Knox Grammar School

2024

Trial Higher School Certificate Examination

Name: ______________________

Teacher: ____________________

Year 12 Extension 1 Mathematics

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using blue or black pen only
- Board approved calculators may be used
- A reference sheet is provided at the back of this paper in addition to the separate yellow document. You may use either.
- In Questions 11-14, show relevant mathematical reasoning and/or calculations.

Teachers:

Mr Bradford (Examiner)

Mr Menzies

Mr Willcocks

Mr Meli

Mr Cheah

Mrs Dempsey

Section I ~ Pages 1-4

- 10 marks
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II ~ Pages 5-11

- 60 marks
- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Section I
Questions 1 – 10 (1 mark for each question)

Read each question and choose an answer A, B, C or D.
Record your answer on the Answer Sheet provided.
Allow about 15 minutes for this section.

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1. Which of the following is the simplest expression for $\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$?

   A) $\frac{1}{2}\ \text{cosec}\ 2\theta$

   B) $2\ \text{cosec}\ 2\theta$

   C) $2\sec 2\theta$

   D) $\frac{1}{2}\sin 2\theta$

2. What is the domain of the function $f(x) = 3\ cos^{-1}\left(1 + \frac{x}{2}\right)$?

   A) $[-4\,,0\,]$

   B) $[\ -4\,,\ 4\,]$

   C) $[-2\,,\ 2\,]$

   D) $[-1\,,\ 1\,]$

3. Which of the following parametric equations represents a parabola with a minimum at $(0\,,2)$?

   A) $x = \ t - \frac{1}{t}\ ,\ \ y = 2t^2 + \frac{2}{t^2}$

   B) $x = \ 2t + \frac{2}{t}\ ,\ \ y = t^2 + \frac{1}{t^2}$

   C) $x = \ 2t - \frac{2}{t},\ \ y = t^2 + \frac{1}{t^2}$

   D) $x = 2t - \frac{2}{t},\ \ y = t^2 - \frac{1}{t^2}$

4. Consider the vector $\underset{\sim}{u} = 5\underset{\sim}{i} - 12\underset{\sim}{j}$.

Which of the following is a vector $\underset{\sim}{v}$ which is perpendicular to $\underset{\sim}{u}$ and has magnitude 26?

A) $-12\underset{\sim}{i} - 5\underset{\sim}{j}$

B) $12\underset{\sim}{i} + 5\underset{\sim}{j}$

C) $24\underset{\sim}{i} - 10\underset{\sim}{j}$

D) $-24\underset{\sim}{i} - 10\underset{\sim}{j}$

5. When the polynomial $P(x)$ is divided by $x + 1$ it gives a remainder n^2, where n is a positive integer.

When $P(x)$ is divided by $(x + 1)(x + 3)$ it gives a remainder $(nx + 6)$.

What is the value of n?

A) 1

B) 2

C) 3

D) 6

6. Consider the vector $\underset{\sim}{a} = \sin 3\lambda\, \underset{\sim}{i} - \cos 3\lambda\, \underset{\sim}{j}$, where λ is a constant such that $0 \leq \lambda \leq \frac{\pi}{4}$.

The length of the projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$, where $\underset{\sim}{b} = \cos\lambda\, \underset{\sim}{i} + \sin\lambda\, \underset{\sim}{j}$, is $\frac{\sqrt{3}}{2}$.

Which of the following is $\underset{\sim}{a}$?

A) $\underset{\sim}{a} = 0\,\underset{\sim}{i} - \underset{\sim}{j}$

B) $\underset{\sim}{a} = \frac{\sqrt{3}}{2}\underset{\sim}{i} - \underset{\sim}{j}$

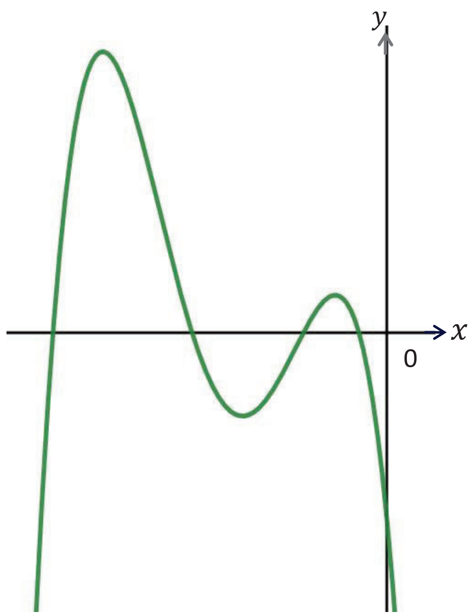
C) $\underset{\sim}{a} = \underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}$

