Mathematics Extension 1

HSC marking feedback 2024

General feedback

Students should:

- show relevant mathematical reasoning and/or calculations
- read the question carefully to ensure that they do not miss important components of the question
- have a clear understanding of key words in the question and recognise the intent of the question and its requirements, such as show, solve, evaluate, hence, calculate, derive
- use the Reference Sheet where appropriate
- ensure the solution is legible and follows a clear sequence
- engage with any stimulus material provided and refer to it in their response when required by the question
- check their solution answers the question
- round off numerical solutions only at the final step of the solution
- construct graphs neatly, with precision and display all relevant information as required by the question
- interpret information presented in graphs across a range of contexts
- understand when to use relevant calculator functions
- carefully note any information in the questions which supplies units of measurement.

Section II

Question 11 (a) (i)

In better responses, students were able to:

- correctly perform algebraic operations on vectors in component form.

Areas for students to improve include:

- solving vector problems involving multiplication by a scalar and vector subtraction
- making proper use of vector notation.

Question 11 (a) (ii)

In better responses, students were able to:

- correctly apply the scalar (dot) product using vector components.

Areas for students to improve include:

- recognising that the scalar (dot) product of two vectors is a scalar and not a vector.

Question 11 (b)

In better responses, students were able to:

- factorise the quadratic and correctly test the regions to determine the solution
- use acceptable notation to display their answer.

Areas for students to improve include:

- demonstrating awareness of the inequality sign in the question
- using graphs to solve quadratic inequalities
- demonstrating proper use of inequalities or brackets to specify intervals.

Question 11 (c)

In better responses, students were able to:

- transform the integral into a simpler form in terms of the given substitution
- correctly simplify the integrand after substitution
- integrate correctly and express their solution in terms of the original variable.

Areas for students to improve include:

- using brackets around binomial terms in the integrand after substitution
- performing integration of terms with fractional indices
- simplifying expressions using the index laws
- expressing their solution in terms of the original variable and not in terms of the substituting variable.

Question 11 (d)

In better responses, students were able to:

- separate the variables in the differential equation
- integrate both sides of the differential equation correctly
- give the answer in the required form ensuring that the constant term of integration was correctly handled.

Areas for students to improve include:

- understanding that differential equations should not be integrated where there are multiple variables on one side
- being mindful of the implications of the restrictions given in the question such as $y > 0$
- using logarithmic laws, such as when solving for y when $lny = f(x)$ and $f(x) = \frac{x^2}{2} + c$.

Question 11 (e)

In better responses, students were able to:

- recognise that $\arcsin(x)^5 = sin^{-1}(x)^5$
- choose the appropriate differentiation rule to be used and applied it correctly.

Areas for students to improve include:

- taking care when simplifying expressions using index laws.

Question 11 (f)

In better responses, students were able to:

- apply the chain rule with relevant pronumerals
- recognise that $\frac{dV}{dt} = 10\ cm^3 s^{-1}$
- find the correct expression for $\frac{dV}{dr}$
- show all necessary working to arrive at the required result.

Areas for students to improve include:

- practising problems involving related rates
- using correct notations to represent rates, for example, $\frac{dV}{dr}$ instead of V'
- showing all necessary steps and ensuring that the last line matches exactly what is being asked in the question.

Question 11 (g)

In better responses, students were able to:

- recognise that the required area is found by taking the difference of the area between two curves
- calculate the area enclosed by the line $y = x$ and the x-axis by either using the correct definite integral or by using the area of a triangle formula
- calculate the area enclosed by the curve $y = sinx$ and the x-axis by using the correct definite integral.

Areas for students to improve include:

- distinguishing between finding the area under a curve and finding the volume when the region under a curve is rotated about any axis
- evaluating or simplifying fractions such as $\frac{\left(\frac{\pi}{2}\right)^2}{2}$
- dealing with double negatives that emerge when expanding brackets during algebraic manipulations and evaluation.

