



Merewether High School

2021

**HIGHER SCHOOL
CERTIFICATE**

Mathematics Extension 1

ASSESSMENT TASK 4

**General
Instructions**

- Reading Time 10 minutes
- Working time – 120 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Section II, show relevant mathematical reasoning and/or calculations

**Total Marks:
70 marks**

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II –60 marks (pages 7–12)

- Attempt Questions 11–16
- Allow about 105 minutes for this section

DO NOT REMOVE THIS EXAMINATION PAPER FROM THE ROOM

Section I

10 Marks

Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10

1. How many arrangements are possible from the word MEREWETHER?

(A) $\frac{10!}{4!}$

(B) $10!4!2!$

(C) $10!$

(D) $\frac{10!}{2!4!}$

2. Given $f(x) = 1 + \sqrt{x}$, what are the domain and range of $f^{-1}(x)$?

(A) $x \geq 0, y \geq 0$

(B) $x \geq 0, y \geq 1$

(C) $x \geq 1, y \geq 0$

(D) $x \geq 1, y \geq 1$

3. Find $\frac{d}{dx}(\cos^{-1} x + 3)^4$

(A) $\frac{-4}{\sqrt{1-x^2}}(\cos^{-1} x + 3)^3$

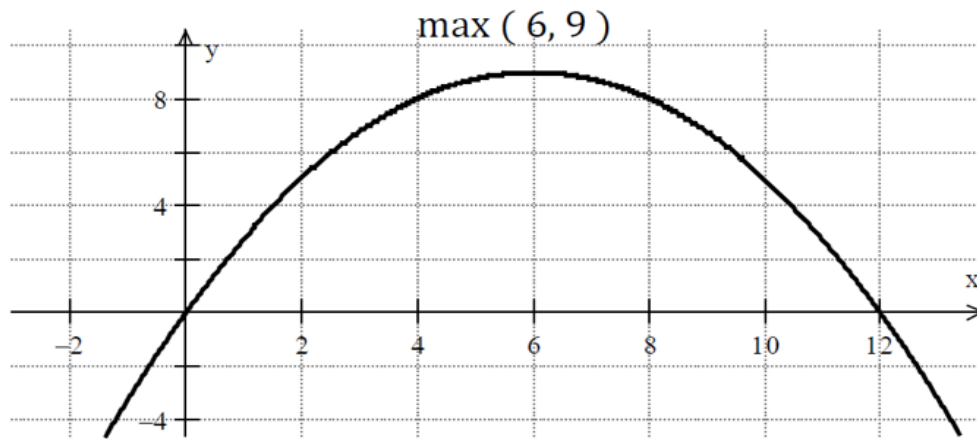
(B) $\frac{4}{\sqrt{1-x^2}}(\cos^{-1} x + 3)^3$

(C) $\frac{-4}{\sqrt{1+x^2}}(\cos^{-1} x + 3)^3$

(D) $\frac{-1}{\sqrt{1+x^2}}(\cos^{-1} x + 3)^3$

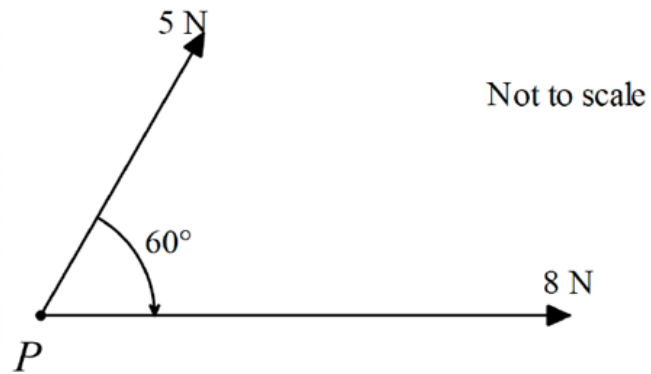
4. Which of the following does NOT describe a Bernoulli random variable?
- (A) An arrow hitting its target.
 - (B) The number of heads when tossing 2 coins.
 - (C) Selecting a packet of salt and vinegar chips from mix of chip types.
 - (D) Guessing the correct answer in an extension 1 multiple choice question.

