ABBOTSLEIGH

**Student's Name:** ______________________

**Student Number:**

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| | | | | | | | | |

**Teacher's Name:**

**2025** **TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

# Mathematics Extension 1

**General Instructions**

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided separately
- Diagrams are not to scale unless indicated
- Ensure your HSC candidate number is on each booklet
- For questions in Section I, answer on the sheet provided
- For questions in Section II,
  - show all necessary working
  - start a new booklet for each question

**Total marks: 70**

**Section I – 10 marks** (pages 3-8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

**Section II – 60 marks** (pages 10-17)

- Attempt Questions 11–15
- Allow about 1 hour and 45 minutes for this section

## Year 11 Mathematics Extension 1 outcomes

**ME11-1** uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses
**ME11-2** manipulates algebraic expressions and graphical functions to solve problems
**ME11-3** applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems
**ME11-4** applies understanding of the concept of a derivative in the solution of problems, including rates of change, exponential growth and decay and related rates of change
**ME11-5** uses concepts of permutations and combinations to solve problems involving counting or ordering
**ME11-7** communicates making comprehensive use of mathematical language, notation, diagrams and graphs

## Year 12 Mathematics Extension 1 outcomes

**ME12-1** applies techniques involving proof or calculus to model and solve problems
**ME12-2** applies concepts and techniques involving vectors and projectiles to solve problems
**ME12-3** applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations
**ME12-4** uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution
**ME12-5** applies appropriate statistical processes to present, analyse and interpret data
**ME12-7** evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

## Section I

**10 Marks**
**Attempt Questions 1–10**
**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Questions 1-10.

---

**1** Given that $\overrightarrow{AD} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{DO} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$, what is $\overrightarrow{OA}$?

A. $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

B. $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$

C. $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

D. $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$

**2** Consider the polynomial $P(x) = 4x^2 + 5x - 10$.

When $(2x - a) \times P(x)$ is divided by $(x - 1)$ the remainder is 3.

What is the value of $a$?

A. $-1$

B. 1

C. 2

D. 5

**Section I continues on page 4**

**3** Which of the following is an anti-derivative of $\dfrac{1}{\sqrt{9-4x^2}}$?

A. $\dfrac{\sin^{-1}\left(\dfrac{2x}{3}\right)}{2}+c$

B. $\dfrac{\sin^{-1}\left(\dfrac{3x}{2}\right)}{3}+c$

C. $\dfrac{\sin^{-1}(2x)}{3}+c$

D. $\dfrac{\sin^{-1}\left(\dfrac{2x}{3}\right)}{6}+c$

**4** A factory produces light switches. Each switch has a 6% chance of being faulty.
A quality control manager selects a random sample of 80 switches.
Let $X$ represent the number of faulty switches in the sample, where $X$ is a binomial variable.
What is the standard deviation of $X$, correct to 2 decimal places?

A. 2.12

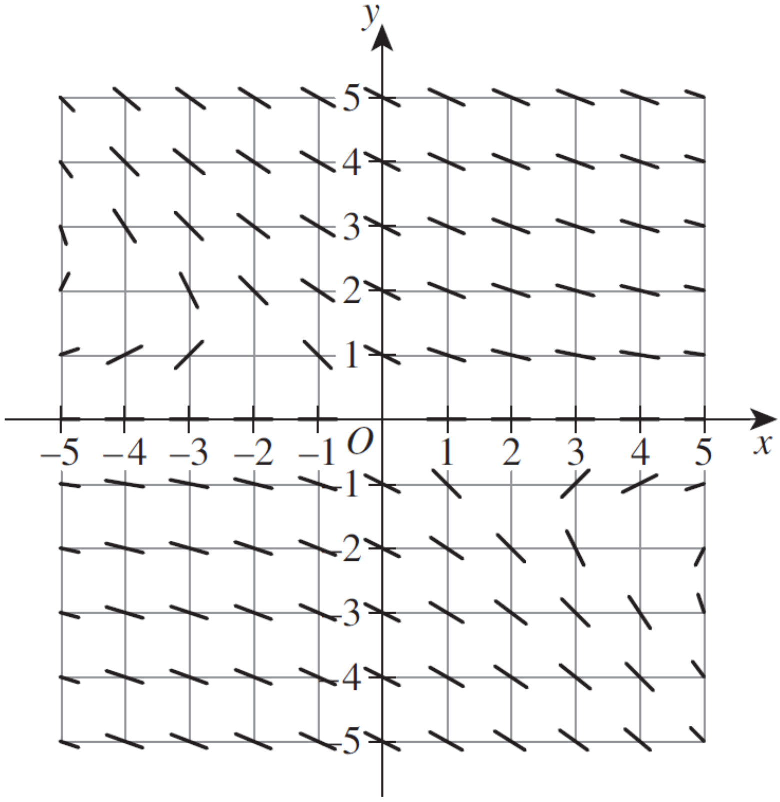
B. 2.13

C. 4.51

D. 4.8

**Section I continues on page 5**

**5** The direction field for a differential equation is shown below.




Which of the differential equations below could give this direction field?

A. $\dfrac{dy}{dx} = \dfrac{y}{2y + x}$

B. $\dfrac{dy}{dx} = -\dfrac{y}{2y + x}$

C. $\dfrac{dy}{dx} = \dfrac{y}{-2y + x}$

D. $\dfrac{dy}{dx} = \dfrac{y}{2y - x}$

**Section I continues of page 6**

**6** The vectors $\underset{\sim}{d}, \underset{\sim}{u}, \underset{\sim}{b}, \underset{\sim}{o}$ are each chosen to be either $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

If each possible selection is equally likely, what is the probability that

$\underset{\sim}{d} + \underset{\sim}{u} + \underset{\sim}{b} + \underset{\sim}{o} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$ ?

A. $\frac{1}{24}$

B. $\frac{3}{16}$

C. $\frac{1}{8}$

D. $\frac{3}{8}$

**7** For the vectors $\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = 9\underset{\sim}{i} + 3\underset{\sim}{j}$, which of the following represents $|\underset{\sim}{a} + \underset{\sim}{b}|$?

A. $\sqrt{37}$

B. $\sqrt{103}$

C. 12

D. 13

**Section I continues on page 7**

**8** The function $f(x)$ exists for all real numbers $x$.

The function $g(x)$ is defined such that $g(x) = f(x) + \dfrac{1}{f(x)}$.

If $f(2) = -1$ and $f'(2) > 0$, which of the following is true about $g(x)$?

A. $g(2) = -2$ and $g'(2) > 0$

B. $g(2) = 1$ and $g'(2) = 0$

C. $g(2) = -2$ and $g'(2) = 0$

D. $g(2) = 1$ and $g'(2) < 0$

**9** A first-order differential equation is given by

$$\frac{dy}{dx} = 2y \ln y$$

Which of the following is a solution to the differential equation?

A. $y = e^{2e^x}$

B. $y = e^{e^{2x}}$

C. $y = e^{e^{x^2}}$

D. $y = 2e^{e^x}$

**Section I continues on page 8**

**10** For a particular function $f(x)$, it is known that $f(0)=0$ and $x \times f(x) > 0$ for all non-zero values of $x$.

It is also known that $\int_{-2}^{2} f(x)\, dx = 4$ and $\int_{-2}^{2} |\, f(x) |\, dx = 8$.

Which of the following gives the value of $\int_{-2}^{0} f(|x|)\, dx$ ?

A. $-2$

B. $2$

C. $4$

D. $6$

**End of Section I**

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## Section II

**60 Marks**

**Attempt Questions 11–15**
**Allow about 1 hour and 45 minutes for this section**

Answer each question in a separate writing booklet. Use extra booklets if needed.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

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**Question 11** (12 marks) Use a SEPARATE writing booklet **Marks**

(a) For the vectors $\underset{\sim}{a} = 10\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} - \underset{\sim}{j}$, evaluate each of the following.

(i) $3\underset{\sim}{a} - \underset{\sim}{b}$ **1**

(ii) $(\underset{\sim}{a} + 2\underset{\sim}{b}) \cdot (\underset{\sim}{a} - 2\underset{\sim}{b})$ **2**

(b) Using the substitution $u = x + 4$, find the exact value of $\displaystyle\int_{-4}^{-3} x(x+4)^{10}\, dx$. **3**

.

(c) In how many different ways can all the letters of the word **TENNANT** be arranged in a line? **2**

(d) Find the constant term in the expansion of $\left(2x - \dfrac{1}{x}\right)^{10}$. **2**

(e) Solve $\sqrt{2}\cos\left(x + \dfrac{\pi}{4}\right) + \sqrt{2}\cos\left(x - \dfrac{\pi}{4}\right) = 1$ for $-\pi \le x \le \pi$. **2**

**End of Question 11**

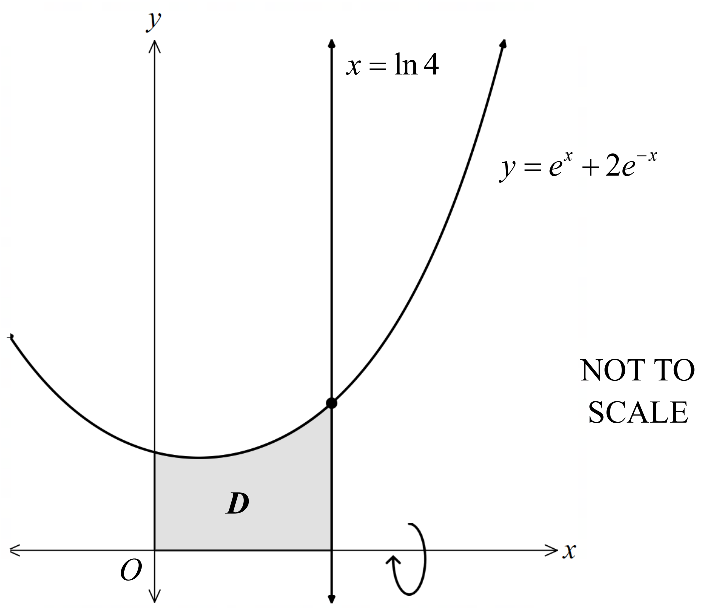
**Question 12** (12 marks) Use a SEPARATE writing booklet **Marks**

(a) (i) Show that $\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)=\frac{2}{1+\sin x}$. **2**

(ii) Hence, or otherwise, evaluate $\int_0^{\pi} \frac{2}{1+\sin x}\,dx$. **2**

(b) The volume of a cylinder with a fixed height $h$ cm, and radius $r$ cm is given by $V=\pi r^2 h$. If the volume is increasing at a rate of $20\text{ cm}^3\text{ s}^{-1}$, show that $\frac{dr}{dt}=\frac{10}{\pi r h}\text{ cm s}^{-1}$. **2**

(c) The region, ***D***, is bounded by the $y$-axis, the $x$-axis, the line with equation $x=\ln 4$ and the curve with equation $y=e^x+2e^{-x}$, as shown in the diagram. **3**




Find the volume of the solid of revolution formed when the region ***D*** is rotated about the $x$-axis. Leave your answer in exact form.

