HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 12 Higher School Certificate

Trial Examination Term 3 2025

STUDENT NUMBER: ______________________________

TEACHER NAME: Ms Guan, Ms Tourikis, Mr Payne, Mrs Sztajer **(circle one)**

STUDENT NAME: ______________________________

General Instructions:

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/ or calculations

Total Marks: 70

Section I – 10 marks (pages 3–7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 9–15)

- Attempt Questions 11–14
- Start each question in a new writing booklet
- Write your student number on every writing booklet
- Allow about 1 hour and 45 minutes for this section

Question	***1-10***	***11***	***12***	***13***	***14***	***Total***
Total	*/10*	*/15*	*/16*	*/14*	*/15*	*/70*

Outcomes assessed: ME 12-1, 12-2, 12-3, 12-4, 12-7

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 What is the domain of the function $y = \cos^{-1}(2x)$?

A. $0 \le x \le \pi$

B. $-2 \le x \le 2$

C. $0 \le x \le \frac{\pi}{2}$

D. $-\frac{1}{2} \le x \le \frac{1}{2}$

2 Which one of the following vectors is perpendicular to $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and has a magnitude of 4?

A. $2\begin{pmatrix} -2 \\ 5 \end{pmatrix}$

B. $\frac{4}{\sqrt{29}}\begin{pmatrix} -2 \\ 5 \end{pmatrix}$

C. $\frac{4}{\sqrt{29}}\begin{pmatrix} 5 \\ 2 \end{pmatrix}$

D. $4\begin{pmatrix} 2 \\ -5 \end{pmatrix}$

3 A four-digit number is formed by choosing four counters from the set $\{9,\ 8,\ 7,\ 6,\ 5,\ 4\}$ and placing them in order. How many numbers greater than 6 000 can be formed?

A. 240

B. 360

C. 840

D. 3024

4 Let $P(x)$ be a polynomial of degree 6. When $P(x)$ is divided by the polynomial $Q(x)$, the remainder is x^2+2.

Which statement is true about the degree of $Q(x)$?

A. The degree of $Q(x)$ must be at least 2.

B. The degree of $Q(x)$ must be 2.

C. The degree of $Q(x)$ must be at least 3.

D. The degree of $Q(x)$ must be 3.

5 Evaluate $\int_0^1 \frac{dx}{\sqrt{25-x^2}}$.

A. $\sin^{-1}\left(\frac{1}{5}\right)$

B. $\frac{1}{5}\sin^{-1}\left(\frac{1}{5}\right)$

C. $\tan^{-1}\left(\frac{1}{5}\right)$

D. $\frac{1}{5}\tan^{-1}\left(\frac{1}{5}\right)$

6 Which of these calculations will give the coefficient of x^3y^9 in the expansion of $(3x-y)^{12}$?

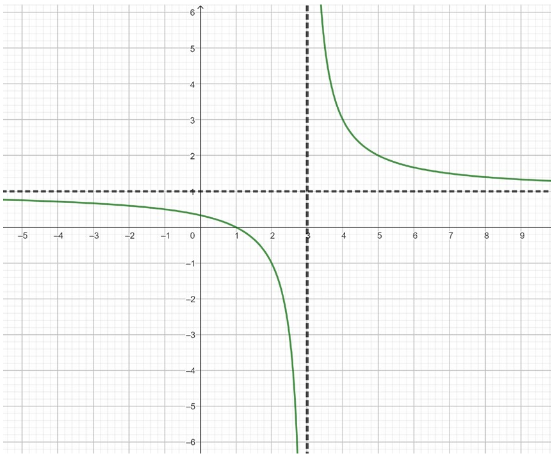
A. $-27\times {}^{12}C_8$

B. $-27\times {}^{12}C_9$

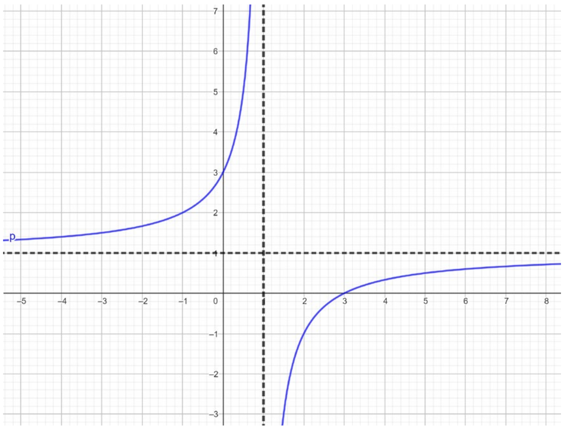
C. $9\times {}^{12}C_8$

D. $27\times {}^{12}C_9$

7 The graph of $f(x)=\dfrac{2}{x-3}+1$ is shown.



A transformation, $y=g(x)$, is applied to $y=f(x)$ to obtain the graph shown below.



What is the equation of $y=g(x)$?