5. Which of the parametric equation represents the parabola below?



- (A) $x = 12t, y = 9t$
- (B) $x = 12t, y = 9 - t^2$
- (C) $x = 6 - 2t, y = 9 - t^2$
- (D) $x = 2t - 6, y = 9t - t^2$

6. Which of the following is the correct magnitude and direction of the resultant force acting on P ?

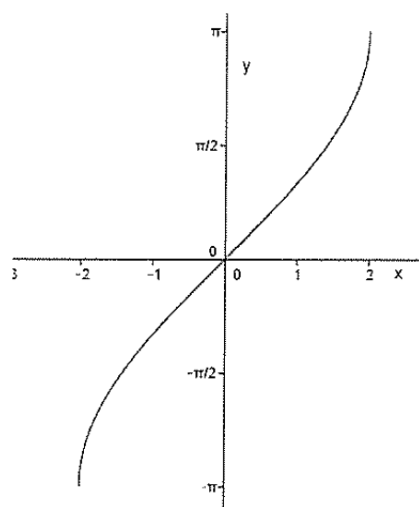


- (A) 11.36 N , $22^\circ 25'$ to the horizontal.
(B) 11.36 N , $67^\circ 35'$ to the horizontal.
(C) 12.58 N , $22^\circ 25'$ to the horizontal.
(D) 12.58 N , $67^\circ 35'$ to the horizontal.

7. Which of the following is a simplified expression for $\frac{\sin 2x}{1 - \cos 2x}$?

- (A) $\sin x$
(B) $\cos x$
(C) $\tan x$
(D) $\cot x$

8. The graph of the following function is best described by what equation?



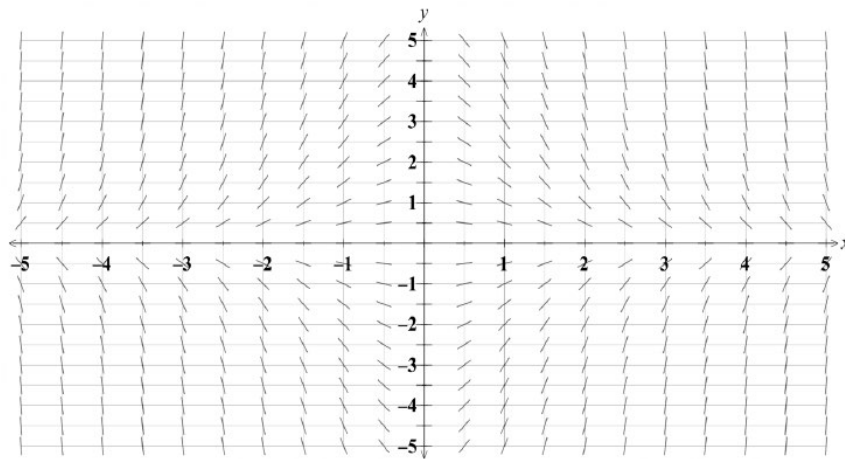
(A) $y = 2 \cos^{-1}(2x)$

(B) $y = 2 \cos^{-1}\left(\frac{x}{2}\right)$

(C) $y = 2 \sin^{-1}\left(\frac{x}{2}\right)$

(D) $y = 2 \sin^{-1}(2x)$

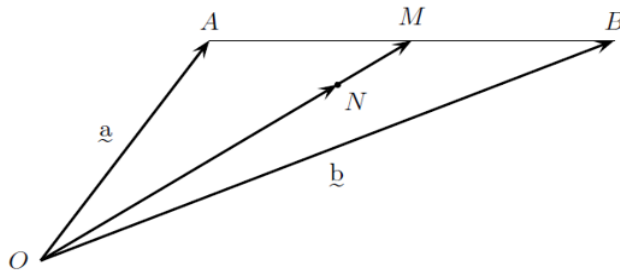
9. Which of the following differential equations is represented by the slope field shown below?



- (A) $\frac{dy}{dx} = xy$
 (B) $\frac{dy}{dx} = -xy$
 (C) $\frac{dy}{dx} = -x^2y$
 (D) $\frac{dy}{dx} = x^2y$

10. In the following diagram, M is the midpoint of the line segment AB , and $\overrightarrow{ON} = \frac{4}{5}\overrightarrow{OM}$.

It is given that $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$.



Which of the following expressions correctly relates $\overrightarrow{NM}$ with $\underline{a}$ and $\underline{b}$?