D) $\underset{\sim}{a} = \underset{\sim}{i} + 0\,\underset{\sim}{j}$

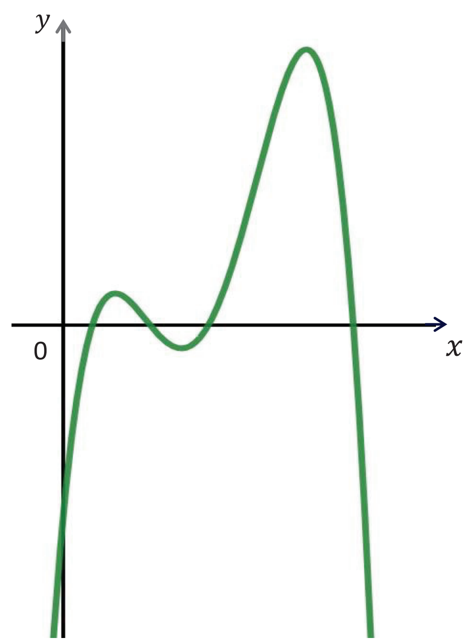
7. Consider the polynomial $f(x) = ax^4 + bx^3 + cx + d$, where the constant a is negative and the constant b is positive.

Which graph could represent $f(x)$?

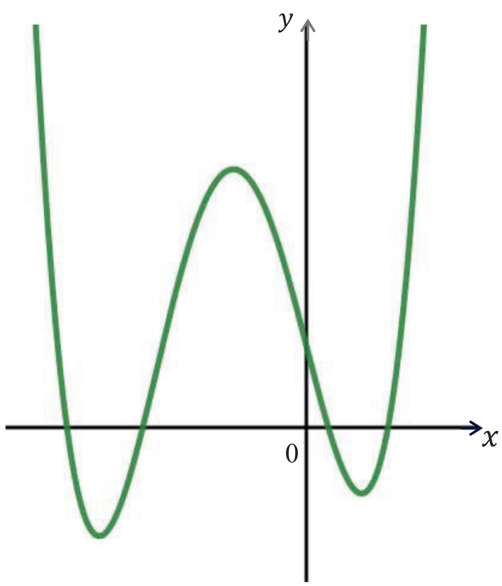
A)



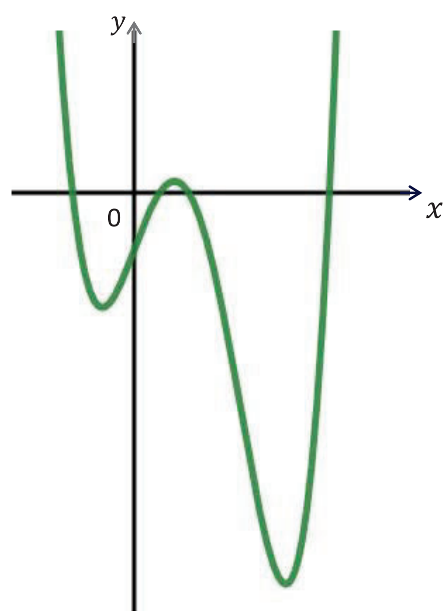
B)



C)



D)



8. What is the value of $\cos^{-1}[\cos(7\pi+\theta)]$, where θ an acute angle?

A) $7\pi+\theta$

B) $\pi+\theta$

C) $\pi-\theta$

D) θ

9. A cup of tea is placed on a table in a room with temperature p.

The temperature of the cup is given by $T = p + re^{-kt}$, where T is the temperature in degrees Celsius C and t is the time in minutes after the cup is placed on the table.

Initially, the cup's temperature was five times the room's temperature and after 15 minutes it became three times the room's temperature.

At what rate was the temperature of the cup decreasing when the temperature of the cup was 30°C more than the room temperature?

A) $1.39°C$ per minute

B) $1.54°C$ per minute

C) $1.62°C$ per minute

D) $1.87°C$ per minute

10. Jared has ten toy cars that are distinguishable by make. Three of these cars are red, two are blue and the remaining have other, different colours.

In how many ways can Jared arrange his cars in a row if the blue cars must be at the end positions and the red cars are to be separated?

A) 80 640

B) 28 800

C) 13 440

D) 2 400

MATHEMATICS EXTENSION 1

Section II

60 marks

Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section.

