2025 HSC Mathematics Extension 1 Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	A
2	B
3	B
4	C
5	D
6	D
7	A
8	A
9	B
10	C

Section II

Question 11 (a)

Criteria	Marks
• Provides correct solution	2
• Rearranges to make x the subject, or equivalent merit	1

Sample answer:

Let $y = f(x)$. Then

$$
\begin{aligned}
y &= 1 - \frac{1}{x-2} \\
y - 1 &= -\frac{1}{x-2} \\
x - 2 &= -\frac{1}{y-1} \\
x &= 2 - \frac{1}{y-1}
\end{aligned}
$$

Swap y and x for the inverse function, $y = 2 - \dfrac{1}{x-1}$.

$\therefore \quad f^{-1}(x) = 2 - \dfrac{1}{x-1}$.

Question 11 (b)

Criteria	Marks
• Provides correct solution	3
• Finds one correct value of θ	2
• Applies double angle formula	1

Sample answer:

$$\sin 2\theta - \sin\theta = 0$$

$$2\sin\theta\cos\theta - \sin\theta = 0$$

$$\sin\theta(2\cos\theta - 1) = 0$$

either $\sin\theta = 0$ or $2\cos\theta - 1 = 0$

$\theta = 0, \pi$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$\theta = 0, \frac{\pi}{3}, \pi$

Question 11 (c)

Criteria	Marks
• Provides correct solution	2
• Applies trigonometric product as sum, or equivalent merit	1

Sample answer:

$$\int \sin 3x \cos x \, dx$$

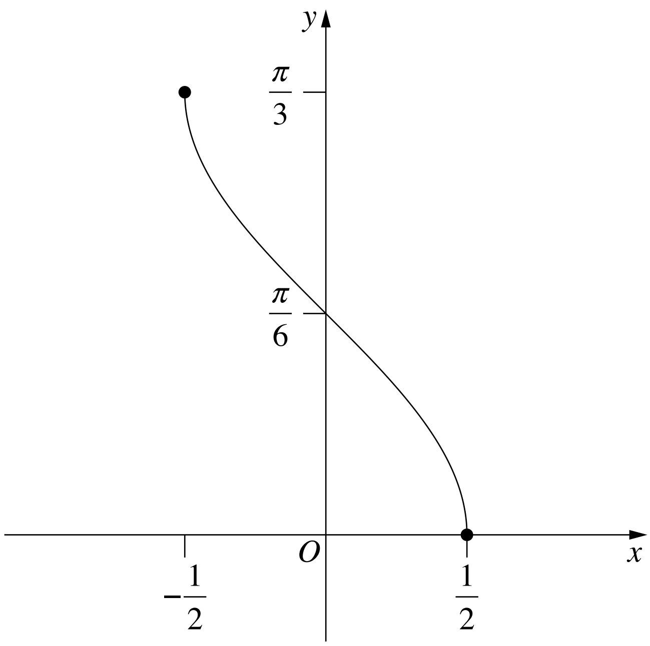
$$= \frac{1}{2}\int \big(\sin(4x) + \sin(2x)\big)\, dx$$

$$= -\frac{\cos(4x)}{8} - \frac{\cos(2x)}{4} + C$$

Question 11 (d)

Criteria	Marks
• Provides correct graph	2
• Provides correct domain and range, or equivalent method	1

Sample answer:



Question 11 (e)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$\begin{pmatrix} 2 \\ 6 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

So $m = 3$ makes the vectors parallel.

Question 11 (f)

Criteria	Marks
• Provides correct solution	2
• Arranges over a common denominator OR • Applies formulas for sums or products of roots, or equivalent merit	1

Sample answer:

$$\frac{1}{\alpha\beta}+\frac{1}{\alpha\gamma}+\frac{1}{\beta\gamma}$$

$$=\frac{\gamma}{\alpha\beta\gamma}+\frac{\beta}{\alpha\beta\gamma}+\frac{\alpha}{\alpha\beta\gamma}$$

$$=\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$$

$$=\frac{-\frac{6}{2}}{\frac{1}{2}}$$

$$=-6$$

Question 11 (g)