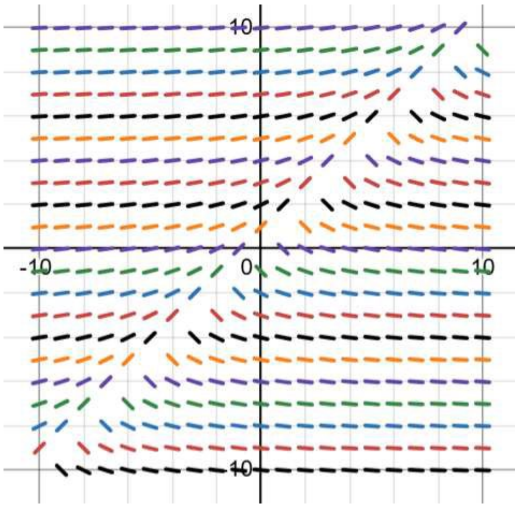
A. $g(x)=f\left(|x|\right)$

B. $g(x)=\sqrt{f(x)}$

C. $g(x)=f(-x)$

D. $g(x)=\dfrac{1}{f(x)}$

8 Which of the following differential equations matches the slope field shown?



A. $\dfrac{dy}{dx} = y - x$

B. $\dfrac{dy}{dx} = x - y$

C. $\dfrac{dy}{dx} = \dfrac{1}{y - x}$

D. $\dfrac{dy}{dx} = \dfrac{1}{x - y}$

9 Which function, $f(x)$, and its inverse function, $f^{-1}(x)$, are equivalent,

i.e. $f(x) = f^{-1}(x)$?

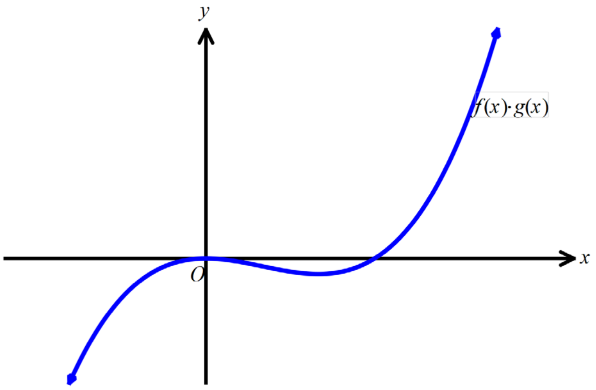
A. $f(x) = \sin|x|$ for $\left[\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right]$

B. $f(x) = x^2$ for $[0, \infty)$

C. $f(x) = |e^x|$ for $(-\infty, \infty)$

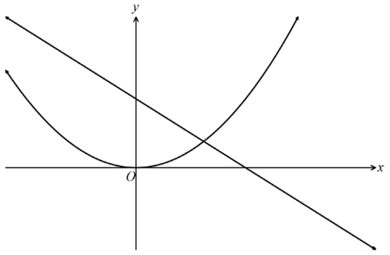
D. $f(x) = \dfrac{1}{x}$ for $(-\infty, 0) \cup (0, \infty)$

10 The diagram shows the graph of $f(x)\times g(x)$.

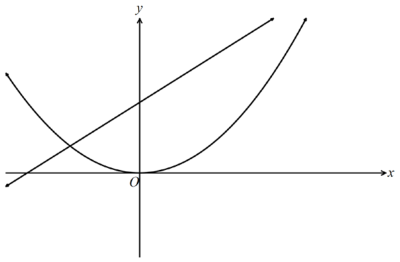


Which one of the following represents the graphs of both $f(x)$ and $g(x)$?

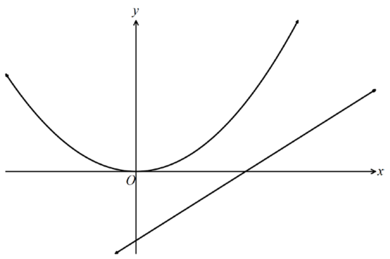
A.



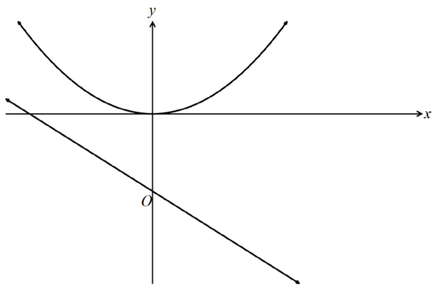
B.



C.



D.



Section II

60 marks
Attempt Questions 11 to 14
Allow about 1 hour and 45 minutes for this section
Instructions

- Answer the questions in the appropriate writing booklet.
- In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start a new writing booklet.

(a) For the vectors $\underset{\sim}{u} = 2\underset{\sim}{i} - \underset{\sim}{j}$ and $\underset{\sim}{v} = \underset{\sim}{i} + 2\underset{\sim}{j}$, evaluate each of the following:

(i) $\underset{\sim}{u} - 3\underset{\sim}{v}$ **1**

(ii) $\left|\underset{\sim}{u} + \underset{\sim}{v}\right|$ **1**

(b) Find the exact value of $\int_1^3 x^2\sqrt{3+x^3}\,dx$ using the substitution $u = 3 + x^3$. **3**

(c) Find the coefficient of x^3 in the expansion of $\left(1 - \frac{x}{3}\right)^9$. **2**

(d) There are two vectors in a plane: $\underset{\sim}{p} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ and $\underset{\sim}{q} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$. **2**

Find the angle between $\underset{\sim}{p}$ and $\underset{\sim}{q}$. Answer correct to the nearest degree.

(e) Write $\sqrt{3}\sin x - 3\cos x$ in the form $R\sin(x + \alpha)$. **3**

(f) Solve $\frac{4}{x+3} \geq 1$. **3**

End of Question 11

Question 12 (16 marks) Start a new writing booklet and use the diagrams provided.

(a) A direction field is to be drawn for the differential equation **2**

$$\frac{dy}{dx} = \frac{2x - y}{y^2 - x}\,.$$

On the answer sheet provided, clearly draw the correct slopes of the direction field at the points P, Q and R, stating the gradient of each interval.