(d) The polynomial $P(x)=8x^4-38x^3+9x^2+ax+b$ has a double root at $x=3$. **3**

Find the values of $a$ and $b$.

**End of Question 12**

**Question 13** (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Use mathematical induction to show that for all integers $n \geq 1$, **3**

$4^{2n} - 1$ is divisible by 3.

(b) Consider the function given by $f(x) = \tan^{-1}\left(\dfrac{\cos x}{\sin x + 1}\right)$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

(i) Find $f'(x)$. **2**

(ii) Hence sketch the graph of $y = f(x)$. **1**

(c) Solve the differential equation $\dfrac{dy}{dx} = x\sqrt{(1-x^2)(1-y^2)}$ given that it **3**

passes through the point $(1, 0)$. Give your answer with $y$ as the subject.

(d) Prove that **3**

$$\frac{1}{{}^{2025}C_{26}} = \frac{26}{25}\left(\frac{1}{{}^{2024}C_{25}} - \frac{1}{{}^{2025}C_{25}}\right)$$

**End of Question 13**

**Question 14** (13 marks) Use a SEPARATE writing booklet **Marks**

(a) A team of ecologists is re-introducing a group of 3000 Komodo dragons to an island in Indonesia. From observations on other islands, the ecologists know that populations of more than 3000 are unsustainable.

They propose the following logistic model of population growth

$$\frac{dK}{dt} = rK(3000 - K)$$

where $K$ is the number of Komodo dragons on the island, $t$ is the time in years and $r$ is a constant.

(i) Ecologists know that when the maximum growth rate occurs, all the Komodo dragons find a mate. Half of these pairs successfully raise one juvenile Komodo dragon. **2**

Show that $r = \dfrac{1}{6000}$.

(ii) The ecologists initially release 500 Komodo dragons on the island. **3**

Use the fact that $\dfrac{1}{K(3000-K)} = \dfrac{1}{3000}\left(\dfrac{1}{K} + \dfrac{1}{3000-K}\right)$ to show

that the population of Komodo dragons can be given by

$$K = \frac{3000}{5e^{-0.5t} + 1}.$$

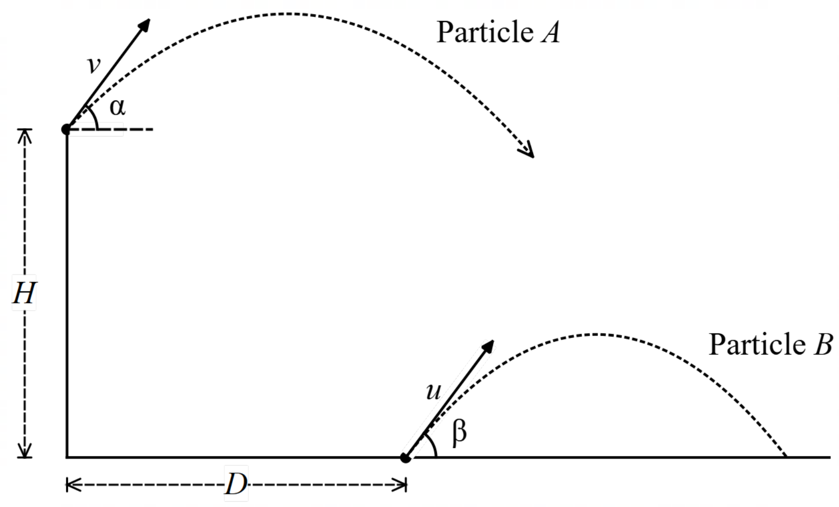
(iii) How long will it take for the Komodo population to reach 95% of the maximum sustainable population? Give your answer correct to 2 decimal places. **1**

**Question 14 continues on page 14**

(b) Particle $A$ is projected from a vertical tower that is $H$ metres above the ground with an initial speed of $v$ m s $^{-1}$ at an acute angle of $\alpha$ with the horizontal plane.

At the same time, from another point $D$ metres horizontally away from the base of the vertical tower, particle $B$ is projected with an initial speed of $u$ m s $^{-1}$ at an acute angle of $\beta$ with the horizontal plane.

This is shown in the diagram below.




Taking the position of the base of the tower as the origin, the positions of the projectiles at time $t$ after they are launched are as follows.

$$\vec{r}_A = \begin{pmatrix} vt\cos\alpha \\ vt\sin\alpha - \frac{g}{2}t^2 + H \end{pmatrix} \qquad \text{Particle } A$$

$$\vec{r}_B = \begin{pmatrix} ut\cos\beta + D \\ ut\sin\beta - \frac{g}{2}t^2 \end{pmatrix} \qquad \text{Particle } B \qquad \text{(DO NOT prove these.)}$$

**Question 14 continues on page 15**

(i) It is known that $u\sin\beta > v\sin\alpha$. (DO NOT prove this.) **3**

Show that for the two particles to collide before either reaches the ground, then:

$$2u\sin\beta(u\sin\beta - v\sin\alpha) > gH\ .$$

(ii) Show, by expanding the right-hand side, that: **1**

$$\sin(2\beta)+\cos(2\beta)=\sqrt{2}\sin\left(2\beta+\frac{\pi}{4}\right).$$

(iii) If $u=v$, $H=\dfrac{u^2}{g}$ and $\alpha=\dfrac{\pi}{2}-\beta$, show that $\dfrac{3\pi}{8}<\beta<\dfrac{\pi}{2}$ **3**

if the two particles collide before hitting the ground.

**End of Question 14**

**Question 15** (11 marks) Use a SEPARATE writing booklet **Marks**

(a) ***You may use the information on page 19 to answer this question.***

At Hogwarts School, the probability of a troll being in the dungeon on any given day is 5%.

Students are becoming disgruntled with the number of trolls in the dungeon and are concerned one may appear during their examination block. They demand a sample size of $n$ is taken and the sample proportion $\hat{p}$ is calculated.

It is assumed that $\hat{p}$ is normally distributed with $E(\hat{p}) = p$ and

$$Var(\hat{p}) = \frac{p(1-p)}{n}.$$

The students will report Hogwarts School to the Ministry of Magic if a troll is in the dungeon during their 10-day examination block. That is, if $\hat{p} \geq 0.1$.

To protect the Hogwarts school's reputation, the Ministry of Magic suggests they choose a sample size so that the chance of students reporting Hogwarts is less than 0.62%.

(i) Show that the mean and standard deviation of the sample proportion are given by $0.05$ and $\sqrt{\dfrac{19}{400n}}$ respectively. **1**

(ii) Hence, or otherwise, find the approximate sample size required. **3**

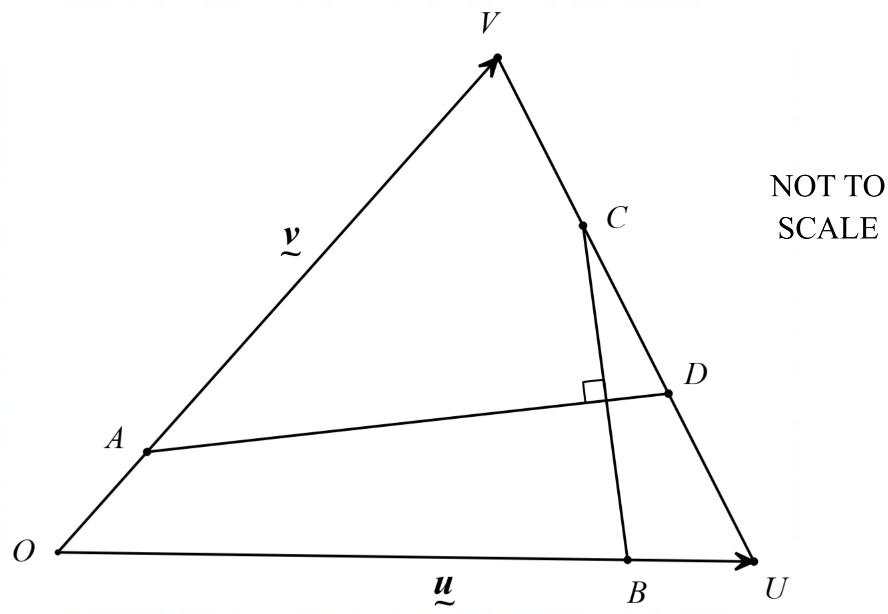
**Question 15 continues on page 17**

(b) Three different points $O$, $V$, and $U$ form an isosceles triangle with unit length sides such that $\left|\overrightarrow{OV}\right| = \left|\overrightarrow{OU}\right| = 1$.

The points $C$ and $D$ are chosen such that $\overrightarrow{VC} = \overrightarrow{CD} = \overrightarrow{DU} = \frac{1}{3}\overrightarrow{VU}$

(that is, the points $C$ and $D$ split the line into thirds.)

