Mathematics Extension 1

HSC marking feedback 2025

General feedback

Students should:

- show relevant mathematical reasoning and/or calculations
- read the question carefully to ensure they do not miss important components of the question
- have a clear understanding of key words in the question and recognise the intent, such as show, solve, evaluate, hence, calculate, derive
- use the Reference Sheet where appropriate
- ensure the solution is legible and follows a clear sequence
- engage with any stimulus material provided and refer to it in their response when required by the question
- check their solution answers the question
- round off numerical solutions only at the final step of the solution
- construct graphs neatly, with precision, and display all relevant information as required by the question
- interpret information presented in graphs across a range of contexts
- understand when to use relevant calculator functions
- carefully note any information in the questions which supplies units of measurement.

Section II

Question 11 (a)

In better responses, students were able to:

- interchange x and y
- use algebraic skills to correctly obtain the inverse function.

Areas for students to improve include:

- recognising the difference between the notation $f^{-1}(x)$ and $[f(x)]^{-1}$
- taking care with negative signs when manipulating the algebraic expression.

Question 11 (b)

In better responses, students were able to:

- use the double angle formula to simplify their expression
- recognise the domain that restrictions had on their solution.

Areas for students to improve include:

- not dividing through by $\sin\theta$ and hence losing some solutions
- using the most logical method for solving an equation, rather than using inefficient methods such as t-formula or half-angles.

Question 11 (c)

In better responses, students were able to:

- recognise the trigonometric product as sums manipulation
- use the Reference Sheet to obtain the correct integrand.

Areas for students to improve include:

- choosing the most efficient method to solve trigonometric equations, rather than writing $\sin(3x)$ as $\sin(2x + x)$ and using the double-angle formula
- using the Reference Sheet to correctly integrate their integrands.

Question 11 (d)

In better responses, students were able to:

- recognise the basic shape of the inverse cosine function
- recognise what effect the numeric values within the inverse function had on the transformations of the graph
- indicate the y-intercept.

Areas for students to improve include:

- sketching larger and more clearly labelled graphs
- recognising the features of the function, for example, not indicating that the graph becomes horizontal or beyond at the y-intercept
- understanding the shape and position of a base inverse cosine function.

Question 11 (e)

In better responses, students were able to:

- identify that one vector is a (positive) scalar multiple of the other.

Areas for students to improve include:

- using the mark allocation as a guide to the amount of working required
- identifying that the question is asking for a value for m and not for perpendicular vectors.

Question 11 (f)

In better responses, students were able to:

- record their expression as a fraction with its lowest common denominator
- use the Reference Sheet to obtain formulas to calculate the sum and product of the roots.

Areas for students to improve include:

- understanding that $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\lambda}\neq\frac{1}{\alpha\beta+\alpha\gamma+\beta\gamma}$
- using the Reference Sheet to obtain correct numeric values for the sum and product of the roots
- ensuring correct application of negative signs on the numeric value of their solution.

Question 11 (g)

In better responses, students were able to:

- use the reference sheet to obtain the correct compound angle formula
- correctly integrate expression in the form $cos(kx)$ for a constant k
- obtain correct numeric solutions for their substitutions into the trigonometric expression.

Areas for students to improve include:

- showing enough working to demonstrate how their answer was obtained
- paying attention to grouping symbols when taking out a common factor like ½, either using brackets or the integral symbol
- obtaining the correct trigonometric identity using the reference sheet.

Question 12 (a)

In better responses, students were able to:

- recognise that the question requires the use of the chain rule, and write down an appropriate chain rule
- substitute correctly in the chain rule
- interpret the negative sign as a decreasing rate.

Areas for students to improve include:

- obtaining a correct chain rule
- reading the question carefully to understand that θ and r are not constants and that this cannot be assumed by differentiating with respect to these
- choosing the correct formula, rather than incorrect formulas, for example, $r\theta, r^2\theta, \pi r^2\theta$, and using the formula listed on the Reference Sheet rather than deriving the correct formula by $\frac{\theta}{2\pi} \times \pi r^2$
- understanding which variable to make the subject.