(b) St Andrews Sports Club manages 16 senior teams. It decides to check the age of all players. Any team that has more than 3 players above the age limit will be penalised. A total of 50 players are found to be above the age limit. **2**
Will any team be penalised? Justify your answer.

(c) Find the equation of the tangent to the curve $y = x \cdot \arcsin(x)$ at the point with coordinates $\left(\frac{1}{2}, \frac{\pi}{12}\right)$. Express your answer in the form $y = mx + c$. **3**

(d) Determine $\displaystyle\lim_{x \to 1} \frac{(x^2 - 1) + \sin(x - 1)}{x - 1}$. **2**

(e) Use mathematical induction to prove that $15^n + 2^{3n} - 2$ is a multiple of 7 for all integers $n > 0$. **3**

Question 12 continues on the next page.

Question 12 continued

(f) In a restaurant with temperature 19°C, pumpkin soup is poured into a bowl.
The temperature of the soup when it is poured into the bowl is 95°C.
At this point, the soup is far too hot to be consumed.

The temperature, T, in degrees Celsius, of the soup, t minutes after it is poured into the bowl, can be modelled using the differential equation $\frac{dT}{dt} = -k\left(T - T_1\right)$, where k is the positive constant of proportionality and T_1 is a constant.

(i) Show that $T = T_1 + Ae^{-kt}$ satisfies the differential equation $\frac{dT}{dt} = -k\left(T - T_1\right)$ **1**

(ii) It takes 10 minutes for the soup to cool to a temperature of 75°C. **2**

Show that $T = 19 + 76e^{\frac{-t}{10}\ln\left(\frac{19}{14}\right)}$.

(iii) The optimal drinking temperature for a hot beverage is 57°C. **1**

Find the value of t when the soup reaches this temperature, giving your answer to the nearest minute.

End of Question 12

Question 13 (14 Marks) Start a new writing booklet.

(a) Consider the function $f(x) = 4 - \ln(x+2)$, for $x \geq -1$. Find the domain and the equation of $y = f^{-1}(x)$. **2**

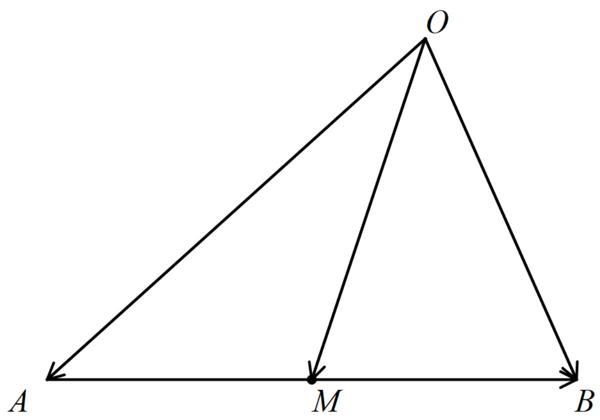
(b) The three solutions of the equation $x^3 - 2x^2 + 3x + 1 = 0$ are denoted by α, β and γ. **2**

Consider the equation $x^3 - x^2 + 7x + C = 0$ whose solutions are:

$$2\alpha - 1,\ 2\beta - 1 \text{ and } 2\gamma - 1.$$

Find the value of *C*, where *C* is an integer.

(c) The diagram below shows ΔOAB. *M* is the midpoint of *AB*. Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$, $\overrightarrow{OM} = \underset{\sim}{m}$ and $\overrightarrow{AB} = \underset{\sim}{p}$.



(i) Write $\underset{\sim}{p}$ and $\underset{\sim}{m}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **2**

(ii) Hence, show that $|\underset{\sim}{p}|^2 + 4|\underset{\sim}{m}|^2 = 2|\underset{\sim}{a}|^2 + 2|\underset{\sim}{b}|^2$. **2**

Question 13 continues on the next page.

(d) The graph below shows a sketch of a part of the curve $y = f(x)$ with parametric equations:

$$x = 3\tan\theta,\ y = 4\cos^2\theta,\ \text{for } 0 \le \theta \le \frac{\pi}{2}.$$

The point P lies on $f(x)$ and has coordinates $(3,\ 2)$. Line l is the normal to $f(x)$ at P. Line l cuts the x-axis at point Q.

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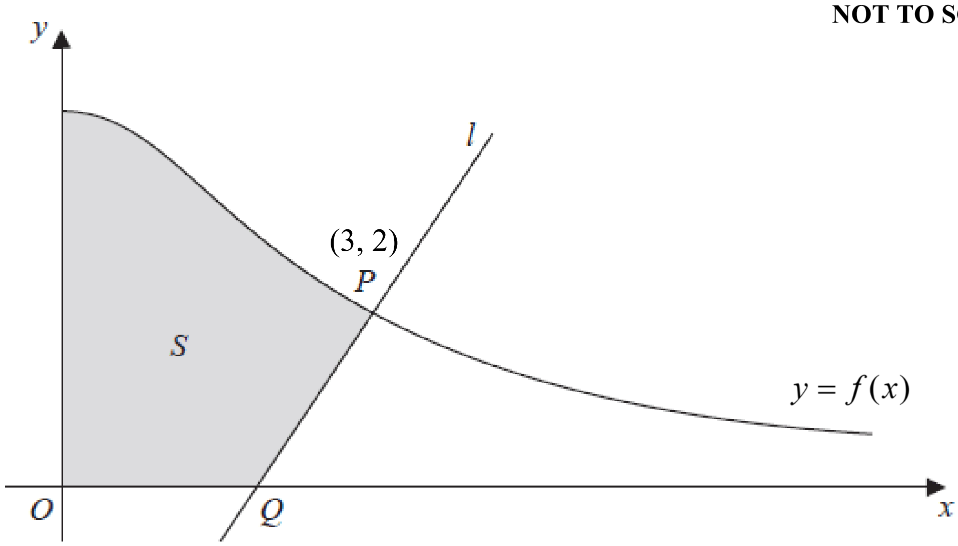


(i) Show that the x coordinate of point Q is $\frac{5}{3}$. **3**

(ii) The finite region S, shown shaded in the graph above, is bounded by the curve $y = f(x)$, the x-axis, the y-axis and the line l. **3**

This shaded region is rotated $360°$ about the x-axis to form a solid of revolution.