Let $\underset{\sim}{v} = \overrightarrow{OV}$ and $\underset{\sim}{u} = \overrightarrow{OU}$, as shown in the diagram.




(i) Show that $\overrightarrow{VD} = \frac{2}{3}\underset{\sim}{u} - \frac{2}{3}\underset{\sim}{v}$, and that $\overrightarrow{UC} = \frac{2}{3}\underset{\sim}{v} - \frac{2}{3}\underset{\sim}{u}$. **1**

(ii) The point $A$ lies on the line $OV$ such that $\overrightarrow{OA} = \lambda \underset{\sim}{v}$ and $0 < \lambda < 1$. **2**

Show that $\overrightarrow{AD} = \frac{2}{3}\underset{\sim}{u} + \left(\frac{1}{3} - \lambda\right)\underset{\sim}{v}$.

(iii) The point $B$ similarly divides the line $OU$ in the same ratio such that $\overrightarrow{BU} = \lambda \underset{\sim}{u}$. **4**

The lines $AD$ and $BC$ are perpendicular, as shown in the diagram.

By first finding an expression for $\overrightarrow{BC}$ in terms of $\underset{\sim}{u}$, $\underset{\sim}{v}$ and $\lambda$, show that $\frac{8}{17} \le \underset{\sim}{u} \cdot \underset{\sim}{v} \le 1$.

**End of Paper**

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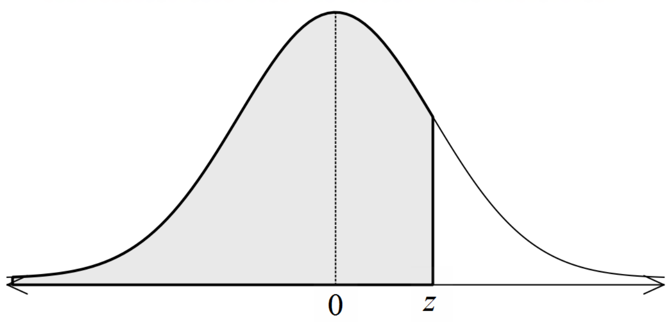
**Table of values $P(Z \le z)$ for the normal distribution $N(0,1)$**




| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| **0.1** | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| **0.2** | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| **0.3** | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| **0.4** | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| **0.5** | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| **0.6** | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| **0.7** | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| **0.8** | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| **0.9** | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| **1.0** | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| **1.1** | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| **1.2** | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| **1.3** | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| **1.4** | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| **1.5** | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| **1.6** | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| **1.7** | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| **1.8** | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| **1.9** | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| **2.0** | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| **2.1** | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| **2.2** | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| **2.3** | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| **2.4** | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| **2.5** | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| **2.6** | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| **2.7** | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| **2.8** | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| **2.9** | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| **3.0** | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |

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ABBOTSLEIGH

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**Student's Name:** Solutions

**Student Number:**

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**Teacher's Name:**

**2025** TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

# Mathematics Extension 1

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**Total marks:** **Section I – 10 marks** (pages 3-8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

**Section II – 60 marks** (pages 9-16)

- Attempt Questions 11–15
- Allow about 1 hour and 45 minutes for this section

## Year 11 Mathematics Extension 1 outcomes

**ME11-1** uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses
**ME11-2** manipulates algebraic expressions and graphical functions to solve problems
**ME11-3** applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problems
**ME11-4** applies understanding of the concept of a derivative in the solution of problems, including rates of change, exponential growth and decay and related rates of change
**ME11-5** uses concepts of permutations and combinations to solve problems involving counting or ordering
**ME11-6** uses appropriate technology to investigate, organise and interpret information to solve problems in a range of contexts
**ME11-7** communicates making comprehensive use of mathematical language, notation, diagrams and graphs

## Year 12 Mathematics Extension 1 outcomes

**ME12-1** applies techniques involving proof or calculus to model and solve problems
**ME12-2** applies concepts and techniques involving vectors and projectiles to solve problems
**ME12-3** applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations
**ME12-4** uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution
**ME12-5** applies appropriate statistical processes to present, analyse and interpret data
**ME12-6** chooses and uses appropriate technology to solve problems in a range of contexts
**ME12-7** evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

## Section I

**10 Marks**
**Attempt Questions 1–10**
**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Questions 1-10.

---

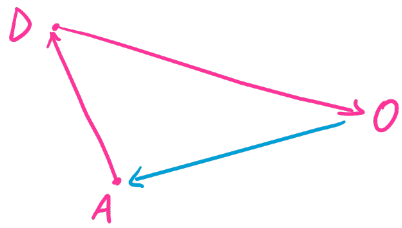
**1** Given that $\overrightarrow{AD} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{DO} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$, what is $\overrightarrow{OA}$?

A. $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

B. $\begin{pmatrix} -4 \\ -1 \end{pmatrix}$

C. $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

D. $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$




$\overrightarrow{OA} = \overrightarrow{OD} + \overrightarrow{DA}$

$\begin{pmatrix} -6 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

**2** Consider the polynomial $P(x) = 4x^2 + 5x - 10$.

When $(2x - a) \times P(x)$ is divided by $(x - 1)$ the remainder is 3.

What is the value of $a$?

A. $-1$

B. 1

C. 2

D. 5

$F(x) = (2x - a)(4x^2 + 5x - 10)$

$F(1) = 3$

$3 = (2(1) - a)(4(1)^2 + 5(1) - 10)$

$3 = (2 - a) \times (-1)$

$3 = a - 2$

$a = 5$

**3** Which of the following is an anti-derivative of $\frac{1}{\sqrt{9-4x^2}}$?

A. $\frac{\sin^{-1}\left(\frac{2x}{3}\right)}{2}+C$

B. $\frac{\sin^{-1}\left(\frac{3x}{2}\right)}{3}+C$

C. $\frac{\sin^{-1}(2x)}{3}+C$

D. $\frac{\sin^{-1}\left(\frac{2x}{3}\right)}{6}+C$

$$\int \frac{1}{\sqrt{9-4x^2}}\,dx$$

$$= \frac{1}{2}\int \frac{2}{\sqrt{(3)-(2x)^2}}\,dx$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + C$$

**4** A factory produces light switches. Each switch has a 6% chance of being faulty.

A quality control manager selects a random sample of 80 switches.

Let $X$ represent the number of faulty switches in the sample, where $X$ is a binomial variable.

What is the standard deviation of $X$, to 2 decimal places?

A. 2.12

B. 2.13

C. 4.51

D. 4.8

$$Var(X) = np(1-p)$$

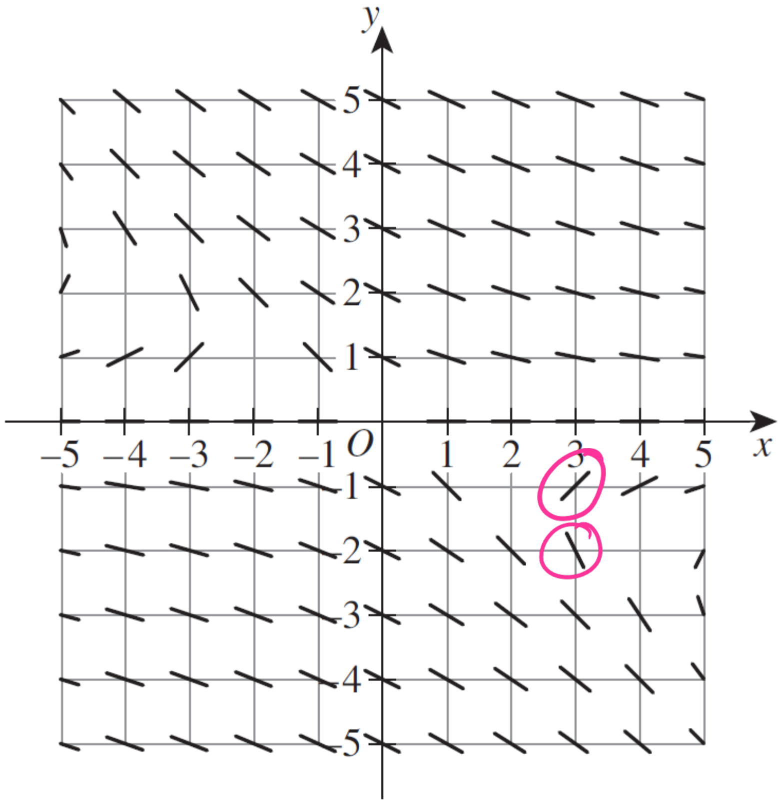
$$= 80 \times 0.06\,(1-0.06)$$

$$= 4.512$$

$$\sigma = \sqrt{4.512}$$

$$= 2.12\,(2dp)$$

**5** The direction field for a differential equation is shown below.




Which of the differential equations below could give this direction field?

A. $\dfrac{dy}{dx} = \dfrac{y}{2y+x}$

B. $\dfrac{dy}{dx} = -\dfrac{y}{2y+x}$

C. $\dfrac{dy}{dx} = \dfrac{y}{-2y+x}$

D. $\dfrac{dy}{dx} = \dfrac{y}{2y-x}$

Consider point $(3,-1)$ where $\dfrac{dy}{dx} > 0$ :

$A: \dfrac{dy}{dx} = \dfrac{-1}{2(-1)+3} = 1$

$B: \dfrac{dy}{dx} = -\dfrac{-1}{2(-1)+3} = 1$

$C: \dfrac{dy}{dx} = \dfrac{-1}{-2(-1)+3} = -\dfrac{1}{5}$ $\therefore$ rule out C

$D: \dfrac{dy}{dx} = \dfrac{-1}{2(-1)+3} = -1$ $\therefore$ rule out D

Now consider $(3,-2)$ where $\dfrac{dy}{dx} < 0$ :

$A: \dfrac{dy}{dx} = \dfrac{-2}{2(-2)+3} = 2$ $\therefore$ rule out A

**6** The vectors $\underset{\sim}{d}, \underset{\sim}{u}, \underset{\sim}{b}, \underset{\sim}{o}$ are each chosen to be either $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

If each possible selection is equally likely, what is the probability that

$\underset{\sim}{d} + \underset{\sim}{u} + \underset{\sim}{b} + \underset{\sim}{o} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}$ ?

$$\begin{pmatrix} 10 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

A. $\frac{1}{24}$

B. $\frac{3}{16}$

C. $\frac{1}{8}$

D. $\frac{3}{8}$

This is a binomial distribution with $p = 0.5$ and $n = 4$ !