- (A) $\overrightarrow{NM} = \frac{4}{5}(\underline{a} - \underline{b})$
 (B) $\overrightarrow{NM} = \frac{2}{5}(\underline{a} + \underline{b})$
 (C) $\overrightarrow{NM} = \frac{1}{10}(\underline{a} + \underline{b})$
 (D) $\overrightarrow{NM} = \frac{1}{10}(\underline{a} - \underline{b})$

Section II
60 marks
Attempt Questions 11-14

Answer each question in a *SEPARATE* answer booklet. Extra writing booklets are available.
In questions 11-14, your responses should show relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a separate writing book

MARKS

(a) For what values of a are the vectors $\begin{pmatrix} a \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 2a-3 \\ 2 \end{pmatrix}$ perpendicular? **3**

(b) Use the substitution $u = e^x$ to integrate $\int \frac{e^x}{1+e^{2x}} dx$. **3**

(c) Solve $\frac{2x}{x-3} \geq 1$ **3**

(d) Find $\int (\cos x - \sin x)(\sin x + \cos x) dx$ **3**

(e) Find the equation of the tangent to $y = \sin^{-1}(\ln(x))$ at $x = 1$. **3**

End of Question 11

Question 12 (15 marks) Use a separate writing book

MARKS

(a) Prove by induction that for all integers $n \geq 1$, that $3^{2n+4} - 2^{2n}$ is divisible by 5 3

(b) Differentiate $x \tan^{-1} x$ and hence show that $\int_0^1 \tan^{-1} x dx = \frac{\pi}{4} - \frac{1}{2} \ln 2$ 3

(c) Find the constant term in the expansion $\left(2x + \frac{3}{x^3}\right)^8$ 2

(d) Solve $\sin 2x = \cos x$ for $0 \leq x \leq 2\pi$ 2

(e) A still lake has experienced a sewerage spill and a circular sludge slick is floating on the surface of the water. 2

The area of the sludge slick is increasing by $0.1m^2$ / minute .

At what rate is the radius increasing when the area is $0.3m^2$ correct to 2 decimal places?

(f) A projectile has an initial velocity vector $\mathbf{v} = \begin{bmatrix} 2\sqrt{3} \\ 2 \end{bmatrix}$, assume gravity $\mathbf{g}_i = -10m/s^2$ 1

(i) What is the initial speed and angle of projection from the horizontal?

(ii) When does the maximum height of the projectile occur? 2

End of Question 12

Question 13 (15 marks) Use a separate writing book

MARKS

(a) Consider vectors $\underline{a} = 4\underline{i} - 5\underline{j}$ and $\underline{b} = -2\underline{i} + 4\underline{j}$.

(i) Find the magnitude and direction of $\underline{a} + \underline{b}$.

2

(ii) Find the resultant vector $2\underline{b} - \underline{a}$.

2

(iii) Calculate the dot product $\underline{a} \bullet \underline{b}$.

1

(iv) Find the angle between the vectors $\underline{a}$ and $\underline{b}$

2

(b) The temperature of a COVID vaccine liquid t minutes after being placed in a freezer is given by the equation $T = -4 + Ae^{-kt}$, where A and k are constants.

(i) Initially the vaccine liquid is at a temperature of 40°C .
Find the value of A .

1

(ii) When the temperature of the liquid vaccine is 26°C the rate of change of the temperature of the liquid is -0.3°C per minute. Show that $k = 0.01$.

2

(iii) Find the time taken for the temperature of the vaccine liquid to fall from 40°C to 6°C . (answer in minutes to one decimal place)

2

(c) Given that α and β are two roots of the polynomial equation $x^3 + px^2 + qx + r = 0$, it is known that one of roots equals the sum of the other two roots.

(i) Show $p = 2(\alpha + \beta)$

1

(ii) Hence or otherwise, prove that $p^3 = 4pq - 8r$.