Criteria	Marks
• Provides correct solution	3
• Integrates expression	2
• Uses compound angle formula	1

Sample answer:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\cos^2(3x)\,dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1+\cos(6x)}{2}\,dx$$

$$=\frac{1}{2}\left[x+\frac{\sin(6x)}{6}\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$=\frac{1}{2}\left(\frac{\pi}{3}+\frac{\sin(2\pi)}{6}\right)-\frac{1}{2}\left(\frac{\pi}{6}+\frac{\sin(\pi)}{6}\right)$$

$$=\frac{\pi}{6}+0-\frac{\pi}{12}-0$$

$$=\frac{\pi}{12}$$

Question 12 (a)

Criteria	Marks
• Provides correct solution	3
• Finds $\frac{dr}{dt}$ in terms of r, or equivalent merit	2
• Finds $\frac{d\theta}{dr}$ OR • Attempts to use chain rule, or equivalent merit	1

Sample answer:

$$\text{Area} = \frac{1}{2}r^2\theta = 10$$

$$\therefore \quad \theta = \frac{20}{r^2}$$

$$\therefore \quad \frac{d\theta}{dr} = -\frac{40}{r^3}$$

$$\frac{d\theta}{dt} = 2 \ \text{(given)}$$

$$\begin{aligned}\therefore \quad \frac{dr}{dt} &= \frac{dr}{d\theta} \times \frac{d\theta}{dt} \\ &= -\frac{r^3}{40} \times 2 \\ &= -\frac{r^3}{20}\end{aligned}$$

When $r = 4$,

$$\begin{aligned}\frac{dr}{dt} &= -\frac{4^3}{20} \\ &= -\frac{16}{5} \\ &= -3.2 \text{ cm s}^{-1}\end{aligned}$$

(That is radius is *decreasing* at a rate of 3.2 cm s^{-1}.)

Question 12 (b)

Criteria	Marks
• Provides correct solution	2
• Provides correct integral, or equivalent merit	1

Sample answer:

The volume is

$$V = \pi \int_1^a \left(\frac{1}{y}\right)^2 dy = \pi\left[-\frac{1}{y}\right]_1^a = \pi - \frac{\pi}{a}$$

Question 12 (c)

Criteria	Marks
• Provides correct proof	3
• Uses inductive assumption, or equivalent merit	2
• Establishes initial statement, or equivalent merit	1

Sample answer:

When $n = 1$

$\text{LHS} = 1 \times (1!) = 1$

$\text{RHS} = (1+1)! - 1 = 2 - 1 = 1$

$\therefore$ True when $n = 1$

Assume true for $n = k$, that is

$$1 \times (1!) + 2 \times (2!) + \cdots + k \times (k!) = (k+1)! - 1$$

Now prove true for $n = k + 1$, that is prove

$$1 \times (1!) + 2 \times (2!) + \cdots + k \times (k!) + (k+1) \times \left((k+1)!\right) = (k+2)! - 1$$

$$\begin{aligned} \text{LHS} &= \left[1 \times (1!) + 2 \times (2!) + \cdots + k \times (k!)\right] + (k+1) \times \left((k+1)!\right) \\ &= \left[(k+1)! - 1\right] + (k+1) \times \left((k+1)!\right) \quad \text{by assumption} \\ &= \left[1 + (k+1)\right](k+1)! - 1 \\ &= (k+2)! - 1 = \text{RHS}. \end{aligned}$$

Hence the statement is also true for $n = k + 1$. So the statement is true for all positive integers by mathematical induction.