Say the success is a vector being chosen as $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$,

then $P\left(\underset{\sim}{d} + \underset{\sim}{u} + \underset{\sim}{b} + \underset{\sim}{o} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}\right)$

$= P(X = 2)$

$= {}^4C_2 \times 0.5^2 \times 0.5^2$

$= \frac{3}{8}$

**7** For the vectors $\underset{\sim}{a} = 3\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = 9\underset{\sim}{i} + 3\underset{\sim}{j}$, which of the following represents $|\underset{\sim}{a} + \underset{\sim}{b}|$?

A. $\sqrt{37}$

B. $\sqrt{103}$

C. 12

D. 13

$\underset{\sim}{a} + \underset{\sim}{b} = 3\underset{\sim}{i} + 2\underset{\sim}{j} + 9\underset{\sim}{i} + 3\underset{\sim}{j}$

$= 12\underset{\sim}{i} + 5\underset{\sim}{j}$

$|\underset{\sim}{a} + \underset{\sim}{b}| = \sqrt{12^2 + 5^2}$

$= \sqrt{169}$

$= 13$

**8** The function $f(x)$ exists for all real numbers $x$.

The function $g(x)$ is defined such that $g(x) = f(x) + \dfrac{1}{f(x)}$.

If $f(2) = -1$ and $f'(2) > 0$, which of the following is true about $g(x)$?

A. $g(2) = -2$ and $g'(2) > 0$

B. $g(2) = 1$ and $g'(2) = 0$

C. $g(2) = -2$ and $g'(2) = 0$

D. $g(2) = 1$ and $g'(2) < 0$

$$g(2) = f(2) + \frac{1}{f(2)}$$
$$= -1 + \frac{1}{-1}$$
$$\boxed{g(2) = -2}$$
$$g(x) = f(x) + [f(x)]^{-1}$$
$$g'(x) = f'(x) - [f(x)]^{-2} \times f'(x)$$
$$= f'(x) - \frac{f'(x)}{[f(x)]^2}$$
$$g'(2) = f'(2) - \frac{f'(2)}{[f(2)]^2}$$
$$= f'(2) - \frac{f'(2)}{[-1]^2}$$
$$= f'(2) - f'(2)$$
$$\boxed{g'(2) = 0}$$

**9** A first-order differential equation is given by

$$\frac{dy}{dx} = 2y \ln y$$

Which of the following is a solution to the differential equation?

A. $y = e^{2e^x}$

B. $y = e^{e^{2x}}$

C. $y = e^{e^{x^2}}$

D. $y = 2e^{e^x}$

A: $y = e^{2e^x} \rightarrow \ln y = 2e^x$
$$\frac{dy}{dx} = e^{2e^x} \times 2e^x$$
$$= y \times \ln y \quad \therefore \text{not } A$$

B: $y = e^{e^{2x}} \rightarrow \ln y = e^{2x}$
$$\frac{dy}{dx} = e^{e^{2x}} \times 2e^{2x}$$
$$= 2e^{e^{2x}} \times e^{2x}$$
$$= 2y \times \ln y \quad \therefore B$$

**10** For a particular function $f(x)$, it is known that $f(0) = 0$ and $x \times f(x) > 0$ for all non-zero values of $x$.

$\therefore$ if $x>0$, $f(x)>0$
if $x<0$, $f(x)<0$

It is also known that $\int_{-2}^{2} f(x)\,dx = 4$ and $\int_{-2}^{2} |f(x)|\,dx = 8$.

Which of the following gives the value of $\int_{-2}^{0} f(|x|)\,dx$?

A. $-2$

B. $2$

C. $4$

D. $6$

$$\int_{-2}^{2} f(x)\,dx = \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

$$\therefore\ 4 = \int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

$$\int_{-2}^{2} |f(x)|\,dx = \int_{-2}^{0} |f(x)|\,dx + \int_{0}^{2} |f(x)|\,dx$$

$$= -\int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

$$\therefore\ 8 = -\int_{-2}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

① + ② :

$$12 = 2\int_{0}^{2} f(x)\,dx$$

$$\therefore\ 6 = \int_{0}^{2} f(x)\,dx$$

$$4 = \int_{-2}^{0} f(x)\,dx + 6$$

$$\therefore\ -2 = \int_{-2}^{0} f(x)\,dx$$

Now consider $f(|x|)$, which is $f(x)$ with anything on the left of the y-axis being replaced by a reflection of anything on the right of the y-axis.

$$\therefore\ \int_{-2}^{0} f(|x|)\,dx = \int_{0}^{2} f(x)\,dx$$

$$= 6$$

## Section II

**60 Marks**
**Attempt Questions 11–15**
**Allow about 1 hour and 45 minutes for this section**

Answer each question in a separate writing booklet. Use extra booklets if needed.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

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**Question 11** (12 marks) Use a SEPARATE writing booklet **Marks**

(a) For the vectors $\underset{\sim}{a} = 10\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} - \underset{\sim}{j}$, evaluate each of the following.

(i) $3\underset{\sim}{a} - \underset{\sim}{b}$ **1**

$= 3(10\underset{\sim}{i} + 2\underset{\sim}{j}) - (\underset{\sim}{i} - \underset{\sim}{j})$
$= 30\underset{\sim}{i} + 6\underset{\sim}{j} - \underset{\sim}{i} + \underset{\sim}{j}$
$= 29\underset{\sim}{i} + 7\underset{\sim}{j}$ 1

(ii) $(\underset{\sim}{a} + 2\underset{\sim}{b}) \cdot (\underset{\sim}{a} - 2\underset{\sim}{b})$ **2**

$(10\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{i} - 2\underset{\sim}{j}) \cdot (10\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{i} + 2\underset{\sim}{j})$
$= (12\underset{\sim}{i} + 0\underset{\sim}{j}) \cdot (8\underset{\sim}{i} + 4\underset{\sim}{j})$ 1
$= (12)(8) + (0)(4)$
$= 96$ 1

(b) Using the substitution $u = x + 4$, find the exact value of $\int_{-4}^{-3} x(x+4)^{10}\,dx$. **3**

Let $u = x + 4 \longrightarrow x = u - 4$

$\frac{du}{dx} = 1$

$\therefore du = dx$

when $x = -3$, $u = -3 + 4$

$u = 1$

when $x = -4$, $u = -4 + 4$

$u = 0$

1

$$\int_{-4}^{3} x(x+4)^{10}\,dx = \int_0^1 (u-4)(u)^{10}\,du \quad 1$$

$$= \int_0^1 u^{11} - 4u^{10}\,du$$

$$= \left[\frac{u^{12}}{12} - \frac{4u^{11}}{11}\right]_0^1$$

$$= \frac{1}{12} - \frac{4}{11} - (0 - 0)$$

$$= -\frac{37}{132} \quad 1$$

(c) In how many different ways can all the letters of the word TENNANT be arranged in a line? **2**

There are 7 letters but we have 2 T's and 3 N's :

1

$$\frac{7!}{\underbrace{2! \times 3!}_{1}} = 420 \text{ ways.}$$

(d) Find the constant term in the expansion of $\left(2x - \frac{1}{x}\right)^{10}$. **2**

General Term :

$$T_k = {}^{10}C_k \, (2x)^k (-x^{-1})^{10-k} (-1)^{10-k}$$

$$= {}^{10}C_k (2^k)(x^k)(x^{-10+k})(-1)^{10-k}$$

$$= {}^{10}C_k (2^k)(x^{2k-10})(-1)^{10-k}$$

for constant term,

$2k - 10 = 0$

$k = 5$ 1 or equivalent working

$\therefore \; T_5 = {}^{10}C_5 (2^5)(x^0)(-1)^5$

$= -8064$ 1

$\therefore$ constant term is -8064.

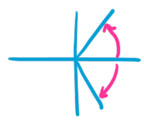
(e) Solve $\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)+\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=1$ for $-\pi \le x \le \pi$. **2**

$\sqrt{2}\cos x\cos\frac{\pi}{4} - \cancel{\sqrt{2}\sin x\sin\frac{\pi}{4}} + \sqrt{2}\cos x\cos\frac{\pi}{4} + \cancel{\sqrt{2}\sin x\sin\frac{\pi}{4}} = 1$ 1

$\sqrt{2}\cos x \times \frac{1}{\sqrt{2}} + \sqrt{2}\cos x \times \frac{1}{\sqrt{2}} = 1$

$2\cos x = 1$

$\cos x = \frac{1}{2}$



Related acute : $\frac{\pi}{3}$

$x = -\frac{\pi}{3}, \frac{\pi}{3}$ 1

**Question 12** (12 marks) Use a SEPARATE writing booklet **Marks**

(a) (i) Show that $\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)=\frac{2}{1+\sin x}$ **2**

$$LHS = \sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$= \frac{1}{\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}$$

$$= \frac{1}{\frac{1}{2}\left(1+\cos\left(2\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right)}$$ (1)

$$= \frac{2}{1+\cos\left(\frac{\pi}{2}-x\right)}$$

$$= \frac{2}{1+\cos\frac{\pi}{2}\cos x+\sin\frac{\pi}{2}\sin x}$$ (1)

$$= \frac{2}{1+0\times\cos x+1\times\sin x}$$

$$= \frac{2}{1+\sin x}$$

$= RHS$ as req.