2

End of Question 13

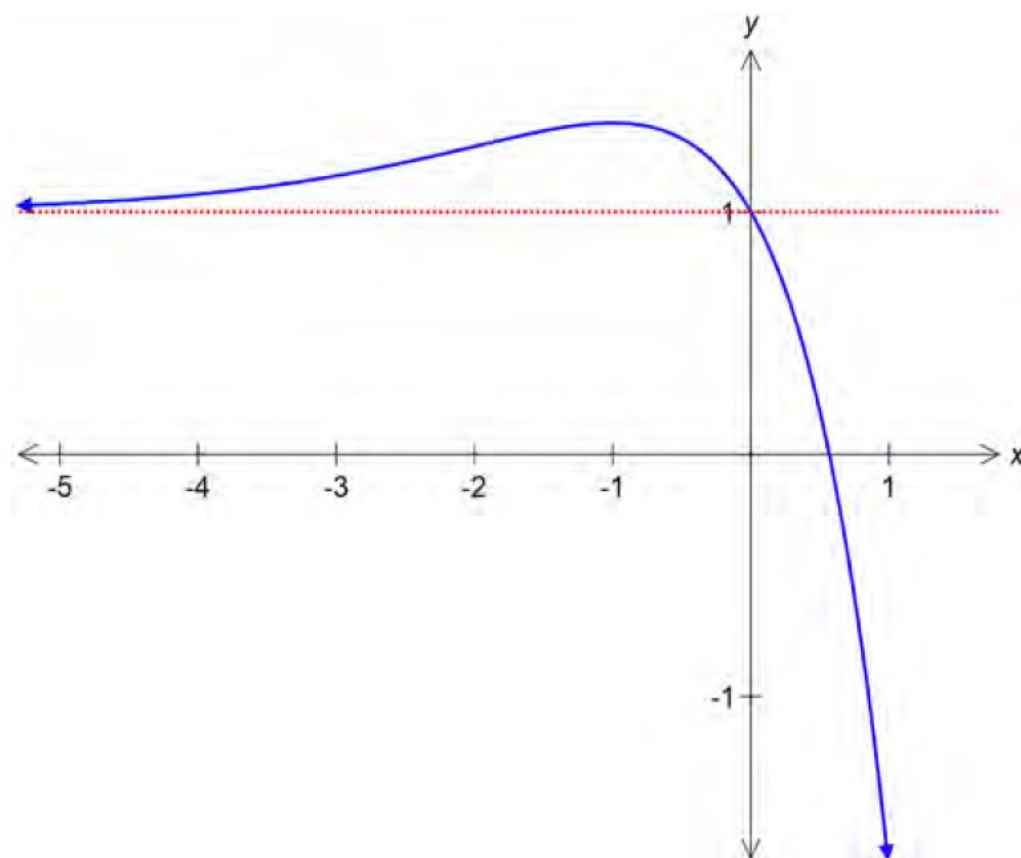
Question 14 (15 marks) Use a separate writing book

MARKS

- (a) If there are 7 people to sit at a round table and two particular people refuse to sit together how many arrangements are then possible?

2

- (b) The graph of the function $y = f(x)$ is shown below.



Sketch the following curve, showing any key features. (on the attached answer sheet)

2

$$y = \frac{1}{f(x)}$$

c) A fair six sided die is tossed 3600 times. The random variable X is the number of even numbered rolls recorded.

(i) Assuming that X can be accurately approximated by a normal distribution, find the values of μ and σ such that $X \sim N(\mu, \sigma^2)$

1

(ii) Find the z scores for throwing 1725 evens and 1845 evens.

1

(iii) Hence use the table below to estimate the probability of throwing between 1845 evens and 1725 evens.

1

First Decimal Place										
z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

(d) Sketch the graph of $y = -\sin(2x)$ between $0 \leq x \leq 2\pi$ also determine the exact volume of solid of revolution when the graph of $y = -\sin(2x)$ is rotated about the x axis between $0 \leq x \leq \pi$.

4

Question 14 continued on next page

MARKS

(e) A financial model in an Economics class is given by $\frac{dy}{dx} = \frac{y(100-y)}{100}$,

4

by showing that $\frac{100}{y(100-y)} = \frac{1}{y} + \frac{1}{100-y}$,

prove that $\frac{dy}{dx} = \frac{y(100-y)}{100}$, given $y=20$ when $x=0$, has a solution

$$y = \frac{100}{1 + 4e^{-x}}.$$

End of Question 14

END OF EXAMINATION



Merewether High School 2021
Year 12 Mathematics Extension 1
Trial HSC

Name: _____

Teacher: _____

Shade the most correct answer.

	A	B	C	D
1	☐	☐	☐	☐
2	☐	☐	☐	☐
3	☐	☐	☐	☐
4	☐	☐	☐	☐
5	☐	☐	☐	☐
6	☐	☐	☐	☐
7	☐	☐	☐	☐
8	☐	☐	☐	☐
9	☐	☐	☐	☐
10	☐	☐	☐	☐

Name: _____

Teacher: _____

Year 12 Mathematics Extension 1

Trial HSC

Question 14 (a) to be completed on this sheet.