Question 12 (a)

In better responses, students were able to:

- apply the scalar (dot) product to vectors expressed in component form
- demonstrate an understanding of the properties of perpendicular vectors
- apply the Factor Theorem to solve a cubic polynomial equation
- use division of polynomials to factorise a cubic polynomial equation.

Areas for students to improve include:

- applying the scalar (dot) product rule to arrive at the correct cubic polynomial equation
- understanding and applying the division of polynomials
- factorising a cubic equation to arrive at possible values of a.

Question 12 (b)

In better responses, students were able to:

- calculate the volume of the solid of revolution formed by rotating a region in the plane about the x-axis
- correctly integrate the function and manipulate the limits of integration to obtain the correct numerical solution.

Areas for students to improve include:

- understanding how to find the volume of the solid of revolution using $V = \pi \int y^2 \, dx$.

Question 12 (c)

In better responses, students were able to:

- recognise the values needed to calculate the standard deviation
- calculate the z-score from the given data
- use the normal distribution table provided to find the required probability.

Areas for students to improve include:

- determining which values to substitute into the z-score formula.

Question 12 (d)

In better responses, students were able to:

- establish the base case
- use the assumption statement to prove the inductive step by manipulating the algebra without error.

Areas for students to improve include:

- understanding the base case occurs when $n = 1$
- rearranging the assumption to substitute into the inductive step
- setting out a proof by mathematical induction including all necessary steps.

Question 12 (e)

In better responses, students were able to:

- solve inequalities containing both absolute value and a variable in the denominator
- identify that solving with absolute values results in two cases
- find the critical points by considering the two cases and testing each region to obtain the correct solution
- solve quadratic equations involving inequalities.

Areas for students to improve include:

- understanding the sign implications if there is a variable in the denominator
- strengthening their algebraic skills in multiplying by the square of the denominator
- understanding that the solution cannot include the value of the variable that makes the denominator zero.

Question 13 (a) (i)

In better responses, students were able to:

- explain explicitly that there exists a horizontal asymptote with S and T opposite it, hence S cannot reach T.

Areas for students to improve include:

- stating that there is a horizontal asymptote at $y = 2000$
- clearly identifying that S and T are opposite sides of the asymptote.

Question 13 (a) (ii)

In better responses, students were able to:

- sketch a smooth concave down graph that follows the horizontal asymptote.

Areas for students to improve include:

- sketching smooth curves
- making sure the curve follows the horizontal asymptote sufficiently
- ensuring their curves do not cross the slopes perpendicularly.

Question 13 (a) (iii)

In better responses, students were able to:

- find the second derivative correctly with respect to P
- equate the second derivative to 0 and solve correctly to get $P = 1000$
- sketch the first derivative correctly as a concave down parabola, then use the turning point to find the maximum value of P.

Areas for students to improve include:

- deriving the first derivative with respect to P rather than t
- applying the implications of the vertex in parabolas
- using quadratic theory $(x = -b/2a)$ to reason the maximum growth rate is half the carrying capacity.

Question 13 (b) (i)

In better responses, students were able to:

- prove with correct trigonometric substitution that LHS = RHS
- use the double angle formulae correctly to arrive at LHS.

Areas for students to improve include:

- knowing their trigonometric identities
- squaring a binomial involving fractions correctly
- working with difficult algebraic expressions.

Question 13 (b) (ii)

In better responses, students were able to:

- use part (i) correctly to substitute the given integrand
- find the correct anti-derivative
- evaluate the limits correctly to arrive at the correct answer.

Areas for students to improve include:

- substituting part (i) correctly in the given integrand
- finding the anti-derivative of $\cos 4x$ correctly
- correctly evaluating a trigonometric function.

Question 13 (c)

In better responses, students were able to:

- understand and apply the projection formula correctly
- obtain both k and p in terms of components of $\underset{\sim}{x}$
- solve simultaneous equations
- arrive at the correct factorised answer of the components.

Areas for students to improve include:

- using the projection formula
- recognising the component must be written as different variables, for example, $\underset{\sim}{x} = \begin{pmatrix} u \\ v \end{pmatrix}$
- solving simultaneous equations with fractions
- understanding the difference between components and vectors.