(ii) Hence, or otherwise, evaluate $\displaystyle\int_0^{\pi} \frac{2}{1+\sin x}\,dx$ **2**

$$\int_0^{\pi} \frac{2}{1+\sin x}\,dx$$

$$= \int_0^{\pi} \sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)\,dx$$

$$= \left[\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{-\frac{1}{2}}\right]_0^{\pi} \quad 1$$

$$= \left[-2\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]_0^{\pi}$$

$$= -2\tan\left(\frac{\pi}{4}-\frac{\pi}{2}\right) - \left(-2\tan\left(\frac{\pi}{4}-0\right)\right)$$

$$= -2\tan\left(-\frac{\pi}{4}\right) + 2\tan\frac{\pi}{4}$$

$$= 2 + 2$$

$$= 4 \quad 1$$

(b) The volume of a cylinder with a fixed height $h$ cm, and radius $r$ cm is given by $V = \pi r^2 h$. If the volume is increasing at a rate of $20 \text{ cm}^3 \text{ s}^{-1}$, show that $\frac{dr}{dt} = \frac{10}{\pi r h} \text{ cm s}^{-1}$. **2**

$$\frac{dV}{dt} = 20$$

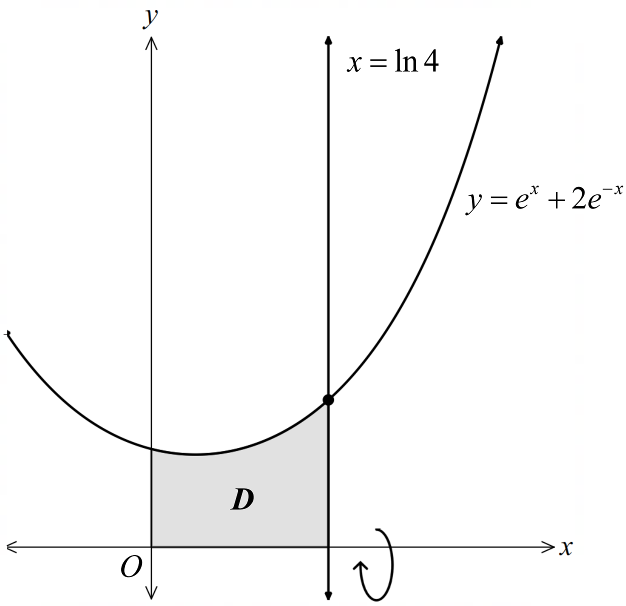
$$\frac{dV}{dr} = 2\pi r h \quad 1$$

$$\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$$

$$\frac{dr}{dt} = 20 \times \frac{1}{2\pi r h} \quad 1$$

$$= \frac{10}{\pi r h} \text{ cm/s} \quad \text{as req.}$$

(c) The region, $D$, is bounded by the $y$-axis, the $x$-axis, the line with equation $x = \ln 4$ and the curve with equation $y = e^x + 2e^{-x}$, as shown in the diagram. **3**




NOT TO SCALE

Find the volume of the solid of revolution formed when the region $D$ is rotated about the $x$-axis. Leave your answer in exact form.

$$V = \pi \int_0^{\ln 4} y^2 \, dx$$

$$= \pi \int_0^{\ln 4} (e^x + 2e^{-x})^2 \, dx$$ 1

$$= \pi \int_0^{\ln 4} e^{2x} + 4e^0 + 4e^{-2x} \, dx$$

$$= \pi \int_0^{\ln 4} e^{2x} + 4 + 4e^{-2x}$$

$$= \pi \left[ \frac{1}{2} e^{2x} + 4x - 2e^{-2x} \right]_0^{\ln 4}$$ 1

$$= \pi \left[ \frac{1}{2} e^{2\ln 4} + 4\ln 4 - 2e^{-2\ln 4} - \left( \frac{1}{2} e^0 + 4(0) - 2e^0 \right) \right]$$

$$= \pi \left[ 8 + 4\ln 4 - \frac{1}{8} - \frac{1}{2} + 2 \right]$$

$$= \pi \left( \frac{75}{8} + 4\ln 4 \right) \ u^3$$ 1

(d) The polynomial $P(x) = 8x^4 - 38x^3 + 9x^2 + ax + b$ has a double root at $x = 3$. **3**

Find the values of $a$ and $b$.

$P(x) = 8x^4 - 38x^3 + 9x^2 + ax + b$

$P'(x) = 32x^3 - 114x^2 + 18x + a$ (1)

Since $x = 3$ is a double root, $P'(3) = 0$:

$0 = 32(3)^3 - 114(3)^2 + 18(3) + a$

$0 = -108 + a$

$a = 108$ (1)

$\therefore P(x) = 8x^4 - 38x^3 + 9x^2 + 108x + b$

Similarly, $P(3) = 0$:

$0 = 8(3)^4 - 38(3)^3 + 9(3)^2 + 108(3) + b$

$0 = 27 + b$

$b = -27$ (1)

**Question 13** (12 marks) Use a SEPARATE writing booklet **Marks**

(a) Use mathematical induction to show that for all integers $n \geq 1$, **3**

$4^{2n} - 1$ is divisible by 3.

Base Case : Show true for $n = 1$

$$4^{2(1)} - 1 = 4^2 - 1$$
$$= 15$$
$$= 3 \times 5$$

$\therefore$ Divisible by 3 for base case $n = 1$ ①

Assume true for $n = k$ :

$4^{2k} - 1 = 3M$ where $M \in \mathbb{N}$

$\therefore 4^{2k} = 3M + 1$ *

Prove true for $n = k + 1$ :

$$4^{2(k+1)} - 1 = 4^{2k+2} - 1$$
$$= 4^{2k} \times 4^2 - 1$$
$$= 16 \times 4^{2k} - 1$$
$$= 16(3M + 1) - 1 \quad ① \text{ from assumption } *$$
$$= 48M + 16 - 1$$
$$= 48M + 15$$
$$= 3(16M + 5) \quad ①$$

which is divisible by 3 for all $M$, $M \in \mathbb{N}$

$\therefore$ Proven true by the principle of mathematical induction

(b) Consider the function given by $f(x) = \tan^{-1}\left(\dfrac{\cos x}{\sin x + 1}\right)$ for $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

(i) Find $f'(x)$. **2**

$$f(x) = \tan^{-1}\left(\frac{\cos x}{\sin x + 1}\right)$$

let $g(x) = \dfrac{\cos x}{\sin x + 1}$

$$u = \cos x \qquad v = \sin x + 1$$

$$u' = -\sin x \qquad v' = \cos x$$

$$g'(x) = \frac{-\sin x(\sin x + 1) - \cos^2 x}{(\sin x + 1)^2}$$

$$= \frac{-\sin^2 x - \sin x - \cos^2 x}{(\sin x + 1)^2}$$

$$= -\frac{(\sin^2 x + \cos^2 x + \sin x)}{(\sin x + 1)^2}$$

$$= -\frac{(1 + \sin x)}{(1 + \sin x)^2}$$

$$= -\frac{1}{1 + \sin x} \qquad 1$$

$$\therefore f'(x) = \frac{\frac{1}{1 + \sin x}}{1 + \left(\frac{\cos x}{1 + \sin x}\right)^2}$$

$$= \frac{-\frac{1}{1 + \sin x}}{1 + \frac{\cos^2 x}{(1 + \sin x)^2}} \times \frac{(1 + \sin x)^2}{(1 + \sin x)^2}$$

$$= \frac{-(1 + \sin x)}{(1 + \sin x)^2 + \cos^2 x}$$

$$= \frac{-(1 + \sin x)}{1 + 2\sin x + \sin^2 x + \cos^2 x}$$

$$= -\frac{(1 + \sin x)}{2 + 2\sin x}$$

$$= -\frac{(1 + \sin x)}{2(1 + \sin x)}$$

$$\therefore f'(x) = -\frac{1}{2} \qquad 1$$

(ii) Hence sketch the graph of $y = f(x)$. **1**

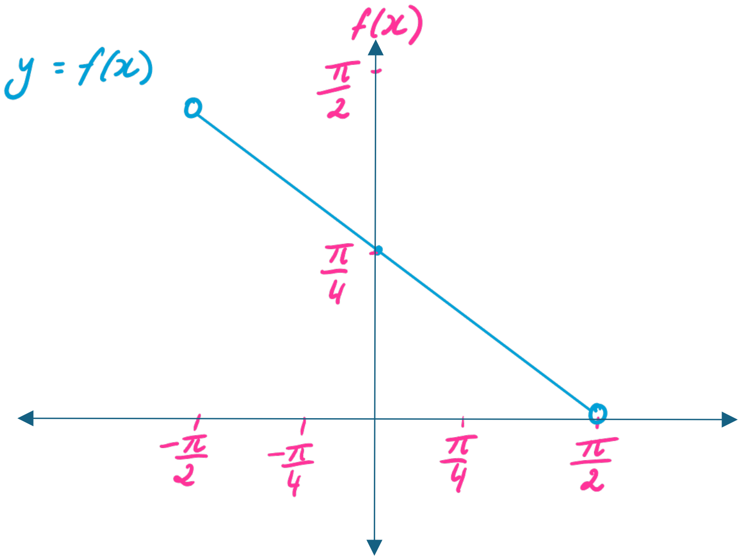
$\therefore f(x) = \tan^{-1}\left(\frac{\cos x}{\sin x + 1}\right) = -\frac{1}{2}x + C$ for $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f(0) = \tan^{-1} \frac{\cos 0}{\sin 0 + 1}$

$= \tan^{-1} 1$

$= \frac{\pi}{4}$ for $[-\pi, \pi]$

$\therefore f(x) = -\frac{1}{2}x + \frac{\pi}{4}$ for $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