By showing $\pi\int_0^3 y^2 dx = 48\pi\int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$, find the exact value of the volume of the solid of revolution.

End of Question 13

Question 14 (15 marks) Start a new writing booklet.

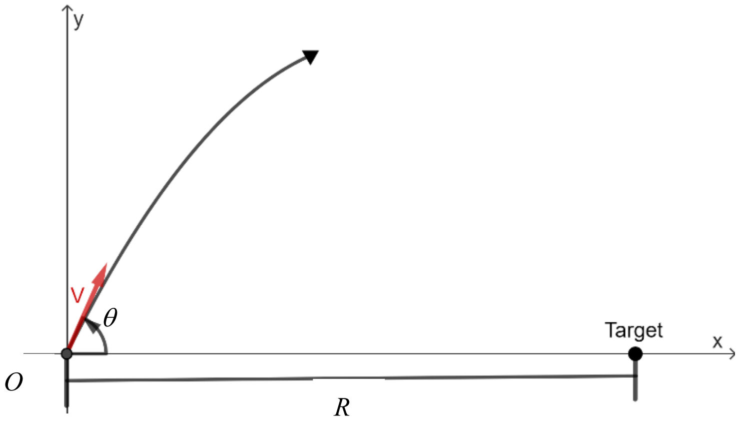
(a) Show that $y = \dfrac{e^x}{|x+1|}$, for $x \neq -1$, is the particular solution to the differential equation $(x+1)\dfrac{dy}{dx} = xy$ that passes through the point (0, 1). **4**

(b) Find the vector that is perpendicular to the vector projection of $\underset{\sim}{v}$ onto $\underset{\sim}{u}$ where $\underset{\sim}{u} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ and $\underset{\sim}{v} = \begin{bmatrix} 7 \\ 4 \end{bmatrix}$. **3**

Question 14 continues on the next page.

Question 14 continued

(c) A projectile is fired from O at an angle θ to the horizontal with initial velocity V m/s. The projectile strikes a target R metres to the right of O on level ground.



The equation of the displacement vector is

$$\underset{\sim}{r}(t) = Vt\cos\theta\underset{\sim}{i} + \left(Vt\sin\theta - \frac{gt^2}{2}\right)\underset{\sim}{j}.$$ (**Do not prove this.**)

(i) If the projectile hits the target, prove that $\tan^2\theta - \left(\frac{2V^2}{gR}\right)\tan\theta + 1 = 0$. **3**

(ii) Show that the target will be hit from two angles of projection, θ_1 and θ_2, **2**

if $R < \frac{V^2}{g}$.

(iii) Let the respective times of flight be t_1 and t_2. **3**

Given $\theta_1 + \theta_2 = 90°$ for the same range, prove that

$$t_1^2 + t_2^2 = \frac{4V^2}{g^2}.$$

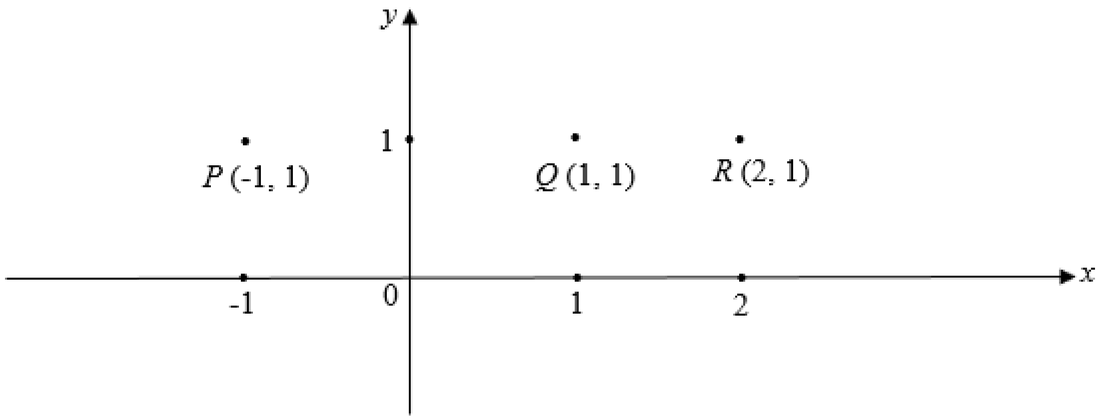
End of paper

Answer sheet for Question 12.

(a) A direction field is to be drawn for the differential equation **2**

$$\frac{dy}{dx}=\frac{2x-y}{y^2-x}.$$

Clearly draw the correct slopes of the direction field at the points P, Q and R, stating the gradient of each interval.