Question 13 (d)

In better responses, students were able to:

- differentiate correctly with respect to u
- expand u^2correctly
- use algebra to factorise the numerator and denominator, to obtain a simple fraction which could be applied to an inverse trigonometric integration
- substitute u back in terms of e^xas their final answer.

Areas for students to improve include:

- differentiating correctly with respect to u
- expanding u^2 correctly as an exponential function
- recognising algebraic manipulations, in particular factorisation to present the integrand as a recognisable simple integration
- rewriting an anti-derivative in terms of e^x.

Question 14 (a)

In better responses, students were able to:

- separate the variables
- integrate correctly and find the value of the constant
- determine the domain and range for the function.

Areas for students to improve include:

- understanding how to integrate terms such as e^{-y}
- ensuring that they include a constant when completing an indefinite integral
- taking care with their algebra, especially when calculating with minus signs
- understanding when an absolute value sign may or may not be needed when using natural logarithms
- understanding how to find the domain and range of a function such as $y = -ln(2 - e^x)$
- understanding the difference between the symbols '[' and '(' when using interval notation
- leaving answers in exact form when they are simple terms such as $ln2$.

Question 14 (b)

In better responses, students were able to:

- demonstrate an understanding of what was required of the original function in order to have an inverse function
- differentiate the function correctly
- recognise that, as the function could have been monotonic increasing or decreasing, they needed to investigate both cases where the derivative may have been exclusively positive or negative
- understand that the numerator of the derivative was a quadratic in x and they should investigate the concavity and translation to determine possible values of k.

Areas for students to improve include:

- explaining the requirement for a function to have an inverse function
- differentiating complex functions
- recognising how to show that a derivative is always positive or negative.

Question 14 (c) (i)

In better responses, students were able to:

- explain concisely, using mathematical language, that both of $tan^{-1}(3x)$ and $tan^{-1}(10x)$ are monotonically increasing functions with a range of $-\frac{\pi}{2} < f(x) < \frac{\pi}{2}$ and so their sum would also be monotonically increasing with a range of $-\pi < f(x) < \pi$
- show that the derivative of $tan^{-1}(3x) + tan^{-1}(10x)$ is always positive and so $tan^{-1}(3x) + tan^{-1}(10x)$ would be monotonically increasing
- graph $y = tan^{-1}(3x) + tan^{-1}(10x)$ neatly, clearly showing the range.

Areas for students to improve include:

- using clear mathematical language to explain a concept
- addressing all components of the question
- understanding that $y = \theta$ would represent a horizontal line through which $y = tan^{-1}(3x) + tan^{-1}(10x)$ would pass only once.

Question 14 (c) (ii)

In better responses, students were able to:

- take the tangent of both sides of the equation and use the relevant compound angle formula to develop a quadratic equation
- solve the quadratic equation to find two solutions
- justify why one of the solutions was not valid in this situation and identify the correct solution.

Areas for students to improve include:

- understanding that the use of the calculator to provide a bold answer is not equivalent to solving an equation
- using the compound angle formula
- factorising complex algebra
- recognising that part (i) of the question clearly stated that the equation would have only one solution.

Question 14 (d)

In better responses, students were able to:

- find an expression for D or D^2
- recognise that the use of D^2 would make their ensuing differentiation easier
- correctly differentiate their expressions for D or D^2
- understand how to show that, for $D(t)$ to be always increasing, their derivative must always be positive.

Areas for students to improve include:

- simplifying algebra with complex expressions for both distance and its derivative
- using notation to represent $\frac{d\left(\left(D(t)\right)^2\right)}{dt}$, $\frac{d\left(D(t)\right)}{dt}$ or other similar derivatives
- differentiating complex expressions, especially those including square root signs
- ensuring that they are differentiating with respect to the correct variable
- showing that an expression such as the derivative is always positive
- recognising a quadratic expression and using the discriminant to show that the expression must always be positive
- understanding that, in a question requiring them to show a result, they should derive the result from the information given.