INSTRUCTIONS

- Answer each question in a separate writing booklet.
- Extra writing booklets are available.
- Your responses should include relevant mathematical reasoning and/or calculations.

Section II

Questions 11 – 14 (60 marks)

Allow about 1 hour 45 minutes for this section.

Question 11 (15 marks)

a) Find the coefficient of x^2 in the expansion of $\left(2x + \frac{1}{x} \right)^{10}$. **2**

b) For what value of p are the vectors $\underset{\sim}{a} = \begin{pmatrix} 3p+1 \\ 3 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 2p \\ 9 \end{pmatrix}$ parallel? **3**

c) Solve the inequality $\frac{1}{x+2} \leq -2$. **3**

d) The probability of tossing a head using a biased coin is 70%.

Layla is to toss this coin 120 times.

Let X be the random variable representing the number of heads she will toss.

This random variable will have a binomial distribution.

i) Find the expected value $E(X)$. **1**

ii) By finding the variance, $\mathrm{Var}(X)$, show that the standard deviation of X is approximately 5. **1**

iii) By using a normal approximation, find the approximate probability that X is between 74 and 96. **2**

You may use the information on page 12 to answer this question.

e) The area of a regular hexagon is $A = \frac{\sqrt{3}}{24}P^2$, where P is its perimeter. **3**

The area of the hexagon is increasing at the rate of $4\sqrt{3}\ \mathrm{cm}^2 s^{-1}$.

At what rate is its perimeter increasing when its perimeter is 40 cm?

Question 12 (15 marks)

a) Rachael has birds in a cage. Each day in December and January she took out two birds to inspect them and then returned them. **2**

On every day of these two months, she never got the same combination of the two birds that had been previously examined.

What is the least number of birds in Rachael's cage?

b) Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} 2\sin 2x\cos x\,dx$ **3**

c) In a fun park there are 6 sliding rides and 8 jumping castles.

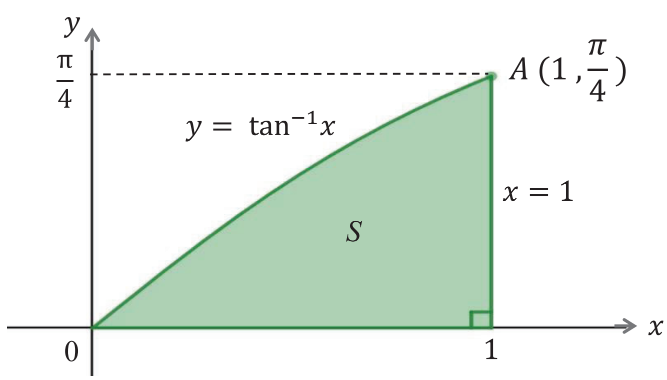
On average, each sliding ride is in use 80% of the time and each jumping castle is in use 70% of the time.

i) Find the probability that at a certain time exactly 5 of the 6 sliding rides are in use. **1**

ii) Find the probability that at a certain time exactly 5 of the 6 sliding rides are in use and 7 of the 8 jumping castles are also in use. **2**

d) In the diagram the graphs of the curve $y = \tan^{-1}x$ and the line $y = \frac{\pi}{4}$ intersect at the point $A\left(1, \frac{\pi}{4}\right)$. **4**

The shaded region S is bounded the curve, the line $x = 1$ and the x axis as shown.



Find the volume of the solid of revolution when the region S is rotated about the y axis.

e) Use mathematical induction to prove that, for any integer $n \geq 1$, **3**

$4 + 7 + 10 + \cdots + (3n + 1) = \frac{n}{2}(3n + 5)$.

Question 13 (15 marks)

a) A recent census found that 40% of the people living on an island are sailors. **3**
A sample of n randomly selected people living on an island was surveyed.

Let $\hat{p}$ be the sample proportion of surveyed people who are sailors.
A normal distribution has been used and an approximate value of $P(\hat{p} \leq 0.42)$ is calculated to be 0.9251.

Use the standard normal distribution and the information on page 12 to find an approximate value for n which is the number of people surveyed in the sample.

Give your answer correct to the nearest whole number.

b) A model for the population, P, of deer in a forest, is

$P = \dfrac{27\,000}{3 - 27^{-\frac{t}{48}}}$, where t is the time in years from today.

i) When will the population of deer drop to 75% of today's population? **2**

ii) According to the model, predict what will be the eventual population of deer. **1**

iii) Sketch the graph of the population P. **2**

iv) By differentiating $P = \dfrac{27\,000}{3 - 27^{-\frac{t}{48}}}$, show that P satisfies **3**

the differential equation $\dfrac{dP}{dt} = \dfrac{1}{16} P \ln 3 \left(1 - \dfrac{P}{9\,000}\right)$.