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Hornsby Girls High School
Year 12 Mathematics Extension 1 HSC Trial 2025 Solutions

Solutions	Marker's Comments
Question 1 **D** $y=\cos^{-1}(2x)$ $-1\le 2x\le 1$ $\frac{-1}{2}\le x\le \frac{1}{2}$	
Question 2 **C** If perpendicular $\perp$, $then\ dot\ product = 0.$ $\binom{-2}{5}\cdot\binom{-2}{5}=4-25\neq 0.$ $\binom{2}{-5}\cdot\binom{-2}{5}=-4-25\neq 0.$ So eliminate option A, B and D. Option C: $\binom{5}{2}\cdot\binom{-2}{5}=-10+10=0.$ $\left\lvert\frac{4}{\sqrt{29}}\binom{5}{2}\right\rvert=\sqrt{\left(\frac{20}{\sqrt{29}}\right)^2+\left(\frac{8}{\sqrt{29}}\right)^2}$ $=\sqrt{\frac{464}{29}}$ $=\sqrt{16}=4$	
Question 3 **A** From the digits 9 8 7 6 5 and 4 numbers between 4 567 and 9 876 can be formed, But we want those greater than 6 000, so: The first digit must be either 9, 8, 7 or 6, ie. 4 possible numbers The 2nd can be any of the remaining 5 numbers The 3rd can be any of the maining 4 numbers The 4th can be any of the remaining 3 numbers Total 4 digit numbers = $4\times5\times4\times3 = 240$	
Question 4 **C** The degree of the remainder must be less than the degree of the quotient. Since the remainder has a degree of 2, the degree of the quotient, $Q(x)$, must be $3,\ 4,\ 5,\ ...$, so it must be at least 3.	

Solutions	Marker's Comments
Question 5 A $\int_0^1 \frac{dx}{\sqrt{25-x^2}}$ $= \left[\sin^{-1}\frac{x}{5}\right]_0^1$ $= \sin^{-1}\frac{1}{5} - \sin^{-1} 0$ $= \sin^{-1}\frac{1}{5}$	
Question 6 B In the expansion of $(3x-y)^{12}$, $t_{r+1} = {}^{12}\mathbf{C}_r.(3x)^{12-r}.(-y)^r$ The term in $x^3\, y^9$ is when $r = 9$ $t_{10} = {}^{12}\mathbf{C}_9.\ (3x)^3\ (-y)^9$ $= -\ {}^{12}\mathbf{C}_9\ .\ (3^3)\ x^3y^9$ since $(-1)^9 = -1$ The coefficient is found from $-\ 27 \times {}^{12}\mathbf{C}_9$	
Question 7 D	
Question 8 C The slope field is $\frac{dy}{dx} = \frac{1}{y-x}$. So, the slope field is not defined everywhere on y = x. So either option C or D. Next, the slope field changes sign below and above y=x line. When y is slightly greater than x (region above y=x line, ie. $y - x > 0$), slope field is positive, and also larger nearer to the line y=x since the slope field is the reciprocal of $y - x$. Correspondingly, slope field was negative below $y = x\ line$.	
Question 9 D The function must be symmetrical about $y = x$ to be its own inverse function. The only function that meets this criterion is D.	
Question 10 C The product is a positive cubic function with a positive root and zero root. Double roots at $x = 0$, one of the functions is a quadratic passing the origin. The other function must be a linear function with positive gradient. Exclude A,D. Check the product of the ordinates. Exclude B.	

SECTION II

Solutions	Marker's Comments
Question 11 (a) (i) $\underset{\sim}{u}-3\underset{\sim}{v}=\left(2\underset{\sim}{i}-\underset{\sim}{j}\right)-3\left(\underset{\sim}{i}+2\underset{\sim}{j}\right)$ $=2\underset{\sim}{i}-3\underset{\sim}{i}-\underset{\sim}{j}-6\underset{\sim}{j}$ $=-\underset{\sim}{i}-7\underset{\sim}{j}$	Question 11 is supposed to be reasonably straightforward and is focused on method. You should be getting 15/15 or close to it for this. Do not rush it. The number of silly errors made by students with calculations cost them easy marks. (a)(i) Mostly well done if students added and subtracted properly
(a) (ii) $\left\|\underset{\sim}{u}+\underset{\sim}{v}\right\|=\left\|2\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{i}+2\underset{\sim}{j}\right\|$ $=\left\|3\underset{\sim}{i}+\underset{\sim}{j}\right\|$ $=\sqrt{3^2+1^2}$ $=\sqrt{10}$	Mostly well done except some students did not evaluate the modulus which is what is the meaning of modulus.
(b) $u=3+x^3$ $\frac{du}{dx}=3x^2,\quad x=1,u=4,\quad x=3,u=30$ $\int_1^3 x^2\sqrt{3+x^3}\,dx=\int_4^{30}\frac{du}{3dx}\sqrt{u}\,dx$ $=\frac{1}{3}\int_4^{30}\sqrt{u}\,du$ $=\frac{1}{3}\left[\frac{2}{3}u^{\frac{3}{2}}\right]_4^{30}$ $=\frac{2}{9}\left(30^{\frac{3}{2}}-4^{\frac{3}{2}}\right)$ $=\frac{2}{9}\left(30\sqrt{30}-8\right)$	Mostly well done except for mistakes with boundaries or simplifying in exact form
(c) $\left(1-\frac{x}{3}\right)^9=\sum_{r=0}^{9}\binom{9}{r}1^{9-r}\left(\frac{-x}{3}\right)^r$ coefficient of $x^3=\binom{9}{3}1^{9-3}\left(\frac{-1}{3}\right)^3$ $=\frac{-28}{9}$	Mostly well done except some got mixed up with term number or sub with $-\frac{x}{3}$ and lost the negative or the over 3

