Mathematics Extension 1

HSC Marking Feedback 2023

General feedback

Students should:

- show relevant mathematical reasoning and/or calculations
- read the question carefully to ensure that they do not miss important components of the question
- have a clear understanding of key words in the question and recognise the intent of the question and its requirements, such as show, solve, evaluate, hence, calculate, derive
- use the Reference Sheet where appropriate
- ensure the solution is legible and follows a clear sequence
- engage with any stimulus material provided and refer to it in their response when required by the question
- check their solution answers the question
- round off numerical solutions only at the final step of the solution
- construct graphs neatly, with precision and display all relevant information as required by the question
- interpret information presented in graphs across a range of contexts
- understand when to use relevant calculator functions
- carefully note any information in the questions which supplies units of measurement.

Section II

Question 11 (a)

In better responses, students were able to:

- change the subject of an equation
- eliminate the parameter and find the Cartesian equation.

Areas for students to improve include:

- solving simultaneous equations.

Question 11 (b)

In better responses, students were able to:

- determine the number of arrangements using factorial notation
- eliminate the repeated letters by division.

Areas for students to improve include:

- identifying repeated letters and eliminating these from the number of arrangements.

Question 11 (c)

In better responses, students were able to:

- apply both the factor theorem and remainder theorem to form equations
- solve simultaneous equations.

Areas for students to improve include:

- applying the remainder theorem correctly
- applying the factor theorem correctly.

Question 11 (d)

In better responses, students were able to:

- use knowledge of inverse trigonometric functions
- rewrite the integrand in a suitable form.

Areas for students to improve include:

- rewriting the integrand to match the Reference Sheet
- recognising the integrand as an inverse trigonometric function.

Question 11 (e)

In better responses, students were able to:

- use the compound angle formula to transform the sum of sin- and cos
- use the relevant t formula to transform the sum of sin and cos-
- use the relevant double angle formula to solve a trigonometric equation.

Areas for students to improve include:

- nominating the correct compounding formula for the equation
- solving a quadratic equation from a t formula substitution
- solving a trigonometric equation over a given domain.

Question 11 (f) (i)

In better responses, students were able to:

- recognise the values needed to calculate $Var(x)$.

Areas for students to improve include:

- connecting the information supplied to a relevant formula.

Question 11 (f) (ii)

In better responses, students were able to:

- calculate the relevant z -score use the z -score table to find the required probability.

Areas for students to improve include:

- determining the values to substitute into the z -score formula
- knowing how to read from the z -score table.

Question 12 (a)

In better responses, students were able to:

- obtain the correct integrand in terms of u
- change the limits in terms of u
- substitute correctly in the anti-derivative to arrive at the correct solution.

Areas for students to improve include:

- using brackets around their factors, so that a correct simplification of terms can occur
- changing the limits with variable x to limits with variable u
- integrating fractional powers correctly.

Question 12 (b)

In better responses, students were able to:

- establish the base assumption steps
- use the exponential laws correctly in the LHS of the inductive step
- arrive at the RHS from the LHS and write a correct conclusion.

Areas for students to improve include:

- testing for $n = k + 1$ on both sides of the equation
- using exponential laws correctly
- equating the LHS to the RHS.

Question 12 (c) (i)

In better responses, students were able to:

- demonstrate the understanding of binomial probability correctly.

Areas for students to improve include:

- applying binomial probability correctly.

Question 12 (c) (ii)

In better responses, students were able to:

- demonstrate the understanding of binomial probability taking into consideration one of the probabilities were not in use.

Areas for students to improve include:

- knowing how to apply the complimentary event in binomial probability.

Question 12 (d)

In better responses, students were able to:

- use the ${}^{n}C_{r} = {}^{n}C_{n-r}$ notation successfully to combine the first two terms of the LHS.

Areas for students to improve include:

- using the symmetry of Pascals Triangle
- applying ${}^{n}C_{r} = {}^{n-1}C_{r-1} + {}^{n-1}C_{r}$ successfully.

Question 12 (e)

In better responses, students were able to:

- obtain the volume of the hyperbola correctly
- obtain the volumes of the cylinder correctly
- calculate the sum of both volumes correctly.

Areas for students to improve include:

- analysing the question by combining volumes as the sum of 2 simpler volumes
- knowing logarithmic laws in terms of ln 0 are undefined
- combining logarithmic laws to obtain a simpler fraction
- knowing how to expand a binomial that has fractional powers
- knowing how to solve for x if it is in the denominator.

Question 13 (a) (i)

In better responses, students were able to:

- find a correct expression for $\frac{dV}{dh}$
- use their knowledge of related rates to link $\frac{dV}{dh}$ and $\frac{dV}{dt}$ to find $\frac{dh}{dt}$
- simplify the algebraic expression to obtain the required result.

Areas for students to improve include:

- showing all steps of their processing.

Question 13 (a) (ii)

In better responses, students were able to:

- solve the differential equation from part (i) to obtain an expression relating time and the height of water in the tank
- clearly explain that when the tank is full, the height of the water will equal the radius of the tank.