Question 12 (d)

Criteria	Marks
• Provides correct solution	3
• Finds the general solution $x = \sin^{-1}\left(\frac{y}{2}\right) + C$, or equivalent merit	2
• Separates variables, or equivalent merit	1

Sample answer:

$$\frac{dy}{dx} = \sqrt{(2-y)(2+y)}$$

$$= \sqrt{4-y^2}$$

$$\therefore \quad \int \frac{dy}{\sqrt{4-y^2}} = \int dx$$

$$x = \sin^{-1}\left(\frac{y}{2}\right) + C$$

When $x = 0,\ y = 1$

$$\therefore \quad 0 = \sin^{-1}\left(\frac{1}{2}\right) + C$$

$$= \frac{\pi}{6} + C$$

$$\therefore \quad C = -\frac{\pi}{6}$$

$$\therefore \quad x = \sin^{-1}\left(\frac{y}{2}\right) - \frac{\pi}{6} \quad \text{or} \quad y = 2\sin\left(x + \frac{\pi}{6}\right)$$

Question 12 (e) (i)

Criteria	Marks
• Provides correct answer	1

Sample answer:

$$2\left(\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x\right) = 2\sin\left(x - \frac{\pi}{6}\right)$$

Question 12 (e) (ii)

Criteria	Marks
• Provides correct answer	2
• Uses part (i) to simplify the equation, or equivalent merit	1

Sample answer:

$$\sqrt{3}\sin x = \cos x + 1$$

$$\sqrt{3}\sin x - \cos x = 1$$

$$2\sin\left(x - \frac{\pi}{6}\right) = 1$$

$$\sin\left(x - \frac{\pi}{6}\right) = \frac{1}{2}$$

$$x - \frac{\pi}{6} = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}.$$

$$\text{So } x = \frac{2\pi}{6} = \frac{\pi}{3} \text{ or } x = \frac{6\pi}{6} = \pi.$$

Question 13 (a)

Criteria	Marks
• Provides correct solution	3
• Obtains $y^2 = 10x + 16$, or equivalent merit	2
• Integrates to obtain $\frac{y^2}{2} = 5x + C$, or equivalent merit	1

Sample answer:

Since $\frac{dy}{dx} = \frac{5}{y}$,

$$\therefore \quad \int y\,dy = \int 5\,dx$$

$$\frac{y^2}{2} = 5x + C$$

When $x = 0$, $y = -4$.

$$\therefore \quad \frac{16}{2} = 0 + C$$

$$C = 8$$

ie $\frac{y^2}{2} = 5x + 8$

$$y^2 = 10x + 16$$

$$y = \pm\sqrt{10x + 16}$$

When $x = 0$, $y = -4$

$$\therefore \quad y = -\sqrt{10x + 16}$$

Question 13 (b)

Criteria	Marks
• Provides correct solution	2
• Obtains 7! unrestricted seating arrangements at a round table, or equivalent merit	1

Sample answer:

There are 7! possible seating arrangements at a round table without restriction.

There are $6! \times 2$ possible seating arrangements where the two people sit together.

Hence, there are $7! - 6! \times 2 = 3600$ seating arrangements where they do not sit together.

Question 13 (c)

Criteria	Marks
• Provides correct solution	4
• Equates the two expressions of $\underset{\sim}{v} \cdot \underset{\sim}{a}$ in terms of t, or equivalent merit	3
• Obtains expression $\underset{\sim}{v} \cdot \underset{\sim}{a} = \frac{4t}{81}$, or equivalent merit	2
• Obtains expression for $\underset{\sim}{v}$	1

Sample answer:

Given $\underset{\sim}{r}(t) = t\underset{\sim}{i} + \frac{t^2}{9}\underset{\sim}{j}$

then $\underset{\sim}{v}(t) = \underset{\sim}{i} + \frac{2t}{9}\underset{\sim}{j}$

and $\underset{\sim}{a}(t) = 0\underset{\sim}{i} + \frac{2}{9}\underset{\sim}{j}$

Using scalar product,

$$\underset{\sim}{v} \cdot \underset{\sim}{a} = \frac{4t}{81}$$

and $\underset{\sim}{v} \cdot \underset{\sim}{a} = |\underset{\sim}{v}| \cdot |\underset{\sim}{a}| \cos\frac{\pi}{4}$

$$= \sqrt{1 + \frac{4t^2}{81}} \cdot \sqrt{\frac{4}{81}} \cdot \frac{1}{\sqrt{2}}$$

Equating,

$$\frac{4t}{81} = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{4t^2}{81}} \cdot \frac{2}{9}$$

$$\frac{2t}{9} = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{4t^2}{81}}$$