(d) $\underset{\sim}{p} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}, \quad \underset{\sim}{q} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$ $\underset{\sim}{p} \cdot \underset{\sim}{q} = \lvert\underset{\sim}{p}\rvert\lvert\underset{\sim}{q}\rvert\cos\theta$ $\cos\theta = \dfrac{\underset{\sim}{p} \cdot \underset{\sim}{q}}{\lvert\underset{\sim}{p}\rvert\lvert\underset{\sim}{q}\rvert}$ $= \dfrac{5\times -3 + -3\times 7}{\sqrt{5^2+3^2}\times\sqrt{3^2+7^2}}$ $= \dfrac{-36}{\sqrt{34}\times\sqrt{58}}$ $= -0.81067...$ $\therefore\ \theta \approx 144°$	Again mostly well done except for calculation errors
(e) $R = \sqrt{\sqrt{3}^2 + 3^2} = \sqrt{12} = 2\sqrt{3}$ $\alpha = \tan^{-1}\dfrac{-3}{\sqrt{3}} = \dfrac{5\pi}{3}$ $\sqrt{3}\sin x - 3\cos x = R\sin(x+\alpha)$ $= \sqrt{12}\sin x\cos\alpha + \sqrt{12}\cos x\sin\alpha$ $= 2\sqrt{3}\sin\left(x + \dfrac{5\pi}{3}\right)$	A lot of mistakes with finding α correctly. It is in 4th quadrant because $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{-3}{\sqrt{3}}$ sin is neg, and cos is pos.
(f) $\dfrac{4}{x+3} \geq 1$ $4(x+3) - (x+3)^2 \geq 0$ $(x+3)(4-x-3) \geq 0$ $(x+3)(1-x) \geq 0$ $(x+3)(x-1) \leq 0$ $-3 < x \leq 1, \quad (x \neq -3)$	Mostly well done except a lot included the -3 in their answer.

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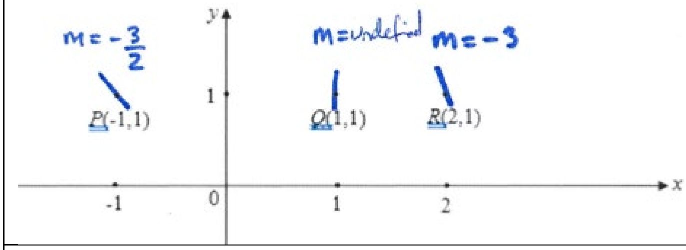
Question 12

(a)



</td><td>Mostly well done however some students didn’t draw a vertical interval for the undefined gradient,</td></tr>
<tr><td>

(b)

Using the pigeonhole principle,

$\frac{50}{16} = 3.125$

Therefore, there must be at least one team with more than 3 players who are over the age limit.

Yes, at least one team will be penalised.

</td><td>Well done.</td></tr>
<tr><td>

(c)

$$\frac{d}{dx} x\sin^{-1} x = \sin^{-1} x + \frac{x}{\sqrt{1-x^2}}$$

$$x = \frac{1}{2}, \quad m = \sin^{-1}\frac{1}{2} + \frac{\frac{1}{2}}{\sqrt{1-\left(\frac{1}{2}\right)^2}}$$

$$= \frac{\pi}{6} + \frac{1}{\sqrt{3}}$$

$$y - \frac{\pi}{12} = \left(\frac{\pi}{6} + \frac{1}{\sqrt{3}}\right)\left(x - \frac{1}{2}\right)$$

$$y = \left(\frac{\pi}{6} + \frac{1}{\sqrt{3}}\right)x - \frac{1}{2\sqrt{3}}$$

</td><td>Well done.</td></tr>
<tr><td>

(d)

$$\lim_{x\to 1}\left(\frac{(x^2-1)}{x-1} + \frac{\sin(x-1)}{x-1}\right)$$

$$= \lim_{x\to 1}\frac{(x^2-1)}{x-1} + \lim_{x\to 1}\frac{\sin(x-1)}{x-1}$$

$$= \lim_{x\to 1}\frac{(x+1)(x-1)}{x-1} + \lim_{(x-1)\to 0}\frac{\sin(x-1)}{x-1}$$

$$= \lim_{x\to 1}(x+1) + 1$$

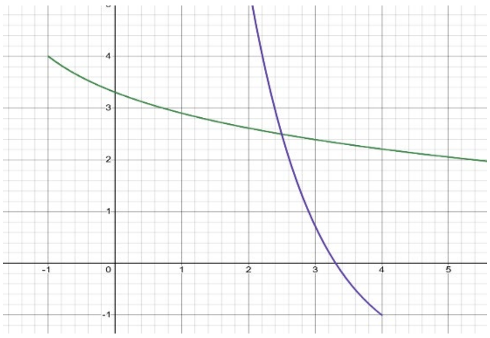
$$= 2 + 1$$

$$= 3$$

</td><td>Poorly done. Many students noticed that the fraction could be split however they didn’t calculate the limit of the second part correctly.</td></tr>
</table>