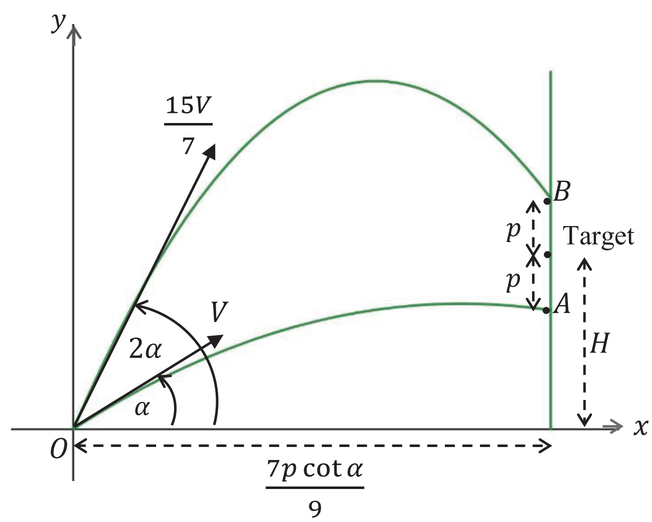
Question 13 continues on page 9

b) Jacob hits two balls at the same time from the origin O towards a target on a vertical wall. **4**

The target is at a horizontal distance $\dfrac{7p\cot\alpha}{9}$ metres and at a vertical distance H metres above the origin.

The initial velocity of one ball is V m/s and its angle of projection is α to the horizontal while the initial velocity of the second ball is $\dfrac{15V}{7}$ m/s and its angle of projection to the horizontal is 2α. After T seconds both balls hit the wall.

The ball with angle of projection α hits the wall at A which is p metres below the target, while the other ball, with angle of projection 2α, hits the wall at B, which is p metres above the target as shown in the diagram below.



The position vectors of the balls t seconds after being hit, are given by

$$\underset{\sim}{r}_{ball_1}(t) = \begin{pmatrix} Vt\cos\alpha \\ -\frac{1}{2}gt^2 + Vt\sin\alpha \end{pmatrix} \quad \text{and}$$

$$\underset{\sim}{r}_{ball_2}(t) = \begin{pmatrix} \frac{15\,V}{7}t\cos 2\alpha \\ -\frac{1}{2}gt^2 + \frac{15\,V}{7}t\sin 2\alpha \end{pmatrix}$$

Show that $\cos\alpha = \frac{5}{6}$.

End of Question 13

Question 14 (15 marks)

a) The resultant force on a particle, which is moving subject to the forces $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$, **3**

has magnitude $2M$ newtons and its direction is on a bearing of $290°$.

The force $\overrightarrow{F_1}$ has magnitude 90 newtons and its direction is on a bearing of $320°$.

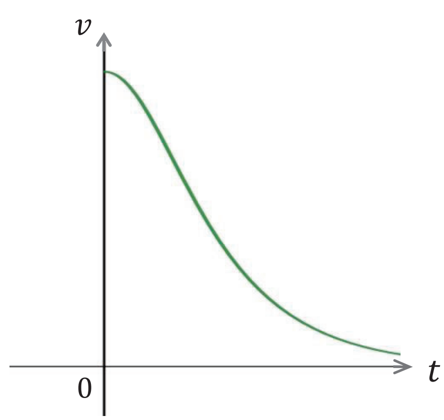
The force $\overrightarrow{F_2}$ has magnitude M newtons.

By drawing an appropriate force diagram, calculate the magnitude and direction of the force $\overrightarrow{F_2}$.

b) A particle P is moving along the x axis. At time $t = 0$ it was at $x = 0$. **3**

Its velocity $v\ m\ s^{-1}$ at a time t is $v = \dfrac{e^{t}}{1+e^{2t}}$.

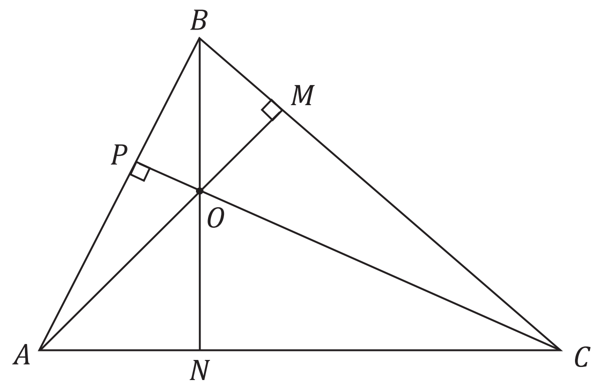
The graph below shows its velocity as a function of time.