Squaring,

$$\frac{4t^2}{81} = \frac{1}{2}\left(1 + \frac{4t^2}{81}\right)$$

$$\frac{8t^2}{81} = 1 + \frac{4t^2}{81}$$

$$\frac{4t^2}{81} = 1$$

$$t^2 = \frac{81}{4}$$

$$t = \pm\frac{9}{2}$$

$$t = \frac{9}{2}, \text{ since } t > 0.$$

Question 13 (d)

Criteria	Marks
• Provides correct solution	4
• Uses $Z = \dfrac{X - 20}{4}$ as the normal approximation and finds $P(X < k) = 0.9332$, or equivalent merit	3
• Attempts to use $Z = \dfrac{X - 20}{4}$ as the normal approximation, or equivalent merit	2
• Obtains $p = \dfrac{1}{5}$ or finds $P(Z \le 1.5) = 0.9332$, or equivalent merit	1

Sample answer:

If p is the probability that a counter is green then $E(X) = np = 100p = 20$ so $p = \dfrac{1}{5}$.

The variance is $\sigma^2 = 100 \times p \times (1 - p)$

$$= 100 \times \frac{1}{5} \times \frac{4}{5} = 16.$$

The probability of selecting at least k green counters is

$$P(X \ge k) = P\left(\frac{X - 20}{4} \ge \frac{k - 20}{4}\right) = P\left(Z \ge \frac{k - 20}{4}\right)$$

where $Z = \dfrac{X - 20}{4} \sim N(0, 1)$.

Since $P(X \ge k) = 0.0668$ then $P(X < k) = 1 - 0.0668 = 0.9332$ and so

$$P\left(Z \le \frac{k - 20}{4}\right) = 0.9332 = P(Z \le 1.5) \text{ by table.}$$

So $1.5 = \dfrac{k - 20}{4}$, hence $k = 26$.

Question 13 (e) (i)

Criteria	Marks
• Provides correct solution	1

Sample answer:

Re-arranging

$$\binom{n-1}{r-1} = \binom{n}{r} - \binom{n-1}{r}$$

Let $r = R + 1$, $n = m + 1$. Therefore

$$\binom{m}{R} = \binom{m+1}{R+1} - \binom{m}{R+1}$$

Question 13 (e) (ii)

Criteria	Marks
• Provides correct solution	2
• Applies part (i), or equivalent merit	1

Sample answer:

Therefore,

$$\begin{aligned}
\text{LHS} &= \binom{2000}{2000} + \left[\binom{2002}{2001} - \binom{2001}{2001}\right] \qquad \text{Using part (i)} \\
&\quad + \left[\binom{2003}{2001} - \binom{2002}{2001}\right] \\
&\quad + \cdots + \left[\binom{2051}{2001} - \binom{2050}{2001}\right] \\
&= \binom{2000}{2000} - \binom{2001}{2001} + \binom{2051}{2001} \\
&= 1 - 1 + \binom{2051}{2001} \\
&= \binom{2051}{2001} \\
&= \text{RHS}
\end{aligned}$$

Question 14 (a)

Criteria	Marks
• Provides correct proof	2
• Creates 6 appropriate buckets, or equivalent merit	1

Sample answer:

Create 6 buckets $[1, 60], [2, 30], [3, 20], [4, 15], [5, 12], [6, 10]$.

If we choose 7 numbers, then by the pigeonhole principle two numbers must be chosen from the same bucket. These two numbers multiply to 60. Hence, the product of all 7 numbers is a multiple of 60.

Question 14 (b)

Criteria	Marks
• Provides correct solution	4
• Equates x-components, and equates y-components with zero, or equivalent merit	3
• Equates x-components, or equates y-components with zero, or equivalent merit	2
• Establishes displacement in either direction for either particle	1

Sample answer:

Let t be the time after the first particle is projected, let the time difference be T seconds and let $OB = h$ metres so $OA = kh$ metres.