(e) For $n=1$, $15^1+2^{3(1)}-2$ $=15+8-2$ $=21$ $=7\times 3$ which is a multiple of 7 $\therefore$ Proven true for $n=1$ Assume true for $n=k$,. $15^k+2^{3(k)}-2=7Q$ where Q is an integer $2^{3k}=7Q-15^k+2$ RTPT for $n=k+1$, i.e. $15^{(k+1)}+2^{3(k+1)}-2$ is a multiple of 7. $15\cdot 15^k+2^3\cdot 2^{3k}-2$ $=15\cdot 15^k+2^3\left(7Q-15^k+2\right)-2$ $=15\cdot 15^k+7\left(8Q\right)-8\cdot 15^k+16-2$ $=7\cdot 15^k+7\left(8Q\right)+14$ $=7\left(15^k+8Q+14\right)$ Which is a multiple of 7. If true for $n=k$, proven true for $n=k+1$. Since true for $n=1$, true for $n=1+1=2$, $n=2+1=3\ldots$ Therefore, by mathematical induction, $15^n+2^{3n}-2$ is a multiple of 7 for all integers $n>0$.	Well done.
(f) (i) $T=T_1+Ae^{-kt}$ $Ae^{-kt}=T-T_1$ $\frac{dT}{dt}=-kAe^{-kt}$ $=-k\left(T-T_1\right)$ $\therefore \frac{dT}{dt}=-k\left(T-T_1\right)$	Well done.

(f) (ii) (g) $T = T_1 + Ae^{-kt}$ $T_1 = 19$ which is the ambient temperature $\therefore T = 19 + Ae^{-kt}$ $t = 0, T = 95$ $95 = 19 + Ae^{-k(0)}$ $A = 95 - 19 = 76$ $\therefore T = 19 + 76e^{-kt}$ $t = 10, T = 75$ $75 = 19 + 76e^{-k(10)}$ $76e^{-10k} = 56$ $e^{-10k} = \frac{56}{76} = \frac{14}{19}$ $-10k = \ln\left(\frac{14}{19}\right)$ $k = \frac{\ln\left(\frac{14}{19}\right)}{-10} = \frac{-\ln\left(\frac{19}{14}\right)}{-10} = \frac{\ln\left(\frac{19}{14}\right)}{10}$ $\therefore T = 19 + 76e^{\frac{-t}{10}\ln\left(\frac{19}{14}\right)}$	Well done.
(iii) $T = 57$ $57 = 19 + 76e^{\frac{-t}{10}\ln\left(\frac{19}{14}\right)}$ $76e^{\frac{-t}{10}\ln\left(\frac{19}{14}\right)} = 38$ $e^{\frac{-t}{10}\ln\left(\frac{19}{14}\right)} = \frac{38}{76} = \frac{1}{2}$ $\frac{-t}{10}\ln\left(\frac{19}{14}\right) = \ln\frac{1}{2}$ $t = \frac{-10\ln 2}{-\ln\left(\frac{19}{14}\right)}$ $= 22.697735...$ ≈ 23	Well done.

Solutions	Marker's Comments
Question 13 (a) $f(x)=4-\ln(x+2),\quad x\ge -1,$ $x=-1, y=4-\ln(-1+2)=4, y\le 4$ To find $f^{-1}(x)$, $x=4-\ln(y+2)$ $\ln(y+2)=4-x$ $y+2=e^{4-x}$ $y=e^{4-x}-2$ $\therefore f^{-1}(x)=e^{4-x}-2$ The domain is $x\le 4$. Or use graphing to justify. 	Some students didn't read the question carefully by finding both $f^{-1}(x)$ and its domain.
(b) For $x^3-2x^2+3x+1=0,$ $\alpha+\beta+\gamma=2$ $\alpha\beta+\beta\gamma+\alpha\gamma=3$ $\alpha\beta\gamma=-1$ $-C=(2\alpha-1)(2\beta-1)(2\gamma-1)$ $=(4\alpha\beta-2\alpha-2\beta+1)(2\gamma-1)$ $=8\alpha\beta\gamma-4\alpha\beta-4\alpha\gamma+2\alpha-4\beta\gamma+2\beta+2\gamma-1$ $=8\alpha\beta\gamma-4(\alpha\beta+\beta\gamma+\alpha\gamma)+2(\alpha+\beta+\gamma)-1$ $=8(-1)-4(3)+2(2)-1$ $=-17$ $\therefore C=17$	Generally, well done. Some students forgot that C must be negative.