Question 12 (b)

In better responses, students were able to:

- set up an appropriate integral and solve
- obtain the equation with respect to y
- attempt a sketch before integrating.

Areas for students to improve include:

- knowing the correct volume formula, rather than omitting π
- identifying that the question is solid of revolution around the y-axis, not around the x-axis.

Question 12 (c)

In better responses, students were able to:

- set out logical, clear proof of mathematical induction
- unroll and relate factorials
- factorise correctly after unrolling the factorial.

Areas for students to improve include:

- testing for the initial statement correctly
- setting out clear steps for $n = k$ and $n = k + 1$ steps
- concluding their inductive step with acknowledging and indicating the LHS = RHS and providing a correct conclusion
- manipulating the factorials to obtain the desired result
- reading the question carefully, and working on both sides of the required result after using brackets correctly, rather than misplacing brackets because of how they appear in the question, that is, $1\times(1!)+2\times(2!)+...+n\times(n!)$, so that when *n* is replaced with *k*, it is written as $(k+1!)$, rather than, for example, $k+1!$ or $k+1(k+1)!$.

Question 12 (d)

In better responses, students were able to:

- separate the variables correctly
- integrate and find C correctly
- find the general solution of the differential equation correctly
- use definite integrals to arrive at the correct C succinctly
- use differential equations of the form $\frac{dy}{dx} = g(y)$ to integrate $\frac{dx}{dy} = \frac{1}{\sqrt{4-y^2}}$ correctly.

Areas for students to improve include:

- separating variables correctly
- using the Reference Sheet provided to integrate trigonometric functions
- setting out their work clearly and neatly
- recognising the difference of squares pattern, rather than simplifying incorrectly or requiring a binomial expansion
- solving the general solution, rather than confusing details in brackets, for example, $x = \sin^{-1}\left(\frac{y}{2}\right) + C$ being rearranged as $\frac{y}{2} = \sin x + C$ change to: $\sin x = \frac{y}{2} + C$, resulting in incorrect solutions.

Question 12 (e) (i)

In better responses, students were able to:

- express the given expression correctly in terms of $2\sin(x-\alpha)$.

Areas for students to improve include:

- converting trigonometric equations to the form $R\sin(x \pm \alpha)$
- ensuring they write in the given form
- ensuring their answer is in radians, as the domain is given in radians.

Question 12 (e) (ii)

In better responses, students were able to:

- use part (i) to solve the given equation correctly ensuring the domain restrictions were maintained.

Areas for students to improve include:

- adding angles correctly in radians, rather than using $\frac{\pi}{6}+\frac{\pi}{6}=0$.

Question 13 (a)

In better responses, students were able to:

- separate the variables and integrate correctly
- use the initial conditions to calculate the constant of integration
- investigate and use the given point to obtain the correct expression for y.

Areas for students to improve include:

- considering the given point to evaluate the constant
- considering the initial conditions to check reasonableness of the solution.

Question 13 (b)

In better responses, students were able to:

- correctly identify $(n-1)!$ seating arrangements when sitting in a circle
- recognise the number of arrangements when grouped together
- consider seating arrangements by allocating and considering options.

Areas for students to improve include:

- considering complementary scenarios
- understanding circle arrangements and considering groupings.

Question 13 (c)

In better responses, students were able to:

- derive the velocity and acceleration functions
- correctly apply the scalar (dot) product using vector components
- solve a quadratic equation involving square roots.

Areas for students to improve include:

- finding the length of a vector in component form
- rejecting negative values of time
- manipulating algebraic fractions involving square roots
- representing vectors on a number plane
- calculating the scalar (dot) product of two vectors
- applying calculus in determining displacement, velocity and acceleration.

Question 13 (d)

In better responses, students were able to:

- use the normal distribution table to find a z-score for a given probability
- calculate the z-score from the given data
- apply the normal approximation to the given data.