$\therefore$ For the particle projected from A, the equations of motion are $x = Ut$ and $y = -\frac{1}{2}gt^2 + kh$ and for the particle projected from B, the equations of motion are $x = V(t-T)$ and $y = -\frac{1}{2}g(t-T)^2 + h$.

The particles land in the same place, so

$$x = Ut = V(t-T)$$

$$\therefore \quad \frac{V}{U} = \frac{t}{t-T}$$

Also, they land on the ground, so

$$y = -\frac{1}{2}gt^2 + kh = 0 \qquad \text{and} \qquad y = -\frac{1}{2}g(t-T)^2 + h = 0$$

$$\therefore \quad kh = \frac{1}{2}gt^2 \qquad\qquad \therefore \quad h = \frac{1}{2}g(t-T)^2$$

$$\frac{2kh}{g} = t^2 \qquad\qquad \frac{2h}{g} = (t-T)^2$$

$$\therefore \quad t^2 = k(t-T)^2$$

$$\frac{t^2}{(t-T)^2} = k$$

$$\therefore \quad \frac{V}{U} = \frac{t}{t-T}$$

$$= \sqrt{k} \qquad \text{since } t > T > 0.$$

Question 14 (c)

Criteria	Marks
• Provides correct solution	3
• Attempts to use compound angle formula, or equivalent merit	2
• Uses the scalar product, or equivalent merit	1

Sample answer:

The hands are perpendicular when the scalar product is zero, which leads to

$$\sin\left(\frac{\pi t}{360}\right)\sin\left(\frac{\pi t}{30}\right)+\cos\left(\frac{\pi t}{360}\right)\cos\left(\frac{\pi t}{30}\right)=0,$$

which simplifies to $\cos\left(\frac{\pi t}{30}-\frac{\pi t}{360}\right)=\cos\left(\frac{11\pi t}{360}\right)=0.$

This means $\frac{11\pi t}{360}=\cdots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \cdots.$

The first time that the hands are perpendicular is when $\frac{11\pi t}{360}=\frac{\pi}{2}$,
which gives $t=\frac{360\times\pi}{2\times 11\times\pi}\approx 16.364$ minutes.

The second time that the hands are perpendicular is when $\frac{11\pi t}{360}=\frac{3\pi}{2}$,
which gives $t=\frac{3\times 360\times\pi}{2\times 11\times\pi}\approx 49.091$ minutes.

Question 14 (d)

Criteria	Marks
• Provides correct solution	3
• Recognises $f'(x)$ is undefined at $x = \frac{\pi}{2}$, or equivalent merit	2
• States $f'(x) = 1$ or $f'(x) = -1$ or $f'(x) = \pm 1$, or equivalent merit	1

Sample answer:

$$f(x) = \cos^{-1}(\sin x)$$

$$\therefore \quad f'(x) = \frac{-\cos x}{\sqrt{1 - \sin^2 x}} = \frac{-\cos x}{|\cos x|} = \pm 1$$

For $0 < x < \frac{\pi}{2}$, $\cos x > 0$ $\quad \therefore \quad f'(x) = -1$

For $\frac{\pi}{2} < x < \pi$, $\cos x < 0$ $\quad \therefore \quad f'(x) = 1$

When $x = \frac{\pi}{2}$, $f'(x)$ is undefined.

$\therefore \quad f'(x) = -1$ for $x \in \left(0, \frac{\pi}{2}\right)$

and $\quad f'(x) = 1$ for $x \in \left(\frac{\pi}{2}, \pi\right)$

Question 14 (e)

Criteria	Marks
• Provides correct solution	3
• Applies $\tan(A+B)$ formula, or equivalent merit	2
• Uses relation between roots and coefficients of polynomials, or equivalent merit	1