Given that the displacement of the particle when $t = k$ seconds is $x = \tan^{-1}\frac{1}{2}$ metres, show that $k = ln\,3$.

Question 14 continues on page 11

c) In triangle ABC shown in the diagram below, AM and CP are the altitudes from A and C to the opposite sides BC and AB respectively.
Let the point of intersection of these two altitudes be O.



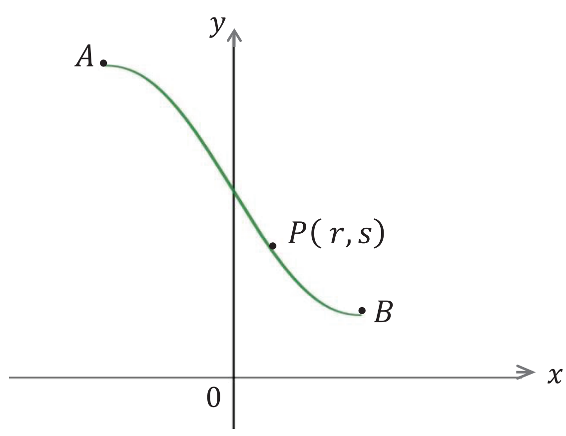
(i) By projecting $\overrightarrow{BO}$ and $\overrightarrow{BC}$ onto $\overrightarrow{BA}$, show that $\overrightarrow{BO} \cdot \overrightarrow{BA} = \overrightarrow{BC} \cdot \overrightarrow{BA}$. **1**

(ii) Show that the line from B to the opposite side AC which passes through the point O, is also an altitude. That is, show $\overrightarrow{BO}$ is perpendicular to $\overrightarrow{AC}$. **3**

d) The function $h(x) = K^{-1}(x)$ is the inverse function of $K(x)$ in the domain $-a \leq x \leq a$, where $a > 0$.

The point $P(r, s)$ which is in the first quadrant, lies on the curve.

$h(x) = 2\cos^{-1}\left(\frac{3x}{x^2 + 4}\right)$ and the gradients of the tangent lines at the end points $A(-a, h(-a))$ and $B(a, h(a))$ are both zero.

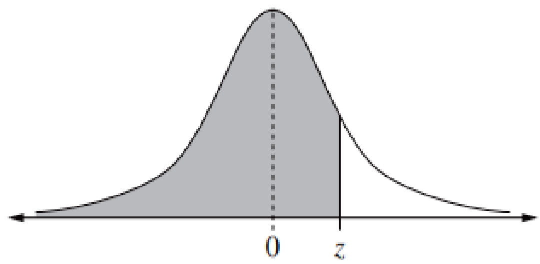


i) Show that $a = 2$. **3**

ii) Given that $6\,[h'(r)]^2 = 2\,[K'(s)]^2 + 1$, show that $K'(s) = -\frac{\sqrt{6}}{2}$. **2**

END OF PAPER

Table of values $P(Z \le z)$ for the normal distribution $N(0,1)$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

NSW Education Standards Authority

2024 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\dfrac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

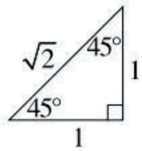
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

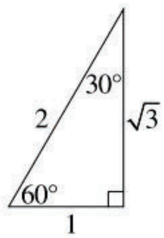
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

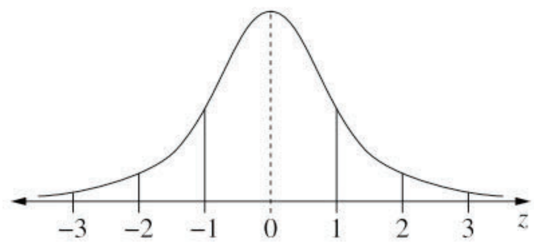
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow \ P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\dfrac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$
$y = \dfrac{u}{v}$	$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\dfrac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\dfrac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\dfrac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\dfrac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\dfrac{dy}{dx} = -\dfrac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\dfrac{dy}{dx} = \dfrac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-\left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+\left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2 + y^2}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^n &= r^n(\cos n\theta + i\sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$