Centre Number

Student Number

NSW Education Standards Authority

2025 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Centre Number and Student Number at the top of this page

Total marks: 70

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 7–13)

- Attempt Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 What is the solution to $|2x+3| < 5$?

A. $-4 < x < 1$

B. $x < -4$ or $x > 1$

C. $-1 < x < 4$

D. $x < -1$ or $x > 4$

2 The projection of $\underset{\sim}{u}$ onto $\underset{\sim}{v}$ is given by $\left(\dfrac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{v}|^2}\right)\underset{\sim}{v}$.

What is the projection of $\underset{\sim}{u} = \underset{\sim}{i} + 2\underset{\sim}{j}$ onto $\underset{\sim}{v} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$?

A. $-\dfrac{4}{5}\left(\underset{\sim}{i} + 2\underset{\sim}{j}\right)$

B. $-\dfrac{4}{13}\left(2\underset{\sim}{i} - 3\underset{\sim}{j}\right)$

C. $-\dfrac{4}{\sqrt{5}}\left(\underset{\sim}{i} + 2\underset{\sim}{j}\right)$

D. $-\dfrac{4}{\sqrt{13}}\left(2\underset{\sim}{i} - 3\underset{\sim}{j}\right)$

3 Consider the integral $\int_{-\frac{5}{2}}^{\frac{5}{2}} \left(\frac{1}{25 - x^2}\right) dx$.

The substitution $x = 5\sin\theta$ is applied.

Which of the following is obtained?

A. $\frac{1}{5}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \operatorname{cosec}\theta \, d\theta$

B. $\frac{1}{5}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec\theta \, d\theta$

C. $\frac{1}{25}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \operatorname{cosec}^2\theta \, d\theta$

D. $\frac{1}{25}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec^2\theta \, d\theta$

4 A Bernoulli random variable X has probability distribution

$$P(x) = \frac{x+1}{3} \text{ for } x = 0, 1.$$

What are the mean and variance of X?

A. $E(X) = \frac{1}{3}, \quad \mathrm{Var}(X) = \frac{2}{9}$

B. $E(X) = \frac{1}{3}, \quad \mathrm{Var}(X) = \frac{2}{3}$

C. $E(X) = \frac{2}{3}, \quad \mathrm{Var}(X) = \frac{2}{9}$

D. $E(X) = \frac{2}{3}, \quad \mathrm{Var}(X) = \frac{2}{3}$

5 How many distinct solutions are there to the equation $\cos 5x + \sin x = 0$ for $0 \leq x \leq 2\pi$?

A. 5

B. 6

C. 9

D. 10

6 Given that a is a non-zero constant, which of the following integrals is equal to zero?

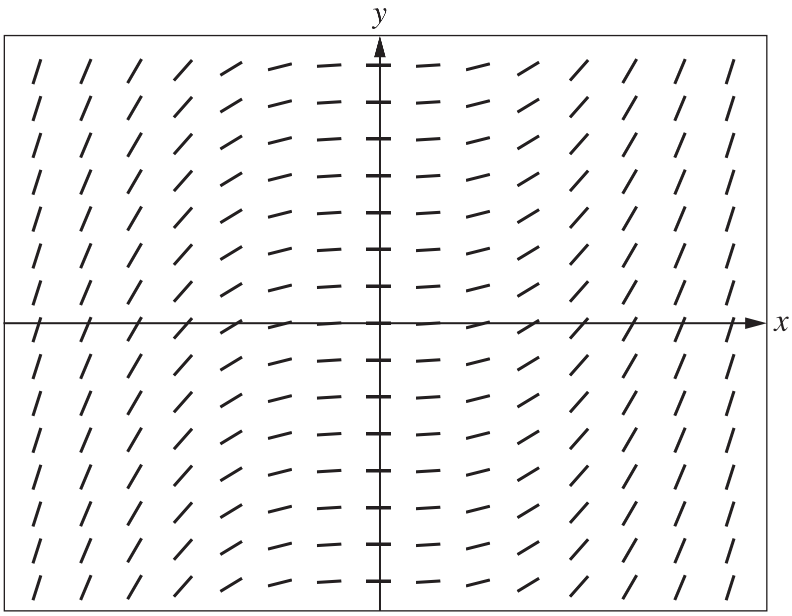
A. $\displaystyle\int_{-a}^{a} x\cos^{-1}(x)\,dx$

B. $\displaystyle\int_{-a}^{a} x^2\cos^{-1}(x)\,dx$

C. $\displaystyle\int_{-a}^{a} x\tan^{-1}(x)\,dx$

D. $\displaystyle\int_{-a}^{a} x^2\tan^{-1}(x)\,dx$

7 A slope field is shown.



Which of the following could be the differential equation represented by the slope field?

A. $\dfrac{dy}{dx} = x^2$

B. $\dfrac{dy}{dx} = x^2 + C,\ C \neq 0$

C. $\dfrac{dy}{dx} = x^3$

D. $\dfrac{dy}{dx} = x^3 + C,\ C \neq 0$

8 Points A and B have non-zero, non-parallel position vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively.

Point C has position vector $\underset{\sim}{c} = 3\underset{\sim}{a} - 2\underset{\sim}{b}$.