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(c) Solve the differential equation $\frac{dy}{dx} = x\sqrt{(1-x^2)(1-y^2)}$ given that it passes through the point $(1, 0)$. Give your answer with $y$ as the subject. **3**

$$\frac{dy}{dx} = x\sqrt{(1-x^2)(1-y^2)}$$

$$\frac{1}{\sqrt{1-y^2}}\,dy = x\sqrt{1-x^2}\,dx$$

$$\int \frac{1}{\sqrt{1-y^2}}\,dy = \int x(1-x^2)^{\frac{1}{2}}\,dx \quad 1$$

$$\int \frac{1}{\sqrt{1-y^2}}\,dy = -\frac{1}{2}\int -2x(1-x^2)^{\frac{1}{2}}\,dx$$

$$\sin^{-1}y = \frac{-\frac{1}{2}(1-x^2)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$\sin^{-1}y = -\frac{1}{3}(1-x^2)^{\frac{3}{2}} + C$$

substitute $x = 1$, $y = 0$:

$$\sin^{-1}0 = -\frac{1}{3}(1-1^2)^{3/2} + C$$

$$0 = 0 + C$$

$$C = 0 \quad 1$$

$$\therefore \sin^{-1}y = -\frac{1}{3}(1-x^2)^{3/2}$$

$$y = \sin\left(-\frac{1}{3}(1-x^2)^{\frac{3}{2}}\right) \quad 1$$

(d) Prove that **3**

$$\frac{1}{{}^{2025}C_{26}} = \frac{26}{25}\left(\frac{1}{{}^{2024}C_{25}} - \frac{1}{{}^{2025}C_{25}}\right)$$

$$LHS = \frac{1}{{}^{2025}C_{26}}$$

$$= 1 \div \frac{2025!}{26!(2025-26)!}$$

$$= \frac{26!\,1999!}{2025!} \quad (1)$$

$$RHS = \frac{26}{25}\left(\frac{1}{{}^{2024}C_{25}} - \frac{1}{{}^{2025}C_{25}}\right)$$

$$= \frac{26}{25}\left(1 \div \frac{2024!}{25!(2024-25)!} - 1 \div \frac{2025!}{25!(2025-25)!}\right)$$

$$= \frac{26}{25}\left(\frac{25!\,1999!}{2024!} - \frac{25!\,2000!}{2025!}\right) \quad (1)$$

$$= \frac{26}{25}\left(\frac{25!\,1999!}{2024!} - \frac{25!\,2000!}{2025\times 2024!}\right)$$

$$= \frac{26}{25}\left(\frac{2025\times 25!\,1999! - 25!\,2000!}{2025!}\right)$$

$$= \frac{26}{25}\left(\frac{2025\times 25!\times 1999! - 25!\times 2000\times 1999!}{2025!}\right)$$

$$= \frac{26}{25}\left(\frac{25!\,1999!\,[2025 - 2000]}{2025!}\right)$$

$$= \frac{26}{\cancel{25}}\left(\frac{25!\,1999! \times \cancel{25}}{2025!}\right)$$

$$= \frac{26\times 25!\,1999!}{2025!} \quad (1)$$

$$= \frac{26!\,1999!}{2025!}$$

$= LHS$ as required.

**Question 14** (13 marks) Use a SEPARATE writing booklet **Marks**

(a) A team of ecologists is re-introducing a group of 3000 Komodo dragons to an island in Indonesia. From observations on other islands, the ecologists know that populations of more than 3000 are unsustainable.

They propose the following logistic model of population growth

$$\frac{dK}{dt} = rK(3000 - K)$$

where $K$ is the number of Komodo dragons on the island, $t$ is the time in years and $r$ is a constant.

(i) Ecologists know that when the maximum growth rate occurs, all the Komodo dragons find a mate. Half of these pairs successfully raise one juvenile Komodo dragon. **2**

Show that $r = \dfrac{1}{6000}$.

Max growth rate occurs when $\frac{dK}{dt}$ is maximised.

Since $\frac{dK}{dt}$ is a concave down quadratic, this occurs when:

$$K = \frac{0 + 3000}{2} \quad \text{(halfway between roots)}$$

$$K = 1500 \quad (1)$$

Which means there will be 750 pairs and hence 375 juveniles being added to the population.

$\therefore \frac{dK}{dt} = 375$ when $K = 1500$.

$$375 = 1500r(3000 - 1500)$$

$$375 = 2\,250\,000r$$

$$r = \frac{375}{2\,250\,000} \quad (1)$$

$$r = \frac{1}{6000} \text{ as required.}$$

(ii) The ecologists initially release 500 Komodo dragons on the island. **3**

Use the fact that $\dfrac{1}{K(3000-K)} = \dfrac{1}{3000}\left(\dfrac{1}{K} + \dfrac{1}{3000-K}\right)$ to show that the population of Komodo dragons can be given by

$$K = \frac{3000}{5e^{-0.5t}+1}.$$

$$\frac{dK}{dt} = \frac{1}{6000}K(3000-K)$$

$$\frac{1}{K(3000-K)}\,dK = \frac{1}{6000}\,dt$$

$$\int \frac{1}{K(3000-K)}\,dK = \int \frac{1}{6000}\,dt$$

$$\frac{1}{3000}\int \frac{1}{K} + \frac{1}{3000-K}\,dK = \int \frac{1}{6000}\,dt \quad \text{(using given fact)}$$

$$\int \frac{1}{K} - \frac{-1}{3000-K}\,dK = \int 0.5\,dt$$

$$\ln|K| - \ln|3000-K| = 0.5t + C \quad 1$$

$$\ln\left|\frac{K}{3000-K}\right| = 0.5t + C$$

at $t=0$, $K=500$:

$$\ln\left|\frac{500}{3000-500}\right| = 0 + C$$

$$\ln\frac{1}{5} = C$$

$$\therefore \quad \ln\left|\frac{K}{3000-K}\right| = 0.5t + \ln\frac{1}{5} \quad 1$$

$$\frac{K}{3000-K} = e^{0.5t+\ln\frac{1}{5}}$$

$$\frac{K}{3000-K} = e^{0.5t} \times e^{\ln\frac{1}{5}}$$

$$\frac{K}{3000-K} = \frac{e^{0.5t}}{5}$$

$$\frac{3000-K}{K} = 5e^{-0.5t}$$

$$3000 - K = 5Ke^{-0.5t}$$

$$3000 = 5Ke^{-0.5t} + K$$

$$3000 = K\left(5e^{-0.5t}+1\right) \quad 1$$

$$K = \frac{3000}{5e^{-0.5t}+1} \quad \text{as required}$$

(iii) How long will it take for the Komodo population to reach 95% of the maximum sustainable population? Give your answer correct to 2 decimal places. **1**

$3000 \times 0.95 = 2850$

let $K = 2850$:

$$2850 = \frac{3000}{5e^{-0.5t}+1}$$

$$5e^{-0.5t} + 1 = \frac{3000}{2850}$$

$$5e^{-0.5t} = \frac{1}{19}$$

$$e^{-0.5t} = \frac{1}{95}$$

$$-0.5t = \ln \frac{1}{95}$$

$$t = \frac{\ln \frac{1}{95}}{-0.5}$$

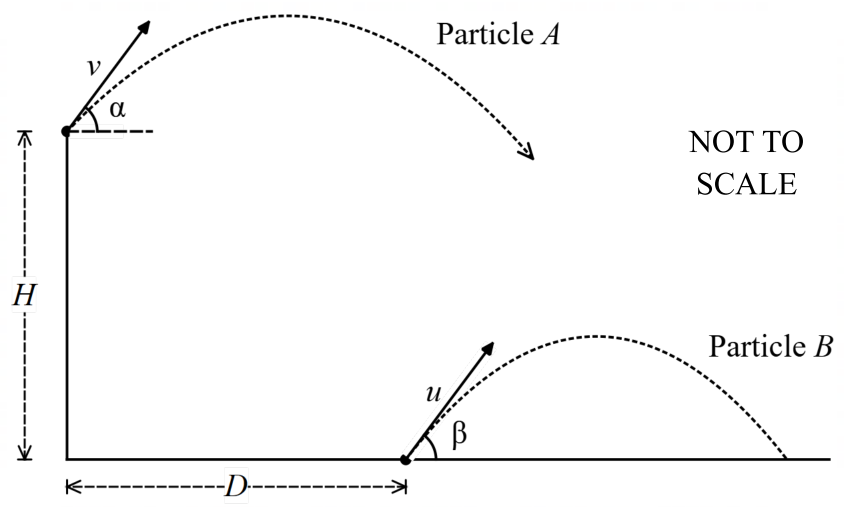
$$t = 9.1077\ldots \quad 1$$

$\therefore$ It will reach 95% of the sustainable population after approx 9.11 years

(b) Particle $A$ is projected from a vertical tower that is $H$ metres above the ground with an initial speed of $v$ m s$^{-1}$ at an acute angle of $\alpha$ with the horizontal plane.

At the same time, from another point $D$ metres horizontally away from the base of the vertical tower, particle $B$ is projected with an initial speed of $u$ m s$^{-1}$ at an acute angle of $\beta$ with the horizontal plane.

This is shown in the diagram below.