Areas for students to improve include:

- knowing how to read a z-score table
- determining the values to substitute into the z-score formula
- understanding what a z-score represents in terms of probability.

Question 13 (e) (i)

In better responses, students were able to:

- apply the correct substitutions $r = R + 1$ and $n = m + 1$ into the given Pascal's triangle relation
- use the expansion of $\binom{n}{r} = \frac{n!}{(n-r)1r!}$ to prove LHS = RHS.

Areas for students to improve include:

- factorising and manipulating factorial notation
- providing justification and reasoning required in a proof.

Question 13 (e) (ii)

In better responses, students were able to:

- apply Pascal's triangle relation to the given expression
- simplify a series
- recognise $\binom{2000}{2000} = \binom{2001}{2001} = 1$.

Areas for students to improve include:

- identifying the terms that can be simplified in a series
- recognising that $\binom{n}{r}$ is defined only if $n \geq r$
- using the result in part (i) to break down terms.

Question 14 (a)

In better responses, students were able to:

- recognise that there were 6 factor pairs, each with a product of 60, and treat them as pigeonholes
- explain that in choosing 7 factors, there would be at least one pair chosen
- identify the prime factors of 60 as 2,2,3,5 and then use the pigeonhole principle to explain clearly why the prime factors of at least 7 factors of 60 were needed in this case.

Areas for students to improve include:

- understanding that the choosing of some factors does not constitute a proof for the general case
- succinctly presenting a mathematical argument.

Question 14 (b)

In better responses, students were able to:

- provide the correct position vectors of both particles, considering that the angle of projection was zero and the particles were not projected from the origin
- recognise that the time of flight was different for particles *A* and *B*
- realise that the horizontal displacements were equal for the respective times of flight
- equate the vertical displacement components to find the respective times of flight.

Areas for students to improve include:

- understanding that when particles are projected at different times, the same value of time cannot be used for both particles
- understanding what 'after *T* seconds' means in the context of the question
- realising that the angle of projection is zero
- undertaking algebraic manipulation, particularly with respect to square roots and cancelling terms
- avoiding quoting standard projectile formulas without adapting them to a particular situation
- writing clearly to distinguish *U* from *V*, *T* from *t*, *r* from *v*, *g* from 9.

Question 14 (c)

In better responses, students were able to:

- recognise that the perpendicular clock hands meant that $\overrightarrow{OA} \cdot \overrightarrow{OB} = 0$ and were able to deftly convert the scaler (dot) product result into a compound angle result
- use the product to sum formulas to arrive at the correct single trigonometric equation

Areas for students to improve include:

- correctly using the Reference Sheet for compound angles and products to sums formulas
- reviewing and practising solving trigonometric equations in a general context.

Question 14 (d)

In better responses, students were able to:

- differentiate the given function correctly and realise from the denominator that the gradient was undefined at $x = \frac{\pi}{2}$
- realise that the denominator was $|cosx|$, rather than $cosx$
- draw the graph of the given function correctly ensuring that they identified the domains for the solution values and the point where the derivative was undefined.

Areas for students to improve include:

- applying the chain rule for differentiation
- using the Reference Sheet correctly for differentiation formulas
- considering the consequences of a zero denominator
- understanding that $\sqrt{cos^2x}$ equals $|cosx|$, rather than $cosx$, and that $|cosx| = \pm cosx$
- carefully noting the marks allocated to a question to determine the work required.

Question 14 (e)

In better responses, students were able to:

- recognise the correct equations relating the roots and coefficients of the given polynomial
- realise that they could apply the $\tan(\alpha + \beta)$ result by pairing to get an expression for $\tan(\alpha + \beta + \gamma)$
- understand how to find the smallest positive angle of $\alpha + \beta + \gamma$ from a negative ratio.

Areas for students to improve include:

- taking care with the signs when using polynomial roots and coefficients relationships
- ensuring accuracy when manipulating complex algebraic expressions.