Areas for students to improve include:

- recognising the need to use the result generated in part (i) of the question
- separating the variables of the differential equation correctly so that the terms of integration match the variable being integrated
- finding the value of any constants generated in their integration
- showing all steps of their processing.

Question 13 (a) (iii)

In better responses, students were able to:

- model the situation to develop a new differential equation representing the rate at which water was draining from the tank
- use their knowledge of related rates to link $\frac{dV}{dh}$ and their new equation $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$
- integrate the rate to obtain the required result.

Areas for students to improve include:

- using the information in the question to determine a new expression for $\frac{dV}{dt}$ when the tank is draining
- separating the variables of the differential equation correctly so that the terms of integration matched the variable being integrated
- finding the value of any constants generated in their integration
- showing all steps of their processing.

Question 13 (b) (i)

In better responses, students were able to:

- clearly show that $cos\theta = \frac{1}{\sqrt{5}}$
- use the result $cos\theta = \frac{1}{\sqrt{5}}$ and the horizontal displacements of the two projectiles to achieve the required result.

Areas for students to improve include:

- using a diagram or trigonometric identities to find a value for $cos\theta$ in exact form
- recognising that the horizontal displacements must be equal when the projectiles collide
- showing all steps of their processing.

Question 13 (b) (ii)

In better responses, students were able to:

- use information from part (i) and the vertical displacements of the projectile to achieve the required result.

Areas for students to improve include:

- recognising that the vertical displacements must be equal when the projectiles collide
- finding an exact value for $sin\theta$ using the methods of part (i)
- showing all steps of their processing.

Question 13 (b) (iii)

In better responses, students were able to:

- find expressions for the velocities of each projectile
- use the $\underset{\sim}{v}_A . \underset{\sim}{v}_B = 0$ result for the projectiles having perpendicular paths to obtain an equation which can be solved
- solve the resulting quadratic equation to obtain the required result.

Areas for students to improve include:

- differentiating with respect to time in order to find the velocities of each projectile
- recognising that H is a constant, its derivative is 0 and that it does not feature as part of the expression for the vertical velocity of projectile *B*
- knowing that the dot product of the velocities of the projectiles will be 0 since their paths are perpendicular at the point of collision
- using algebraic skills to simplify the resulting quadratic equation formed from the dot product
- factorising or using the quadratic formula to solve the equation
- showing all steps of their processing.

Question 13 (b) (iv)

In better responses, students were able to:

- find the time to maximum height for projectile A
- use time to find the maximum height
- express their maximum height in terms of H.

Areas for students to improve include:

- recognising that maximum height will occur when the vertical velocity is 0
- using the correct expression for maximum height
- substituting their value for time and using algebraic skills to simplify the resulting expression
- understanding the intent of the phrase 'Give your answer in terms of H'.

Question 14 (a) (i)

In better responses, students were able to:

- provide clear explanations
- express answers clearly and unambiguously using appropriate mathematical notations.

Areas for students to improve include:

- knowing what the term 'explain' means in the context of the problem
- using proper mathematical terms.

Question 14 (a) (ii)

In better responses, students were able to:

- correctly use the chain rule to find an expression for the gradient of the inverse function
- relate the tangents to corresponding points on a function and its inverse function.

Areas for students to improve include:

- relating the coordinates of a point on a function to the coordinates of a corresponding point on the inverse of that function
- finding derivatives of inverse functions using algebraic methods as well as the geometrical approach.

Question 14 (b) (i)

In better responses, students were able to:

- substitute and use the correct algebraic techniques to arrive at the given formula.

Areas for students to improve include:

- showing full working before arriving at the required equation
- performing algebraic manipulations with directional signs correctly

- realising that finding points of intersection involves solving simultaneous equations.

Question 14 (b) (ii)

In better responses, students were able to:

- correctly interpret information given in graphical form
- understand that the question required locating a double root
- recognise from the graph the constraints on the variables x and c.

Areas for students to improve include:

- knowing how and when to use the concept of multiple roots in polynomials
- demonstrating how the interaction of geometric shapes can be depicted on a graph
- estimating graphically the values of variables to meet specific conditions.

Question 14 (c) (i)

In better responses, students were able to:

- understand the significance of the given property and use it to arrive at the required proof
- demonstrate skills in working with vectors and arrive at the required result using a variety of equivalent proofs.

Areas for students to improve include:

- constructing and clearly labelling diagrams with all the relevant information
- using correct vector notation
- knowing that in 'show that...' questions the last line should be what they are asked to show
- knowing how to find the projection of a vector and its application in unfamiliar situations.

Question 14 (c) (ii)

In better responses, students were able to:

- make effective use of the information in part (i) to facilitate the solution of the problem
- use a combination of skills in trigonometry, calculus, and vectors to solve the problem.

Areas for students to improve include:

- writing down all they know about the question in the hope that may trigger some key thought processes that they may earlier not have realised
- improving problem-solving skills by practising more complex questions in vector applications
- knowing how the area and its rate of changes, varies with the angle t.