas for students to improve include:

- understanding how to use each of the pieces of supplied information and what they represented with respect to $E(x)$ and $Var(x)$
- reading the question carefully, and using all columns of the supplied z - score table
- developing algebraic skills to reorganise an expression for z - score into a quadratic equation which can be solved.