Taking the position of the base of the tower as the origin, the positions of the projectiles at time $t$ after they are launched are as follows.

$$\vec{r}_A = \begin{pmatrix} vt\cos\alpha \\ vt\sin\alpha - \frac{g}{2}t^2 + H \end{pmatrix} \qquad \text{Particle } A$$

$$\vec{r}_B = \begin{pmatrix} ut\cos\beta + D \\ ut\sin\beta - \frac{g}{2}t^2 \end{pmatrix} \qquad \text{Particle } B \qquad \text{(Do NOT prove these)}$$

(i) It is known that $u\sin\beta > v\sin\alpha$. (Do NOT prove this). **3**

Show that for the two particles to collide before either reaches the ground,

$$2u\sin\beta(u\sin\beta - v\sin\alpha) > gH$$

One possible way to do this (students had many valid methods!):

Particle hits the ground when $y_A$ or $y_B = 0$. Let $y_B = 0$:

$$ut\sin\beta - \frac{gt^2}{2} = 0$$

$$u\sin\beta - \frac{gt}{2} = 0$$

$$u\sin\beta = \frac{gt}{2}$$

$$t = \frac{2u\sin\beta}{g} \qquad 1$$

Particles will collide when $x_A = x_B$ and $y_A = y_B$. Solve for $y_A = y_B$:

$$vt\sin\alpha + H = ut\sin\beta$$

$$H = ut\sin\beta - vt\sin\alpha$$

$$H = t(u\sin\beta - v\sin\alpha)$$

$$t = \frac{H}{u\sin\beta - v\sin\alpha} \qquad 1$$

For particles to collide before hitting the ground, we have $t_{collision} < t_{hits\ ground}$:

$$\therefore \quad \frac{H}{u\sin\beta - v\sin\alpha} < \frac{2u\sin\beta}{g}$$

$$\frac{gH}{u\sin\beta - v\sin\alpha} < 2u\sin\beta$$

Since $u\sin\beta > v\sin\alpha$ from question, we know that $u\sin\beta - v\sin\alpha > 0$ and $\therefore$ we can multiply across without worrying about flipping the inequality sign: 1

$$gH < 2u\sin\beta(u\sin\beta - v\sin\alpha)$$

$$2u\sin\beta(u\sin\beta - v\sin\alpha) > gH \quad \text{as req.}$$

(ii) Show, by expanding the right-hand side, that **1**

$$\sin(2\beta) + \cos(2\beta) = \sqrt{2}\sin\left(2\beta + \frac{\pi}{4}\right).$$

$RHS = \sqrt{2}\sin\left(2\beta + \frac{\pi}{4}\right)$

$= \sqrt{2}\left(\sin 2\beta \cos\frac{\pi}{4} + \cos 2\beta \sin\frac{\pi}{4}\right)$ 1

$= \cancel{\sqrt{2}}\left(\sin 2\beta \times \frac{1}{\cancel{\sqrt{2}}} + \cos 2\beta \times \frac{1}{\cancel{\sqrt{2}}}\right)$

$= \sin 2\beta + \cos 2\beta$

$= LHS$ as req.

(iii) If $u = v$, $H = \dfrac{u^2}{g}$ and $\alpha = \dfrac{\pi}{2} - \beta$, show that $\dfrac{3\pi}{8} < \beta < \dfrac{\pi}{2}$ if the two particles collide before hitting the ground. **3**

Sub known facts into inequality from (i):

$$2u \sin\beta \left( u \sin\beta - u \sin\left(\frac{\pi}{2} - \beta\right)\right) > g \times \frac{u^2}{g}$$

$$2u \sin\beta \left( u \sin\beta - \underbrace{u \cos\beta}_{\text{using } \sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta} \right) > u^2 \qquad 1$$

$$2\cancel{u^2} \sin^2\beta - 2\cancel{u^2} \sin\beta\cos\beta > \cancel{u^2}$$

$$2\sin^2\beta - 2\sin\beta\cos\beta > 1$$

$$\cancel{2}\left(\cancel{\tfrac{1}{2}}(1 - \cos 2\beta)\right) - \cancel{2}\left(\cancel{\tfrac{1}{2}}(\sin(\beta+\beta) + \sin(\beta-\beta))\right) > 1$$

$$\cancel{1} - \cos 2\beta - (\sin 2\beta + \sin 0) > \cancel{1}$$

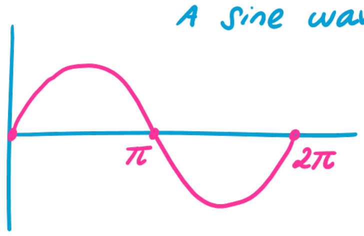
$$-\cos 2\beta - \sin 2\beta > 0$$

$$\sin 2\beta + \cos 2\beta < 0$$

$$\sqrt{2} \sin\left(2\beta + \frac{\pi}{4}\right) < 0 \qquad \text{from (ii)} \qquad 1$$

$$\sin\left(2\beta + \frac{\pi}{4}\right) < 0$$

A sine wave is below $x$-axis between $\pi$ and $2\pi$




$\therefore$ We need to solve for:

$$\pi < 2\beta + \frac{\pi}{4} < 2\pi$$

$$\frac{3\pi}{4} < 2\beta < \frac{7\pi}{4}$$

$$\frac{3\pi}{8} < \beta < \frac{7\pi}{8}$$

But since $\beta$ is acute, we have: 1

$$\frac{3\pi}{8} < \beta < \frac{\pi}{2}$$

**Question 15** (11 marks) Use a SEPARATE writing booklet **Marks**

(a) *You may use the information on page 17 to answer this question.*

At Hogwarts School, the probability of a troll being in the dungeon on any given day is 5%.

Students are becoming disgruntled with the number of trolls in the dungeon and are concerned one may appear during their examination block. They demand a sample size of $n$ is taken and the sample proportion $\hat{p}$ is calculated.

It is assumed that $\hat{p}$ is normally distributed with $E(\hat{p}) = p$ and

$Var(\hat{p}) = \dfrac{p(1-p)}{n}$.

The students will report Hogwarts School to the Ministry of Magic if a troll is in the dungeon during their 10-day examination block. That is, if $\hat{p} \geq 0.1$.

To protect the Hogwarts school's reputation, the Ministry of Magic suggests they choose a sample size so that the chance of students reporting Hogwarts is less than 0.62%.

(i) Show that mean and standard deviation of the sample proportion is given by $0.05$ and $\sqrt{\dfrac{19}{400n}}$ respectively. **1**

$E(\hat{p}) = p$

$= 0.05$

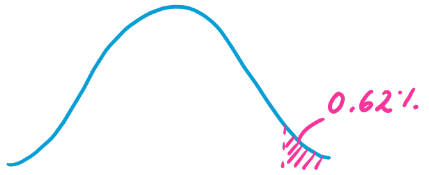
$Var(\hat{p}) = \dfrac{p(1-p)}{n}$

$= \dfrac{0.05(1-0.05)}{n}$

$= \dfrac{19}{400n}$ $\Rightarrow \therefore \sigma_{\hat{p}} = \sqrt{\dfrac{19}{400n}}$ 1

(ii) Hence, or otherwise, find the approximate sample size required. **3**

i.e. find $n$ such that $P(\hat{p} \geq 0.1) < 0.0062$




Consider $P(p \geq 0.1) = 0.0062$

$\therefore P(p < 0.1) = 1 - 0.0062$

$= 0.9938$

Reading backwards from z-table,

$P(z \leq 2.5) = 0.9938$

$\therefore z = 2.5$ 1

Since $z = 2.5$,

$0.1 = \mu + 2.5\sigma$

$0.1 = 0.05 + 2.5\sqrt{\frac{19}{400n}}$ 1

$0.05 = 2.5\sqrt{\frac{19}{400n}}$

$0.02 = \sqrt{\frac{19}{400n}}$

$0.0004 = \frac{19}{400n}$

$400n = \frac{19}{0.0004}$

$n = 118.75$ 1

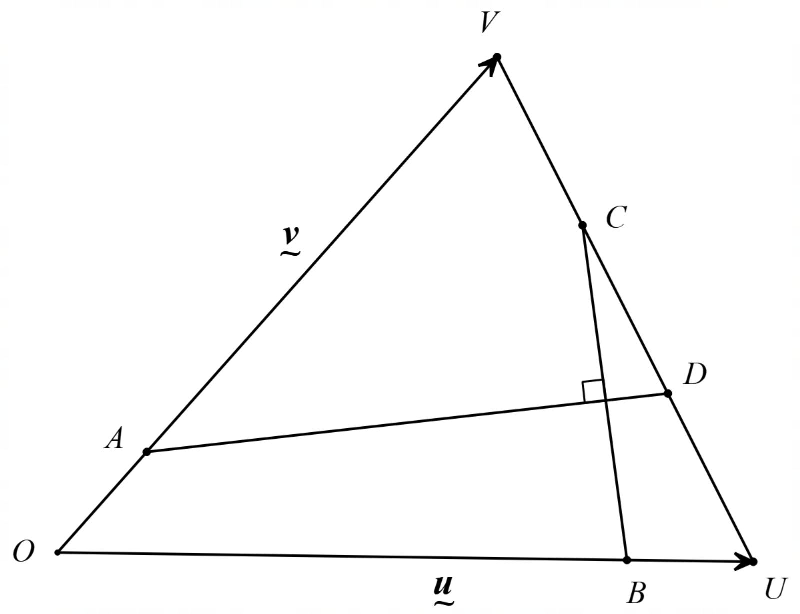
$\therefore$ The sample size should be approximately 119

Question 15 (continued) **Marks**

(b) Three different points $O$, $V$, and $U$ form an isosceles triangle with unit length sides such that $\left|\overrightarrow{OV}\right| = \left|\overrightarrow{OU}\right| = 1$.

The points $C$ and $D$ are chosen such that $\overrightarrow{VC} = \overrightarrow{CD} = \overrightarrow{DU} = \frac{1}{3}\overrightarrow{VU}$ (that is, it splits the line into thirds).

Let $\underset{\sim}{v} = \overrightarrow{OV}$ and $\underset{\sim}{u} = \overrightarrow{OU}$, as shown in the diagram.