Sample answer:

$$\tan\alpha + \tan\beta + \tan\gamma = -b$$

$$\tan\alpha\tan\beta + \tan\alpha\tan\gamma + \tan\beta\tan\gamma = c$$

$$\tan\alpha\tan\beta\tan\gamma = 1 - b - c$$

$$\tan(\alpha+\beta+\gamma) = \frac{\tan\alpha + \tan(\beta+\gamma)}{1 - \tan\alpha\tan(\beta+\gamma)}$$

$$= \frac{\tan\alpha + \dfrac{\tan\beta + \tan\gamma}{1 - \tan\beta\tan\gamma}}{1 - \tan\alpha\left(\dfrac{\tan\beta + \tan\gamma}{1 - \tan\beta\tan\gamma}\right)}$$

$$= \frac{\tan\alpha - \tan\alpha\tan\beta\tan\gamma + \tan\beta + \tan\gamma}{1 - \tan\beta\tan\gamma - \tan\alpha\tan\beta - \tan\alpha\tan\gamma}$$

$$= \frac{(\tan\alpha + \tan\beta + \tan\gamma) - \tan\alpha\tan\beta\tan\gamma}{1 - (\tan\alpha\tan\beta + \tan\alpha\tan\gamma + \tan\beta\tan\gamma)}$$

$$= \frac{-b - (1 - b - c)}{1 - c}$$

$$= -\frac{(1-c)}{1-c}$$

$$= -1$$

$$\therefore \quad \alpha + \beta + \gamma = \cdots, -\frac{\pi}{4}, \frac{3\pi}{4}, \cdots$$

Smallest positive value is $\dfrac{3\pi}{4}$.

2025 HSC Mathematics Extension 1 Mapping Grid

Section I

Question	Marks	Content	Syllabus outcomes
1	1	ME-F1 Further Work with Functions	ME11-2
2	1	ME-V1 Introduction to Vectors	ME12-2
3	1	ME-C2 Further Calculus Skills	ME12-1
4	1	ME-S1 The Binomial Distribution	ME12-5
5	1	ME-T3 Trigonometric Equations	ME12-3
6	1	ME-T1 Inverse Trigonometric Functions	ME11-3
7	1	ME-C3 Applications of Calculus	ME12-4
8	1	ME-V1 Introduction to Vectors	ME12-2
9	1	ME-V1 Introduction to Vectors	ME12-2
10	1	ME-C2 Further Calculus Skills	ME12-4

Section II

Question	Marks	Content	Syllabus outcomes
11 (a)	2	ME-F1 Further Work with Functions	ME11-1
11 (b)	3	ME-T3 Trigonometric Equations	ME12-3
11 (c)	2	ME-T2 Further Trigonometric Identities	ME11-3
11 (d)	2	ME-T1 Inverse Trigonometric Functions	ME11-3
11 (e)	1	ME-V1 Introduction to Vectors	ME12-2
11 (f)	2	ME-F2 Polynomials	ME11-2
11 (g)	3	ME-C2 Further Calculus Skills	ME12-1
12 (a)	3	ME-C1 Rates of Change	ME11-4
12 (b)	2	ME-C3 Applications of Calculus	ME12-4
12 (c)	3	ME-P1 Proof by Mathematical Induction	ME12-1
12 (d)	3	ME-C3 Applications of Calculus	ME12-4
12 (e) (i)	1	ME-T3 Trigonometric Equations	ME12-3
12 (e) (ii)	2	ME-T3 Trigonometric Equations	ME12-3
13 (a)	3	ME-C3 Applications of Calculus	ME12-4
13 (b)	2	ME-A1 Working with Combinatorics	ME11-5
13 (c)	4	ME-V1 Introduction to Vectors	ME12-2
13 (d)	4	ME-S1 The Binomial Distribution	ME12-5
13 (e) (i)	1	ME-A1 Working with Combinatorics	ME11-5
13 (e) (ii)	2	ME-A1 Working with Combinatorics	ME11-5
14 (a)	2	ME-A1 Working with Combinatorics	ME11-5
14 (b)	4	ME-V1 Introduction to Vectors	ME12-2

Question	Marks	Content	Syllabus outcomes
14 (c)	3	ME-V1 Introduction to Vectors ME-T3 Trigonometric Equations	ME12-2, ME12-3
14 (d)	3	ME-C2 Further Calculus Skills	ME12-1
14 (e)	3	ME-F2 Polynomials ME-T3 Trigonometric Equations	ME12-3