The points A, B and C lie on the same line.

Which of the following must be true?

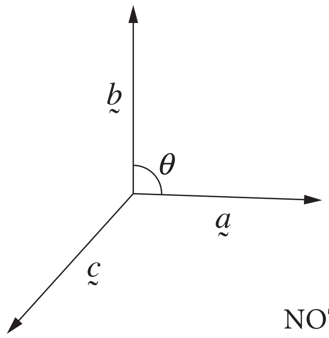
A. Point A always lies between Points B and C.

B. Point B always lies between Points A and C.

C. Point C always lies between Points A and B.

D. The order of the points cannot be determined.

9 The vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ have magnitudes 3, 5 and 7 respectively.



NOT TO SCALE

Given that $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} = \underset{\sim}{0}$, what is the size of angle θ between $\underset{\sim}{a}$ and $\underset{\sim}{b}$?

A. $\dfrac{\pi}{6}$

B. $\dfrac{\pi}{3}$

C. $\dfrac{2\pi}{3}$

D. $\dfrac{5\pi}{6}$

10 For the function $f(x)$, it is known that $f(3) = 1$, $f'(3) = 2$ and $f''(3) = 4$.

Let $g(x) = f^{-1}(x)$.

What is the value of $g''(1)$?

A. $\dfrac{1}{4}$

B. $-\dfrac{1}{4}$

C. $-\dfrac{1}{2}$

D. -1

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet

(a) Find the inverse function, $f^{-1}(x)$, of the function $f(x) = 1 - \dfrac{1}{x-2}$. **2**

(b) Solve $\sin 2\theta - \sin\theta = 0$ for $0 \le \theta \le \pi$. **3**

(c) Find $\displaystyle\int \sin 3x \cos x \, dx$. **2**

(d) Sketch the graph of $y = \dfrac{1}{3}\cos^{-1}(2x)$. **2**

(e) For what value of m is the vector $\begin{pmatrix} 1 \\ m \end{pmatrix}$ parallel to the vector $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$? **1**

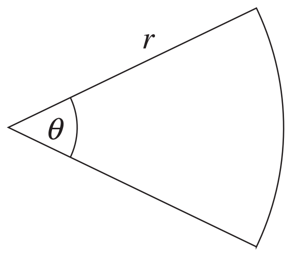
(f) The roots of $2x^3 + 6x^2 + x - 1 = 0$ are α, β and γ. **2**

What is the value of $\dfrac{1}{\alpha\beta} + \dfrac{1}{\alpha\gamma} + \dfrac{1}{\beta\gamma}$?

(g) Evaluate $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^2(3x)\, dx$. **3**

Question 12 (14 marks) Use the Question 12 Writing Booklet

(a) The radius, r cm, and angle, θ radians, of a sector vary in such a way that its area remains a constant 10 cm^2. **3**



The angle θ is increasing at a constant rate of 2 radians per second.

Find the rate at which the radius is changing when the radius is 4 cm.

(b) Consider the region bounded by the hyperbola $y = \dfrac{1}{x}$, the y-axis and the lines $y = 1$ and $y = a$ for $a > 1$. **2**

Find the volume of the solid of revolution formed when the region is rotated about the y-axis.

(c) Prove by mathematical induction that **3**

$$1 \times (1!) + 2 \times (2!) + \cdots + n \times (n!) = (n+1)! - 1$$

for integers $n \geq 1$.

(d) Find the solution of $\dfrac{dy}{dx} = \sqrt{(2-y)(2+y)}$, given that $y = 1$ when $x = 0$. **3**

(e) (i) Express $\sqrt{3}\sin x - \cos x$ in the form $2\sin(x - \alpha)$, where $0 < \alpha < \dfrac{\pi}{2}$. **1**

(ii) Hence, or otherwise, solve $\sqrt{3}\sin x = \cos x + 1$, where $0 \leq x \leq 2\pi$. **2**

Question 13 (16 marks) Use the Question 13 Writing Booklet

(a) It is given that $\dfrac{dy}{dx} = \dfrac{5}{y}$ and $y = -4$ when $x = 0$. **3**

Find y as a function of x.

(b) Eight guests are to be seated at a round table. If two of these guests refuse to sit next to each other, how many seating arrangements are possible? **2**

(c) At time t, a particle has position vector $\underset{\sim}{r}(t) = t\underset{\sim}{i} + \dfrac{t^2}{9}\underset{\sim}{j}$, velocity vector $\underset{\sim}{v}(t)$ and acceleration vector $\underset{\sim}{a}(t)$. **4**

Find the time when the angle between $\underset{\sim}{v}(t)$ and $\underset{\sim}{a}(t)$ is $\dfrac{\pi}{4}$.