(i) Show that $\overrightarrow{VD} = \frac{2}{3}\underset{\sim}{u} - \frac{2}{3}\underset{\sim}{v}$, and that $\overrightarrow{UC} = \frac{2}{3}\underset{\sim}{v} - \frac{2}{3}\underset{\sim}{u}$. **1**

$\overrightarrow{VU} = -\underset{\sim}{v} + \underset{\sim}{u}$ and $\overrightarrow{UV} = \underset{\sim}{v} - \underset{\sim}{u}$

$\overrightarrow{VD} = \frac{2}{3}\overrightarrow{VU}$

$\therefore \overrightarrow{VD} = \frac{2}{3}(-\underset{\sim}{v} + \underset{\sim}{u})$

$= \frac{2}{3}\underset{\sim}{u} - \frac{2}{3}\underset{\sim}{v}$

1

Similarly, $\overrightarrow{UC} = \frac{2}{3}\overrightarrow{UV}$

$\therefore \overrightarrow{UC} = \frac{2}{3}(\underset{\sim}{v} - \underset{\sim}{u})$

$= \frac{2}{3}\underset{\sim}{v} - \frac{2}{3}\underset{\sim}{u}$

(ii) The point $A$ lies on the line $OV$ such that $\overrightarrow{OA} = \lambda \underset{\sim}{v}$ and $0 < \lambda < 1$. **2**

Show that $\overrightarrow{AD} = \frac{2}{3}\underset{\sim}{u} + \left(\frac{1}{3} - \lambda\right)\underset{\sim}{v}$.

$\overrightarrow{OA} = \lambda \underset{\sim}{v}$ $\quad \therefore \overrightarrow{AV} = (1-\lambda)\underset{\sim}{v}$ $\quad 1$

$\overrightarrow{AD} = \overrightarrow{AV} + \overrightarrow{VD}$

$= (1-\lambda)\underset{\sim}{v} + \frac{2}{3}\underset{\sim}{u} - \frac{2}{3}\underset{\sim}{v}$

$= \frac{2}{3}\underset{\sim}{u} + \left(1 - \lambda - \frac{2}{3}\right)\underset{\sim}{v}$ $\quad 1$

$= \frac{2}{3}\underset{\sim}{u} + \left(\frac{1}{3} - \lambda\right)\underset{\sim}{v}$ as req.

(iii) The point $B$ similarly divides the line $OU$ in the same ratio such that $\overrightarrow{BU} = \lambda \underset{\sim}{u}$. **4**

The lines $AD$ and $BC$ are perpendicular, as shown in the diagram.

By first finding an expression for $\overrightarrow{BC}$ in terms of $\underset{\sim}{u}$, $\underset{\sim}{v}$ and $\lambda$, show that $\frac{8}{17} \le \underset{\sim}{u} \cdot \underset{\sim}{v} \le 1$.

Since $AD \perp BC$, $\overrightarrow{AD} \cdot \overrightarrow{BC} = 0$

$$\overrightarrow{BC} = \overrightarrow{BU} + \overrightarrow{UC}$$

$$= \lambda \underset{\sim}{u} + \frac{2}{3}\underset{\sim}{v} - \frac{2}{3}\underset{\sim}{u}$$

$$= \left(\lambda - \frac{2}{3}\right)\underset{\sim}{u} + \frac{2}{3}\underset{\sim}{v}$$

$$\therefore \left(\frac{2}{3}\underset{\sim}{u} + \left(\frac{1}{3} - \lambda\right)\underset{\sim}{v}\right) \cdot \left(\left(\lambda - \frac{2}{3}\right)\underset{\sim}{u} + \frac{2}{3}\underset{\sim}{v}\right) = 0 \quad 1$$

$$\frac{2}{3}\left(\lambda - \frac{2}{3}\right)\underset{\sim}{u} \cdot \underset{\sim}{u} + \frac{4}{9}\underset{\sim}{u} \cdot \underset{\sim}{v} + \left(\frac{1}{3} - \lambda\right)\left(\lambda - \frac{2}{3}\right)\underset{\sim}{u} \cdot \underset{\sim}{v}$$

$$+ \frac{2}{3}\left(\frac{1}{3} - \lambda\right)\underset{\sim}{v} \cdot \underset{\sim}{v} = 0$$

$$\left(\frac{2}{3}\lambda - \frac{4}{9}\right)|\underset{\sim}{u}|^2 + \frac{4}{9}\underset{\sim}{u} \cdot \underset{\sim}{v} + \left(\frac{1}{3}\lambda - \frac{2}{9} - \lambda^2 + \frac{2}{3}\lambda\right)\underset{\sim}{u} \cdot \underset{\sim}{v}$$

$$+ \left(\frac{2}{9} - \frac{2}{3}\lambda\right)|\underset{\sim}{v}|^2 = 0$$

and from question, we know that $|\overrightarrow{OV}| = |\overrightarrow{OU}| = 1$

$\therefore$ $|\underset{\sim}{u}| = |\underset{\sim}{v}| = 1$ :

$$\cancel{\frac{2}{3}\lambda} - \frac{4}{9} + \frac{4}{9}\underset{\sim}{u} \cdot \underset{\sim}{v} + \left(\lambda - \frac{2}{9} - \lambda^2\right)\underset{\sim}{u} \cdot \underset{\sim}{v}$$

$$+ \frac{2}{9} - \cancel{\frac{2}{3}\lambda} = 0 \quad 1$$

$$\left(\lambda - \frac{2}{9} - \lambda^2\right)\underset{\sim}{u} \cdot \underset{\sim}{v} + \frac{4}{9}\underset{\sim}{u} \cdot \underset{\sim}{v} - \frac{2}{9} = 0$$

$$\left(\lambda - \frac{2}{9} - \lambda^2\right) \underset{\sim}{u}\cdot\underset{\sim}{v} + \frac{4}{9}\underset{\sim}{u}\cdot\underset{\sim}{v} = \frac{2}{9}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v}\left[\lambda - \frac{2}{9} - \lambda^2 + \frac{4}{9}\right] = \frac{2}{9}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v}\left[\lambda - \lambda^2 + \frac{2}{9}\right] = \frac{2}{9}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v}\left[\lambda^2 - \lambda - \frac{2}{9}\right] = -\frac{2}{9}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v}\left[\lambda^2 - \lambda + \frac{1}{4} - \frac{2}{9} - \frac{1}{4}\right] = -\frac{2}{9}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v}\left[\left(\lambda - \frac{1}{2}\right)^2 - \frac{17}{36}\right] = -\frac{2}{9}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = -\frac{2}{9\left(\lambda - \frac{1}{2}\right)^2 - \frac{17}{4}} \quad 1$$

Consider the quadratic in the denominator.
This has a minimum when $\lambda = \frac{1}{2}$, which will also minimise $\underset{\sim}{u}\cdot\underset{\sim}{v}$:

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = -\frac{2}{9\left(\frac{1}{2} - \frac{1}{2}\right)^2 - \frac{17}{4}}$$

$$= -\frac{2}{-\frac{17}{4}}$$

$$= \frac{8}{17}$$

$$\therefore \underset{\sim}{u}\cdot\underset{\sim}{v} \geq \frac{8}{17}$$

Now consider max value for $\underset{\sim}{u}\cdot\underset{\sim}{v}$:

we know that $\underset{\sim}{u}\cdot\underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta$ and since $|\underset{\sim}{u}| = |\underset{\sim}{v}| = 1$ from question,

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \cos\theta$$

Since $-1 \leq \cos\theta \leq 1$,

$$-1 \leq \underset{\sim}{u}\cdot\underset{\sim}{v} \leq 1$$

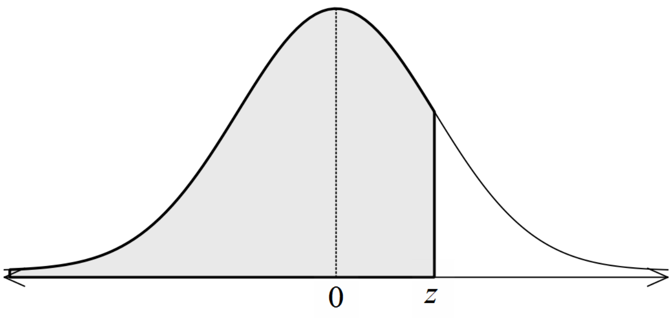
1

and therefore the max value of $\underset{\sim}{u}\cdot\underset{\sim}{v}$ is 1 !

Combining our inequalities, we have

$$\frac{8}{17} \leq \underset{\sim}{u}\cdot\underset{\sim}{v} \leq 1 \quad \text{as req.}$$

**Table of values** $P(Z \le z)$ **for the normal distribution** $N(0,1)$




| **Z** | **0.00** | **0.01** | **0.02** | **0.03** | **0.04** | **0.05** | **0.06** | **0.07** | **0.08** | **0.09** |
|---|---|---|---|---|---|---|---|---|---|---|
| **0.0** | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| **0.1** | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| **0.2** | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| **0.3** | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| **0.4** | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| **0.5** | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| **0.6** | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| **0.7** | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| **0.8** | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| **0.9** | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| **1.0** | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| **1.1** | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| **1.2** | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| **1.3** | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| **1.4** | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| **1.5** | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| **1.6** | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| **1.7** | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| **1.8** | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| **1.9** | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| **2.0** | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| **2.1** | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| **2.2** | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| **2.3** | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| **2.4** | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| **2.5** | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| **2.6** | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| **2.7** | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| **2.8** | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| **2.9** | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| **3.0** | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |

Some notes regarding q 15a (ii)

The 10-day test didn't want a troll at any of their tests, so that means they want the probability of a troll appearing to be less than 0.62%.
That is, $P(X \geq 1) < 0.62\%$
So, in the context of proportion and that it is 10-days.

This becomes, $P(p \geq 10\%) < 0.62\%$

So, really the school is no longer looking to the context of 10-days, and instead trying to see over what time span is needed to assure this,

That is, $P(X \geq 10\%\ of\ n) < 0.62\%$

After doing calculations, you find that $n = 119$

So, $10\%\ of\ 119 = 11.9$

So, the binomial would look like $X \sim Bin(119, 0.05)$
The Normal, $Y \sim N(5.95, 2.41^2)$

And so really, $P(Y \geq 11.9) < 0.62\%$

But this does mean $P(Y \geq 11)$ or $P(Y \geq 10)$ is greater than that $0.62\%$ - so this is some wizardry by the school to seem as though there won't be trolls, but they had to massively increase the number of days the test would take place on…

The numbers have been truncated a little, but you see the Probability comes out very close to 0.62%.
https://homepage.divms.uiowa.edu/~mbognar/applets/normal.html