(d) A bag contains counters, some of which are green. **4**

One hundred trials of an experiment are run. In each trial, one counter is selected from the bag at random and its colour noted. The counter is returned to the bag after each trial.

Let X be the random variable representing the number of times that a green counter is selected.

Given that $E(X) = 20$ and $P(X \geq k) = 0.0668$, find the value of k. You may use the standard normal approximation and the information on page 13.

Question 13 continues on page 10

Question 13 (continued)

(e) (i) The Pascal's triangle relation can be expressed as **1**

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}. \quad \text{(Do NOT prove this.)}$$

Show that $\binom{m}{R} = \binom{m+1}{R+1} - \binom{m}{R+1}$.

(ii) Hence, or otherwise, prove that **2**

$$\binom{2000}{2000} + \binom{2001}{2000} + \binom{2002}{2000} + \cdots + \binom{2050}{2000} = \binom{2051}{2001}.$$

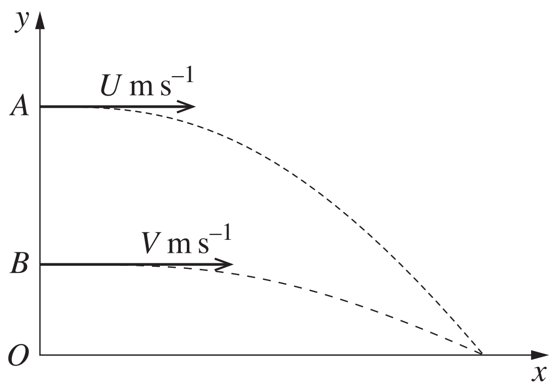
End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet

(a) Prove that the product of any seven distinct factors of 60 must be a multiple of 60. **2**

(b) Points A and B lie vertically above the origin. Point A is higher than point B such that $\dfrac{OA}{OB} = k$, where $k > 1$. **4**

A particle is projected horizontally from point A with velocity $U\ \mathrm{m\,s^{-1}}$. After T seconds, another particle is projected horizontally from point B with velocity $V\ \mathrm{m\,s^{-1}}$. The two particles land on the ground in the same place.



Show that the ratio $\dfrac{V}{U}$ depends only on k.

(c) The hands of an analogue clock are OA and OB, **3**

where A is $\left(\sin\left(\dfrac{\pi t}{360}\right), \cos\left(\dfrac{\pi t}{360}\right)\right)$, B is $\left(2\sin\left(\dfrac{\pi t}{30}\right), 2\cos\left(\dfrac{\pi t}{30}\right)\right)$,

O is the origin, and $t \geq 0$ is the number of minutes past midnight.

Find the values of t when the hands are perpendicular for the first and second time after midnight. Give your answers to 3 decimal places.

Question 14 continues on page 12

(d) The function $f(x)$ is defined by $f(x) = \cos^{-1}(\sin x)$ in the domain $(0, \pi)$. **3**

Find $f'(x)$ for those values of x where it is defined.

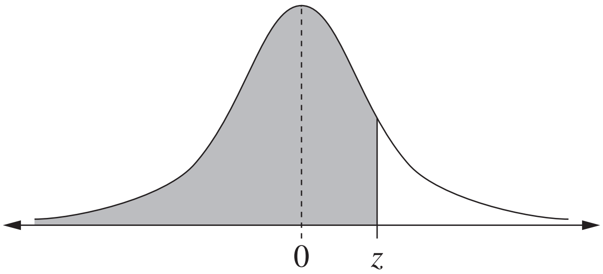
(e) It is given that $\tan\alpha$, $\tan\beta$ and $\tan\gamma$ are the three real roots of the polynomial equation $x^3 + bx^2 + cx - 1 + b + c = 0$, where b and c are real numbers and $c \neq 1$. **3**

Find the smallest positive value of $\alpha + \beta + \gamma$.

End of paper

Use the information below to answer Question 13 (d).

Table of values $P(Z \le z)$ for the normal distribution $N(0, 1)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995

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2025 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}\left[2a + (n - 1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab\sin C$$

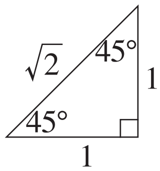
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

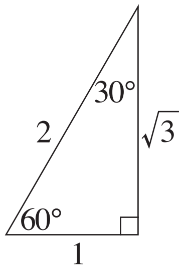
$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$





Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

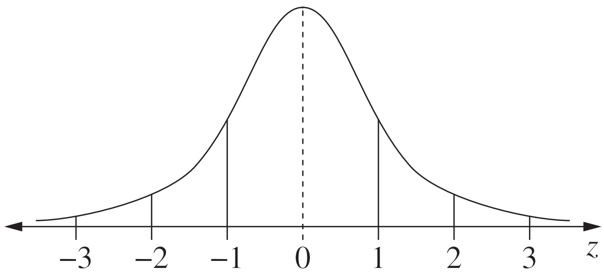
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \quad P(X = x)$$

$$= \binom{n}{x} p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = n f'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)\, f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)\, f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1 + \left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$