c. (i) $\underset{\sim}{p} = \underset{\sim}{b} - \underset{\sim}{a}$ $\underset{\sim}{m} = \underset{\sim}{a} + \frac{\underset{\sim}{p}}{2}$ $= \underset{\sim}{a} + \frac{\underset{\sim}{b} - \underset{\sim}{a}}{2}$ $= \frac{\underset{\sim}{a} + \underset{\sim}{b}}{2}$	Very well done! Some students didn't answer in terms of $\underset{\sim}{a}$ *and* $\underset{\sim}{b}$.
(c) (ii) $LHS = \lvert\underset{\sim}{p}\rvert^2 + 4\lvert\underset{\sim}{m}\rvert^2$ $= \underset{\sim}{p} \cdot \underset{\sim}{p} + 4\underset{\sim}{m} \cdot \underset{\sim}{m}$ $= (\underset{\sim}{b} - \underset{\sim}{a}) \cdot (\underset{\sim}{b} - \underset{\sim}{a}) + 4\left(\frac{\underset{\sim}{a} + \underset{\sim}{b}}{2}\right) \cdot \left(\frac{\underset{\sim}{a} + \underset{\sim}{b}}{2}\right)$ $= \underset{\sim}{b} \cdot \underset{\sim}{b} - \underset{\sim}{b} \cdot \underset{\sim}{a} - \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{a} + 4\frac{\underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{a} + \underset{\sim}{b} \cdot \underset{\sim}{b}}{4}$ $= \underset{\sim}{b} \cdot \underset{\sim}{b} - 2\underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{a} + 2\underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{b}$ $= \underset{\sim}{b} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{b} \cdot \underset{\sim}{b}$ $= 2\underset{\sim}{a} \cdot \underset{\sim}{a} + 2\underset{\sim}{b} \cdot \underset{\sim}{b}$ $= 2\lvert\underset{\sim}{a}\rvert^2 + 2\lvert\underset{\sim}{b}\rvert^2$	It's a "show" question. Students need to write sufficient steps to justify the final answer which is given. The following mistakes are common and should be **avoided**! $\lvert\underset{\sim}{b} - \underset{\sim}{a}\rvert^2 = (\underset{\sim}{b} - \underset{\sim}{a})^2$ $= \underset{\sim}{b}^2 - 2\underset{\sim}{a}\underset{\sim}{b} + \underset{\sim}{a}^2$ $\lvert\underset{\sim}{b} - \underset{\sim}{a}\rvert^2 = \underset{\sim}{b}^2 - 2\underset{\sim}{a}\underset{\sim}{b} + \underset{\sim}{a}^2$ $\lvert\underset{\sim}{b} - \underset{\sim}{a}\rvert^2 = \sqrt{\underset{\sim}{b} - \underset{\sim}{a}}^2$
(d) (i) $x = 3\tan\theta,\ y = 4\cos^2\theta, \quad 0 \le \theta \le \frac{\pi}{2}$ $x = 3, \quad 3\tan\theta = 3, \quad \tan\theta = 1, \quad \theta = \frac{\pi}{4}, \quad 0 \le \theta \le \frac{\pi}{2}$ $or: 4\cos^2\theta = 2, \quad \cos\theta = \frac{\pm 1}{\sqrt{2}}, \quad \theta = \frac{\pi}{4}, \quad 0 \le \theta \le \frac{\pi}{2}$ $\frac{dx}{d\theta} = 3\sec^2\theta, \frac{dy}{d\theta} = 4 \times 2\cos\theta(-\sin\theta) = -8\sin\theta\cos\theta$ $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = -8\sin\theta\cos\theta \times \frac{\cos^2\theta}{3} = \frac{-8\sin\theta\cos^3\theta}{3}$ $\theta = \frac{\pi}{4}, \quad m = \frac{dy}{dx} = \frac{-8\sin\frac{\pi}{4}\cos^3\frac{\pi}{4}}{3} = \frac{-8\frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}\right)^3}{3} = \frac{-2}{3}$ $m_{normal} = \frac{3}{2},$ $\therefore$ the normal passing (3,2) is: $y - 2 = \frac{3}{2}(x - 3)$ $y = 0, -2 = \frac{3}{2}(x - 3), x = \frac{-4}{3} + 3 = \frac{5}{3}$	The alternative method is to convert the parametric equations into Cartesian equation and differentiate. $x^2 = 9\tan^2\theta, y = 16\cos^2\theta$ $\tan^2\theta = \frac{x^2}{9}, \quad \sec^2\theta = \frac{16}{y}$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \frac{x^2}{9} = \frac{16}{y}$ $y = \frac{36}{9 + x^2}$ $\frac{dy}{dx} = \frac{-72x}{(9 + x^2)^2}$ $x = 3, \quad \frac{dy}{dx} = \frac{-72(3)}{(9 + (3)^2)^2} = \frac{-2}{3}$ $m_{normal} = \frac{3}{2}$ $y - 2 = \frac{3}{2}(x - 3)$ $y = 0, -2 = \frac{3}{2}(x - 3), x = \frac{-4}{3} + 3 = \frac{5}{3}$

(d) (ii) $x = 3\tan\theta,\ y = 4\cos^2\theta,\quad 0 \le \theta \le \frac{\pi}{2}$ $x = 0,\ \theta = 0$ $x = 3,\ \theta = \frac{\pi}{4}$ $V = \pi\int_0^3 y^2 dx - V_{cone}$ $= \pi\int_0^{\frac{\pi}{4}} \left(4\cos^2\theta\right)^2 \frac{dx}{d\theta} d\theta - \frac{1}{3}\pi r^2 h$ $= 16\pi\int_0^{\frac{\pi}{4}} \cos^4\theta \cdot 3\sec^2\theta d\theta - \frac{1}{3}\pi 2^2\left(3 - \frac{5}{3}\right)$ $= 16\pi \times 3\int_0^{\frac{\pi}{4}} \cos^2\theta d\theta - \frac{1}{3}\pi 2^2\left(3 - \frac{5}{3}\right)$ $= 16\pi \times 3 \times \frac{1}{2}\int_0^{\frac{\pi}{4}} (\cos 2\theta + 1) d\theta - \frac{16\pi}{9}$ $= 24\pi\left[\frac{\sin 2\theta}{2} + \theta\right]_0^{\frac{\pi}{4}} - \frac{16\pi}{9}$ $= 24\pi\left(\frac{\sin 2\frac{\pi}{4}}{2} + \frac{\pi}{4} - 0\right) - \frac{16\pi}{9}$ $= 24\pi\left(\frac{1}{2} + \frac{\pi}{4}\right) - \frac{16\pi}{9}$ $= 12\pi + 6\pi^2 - \frac{16\pi}{9}$ $= \frac{92\pi}{9} + 6\pi^2 \quad \text{unit}^3$	Many students could evaluate $48\pi\int_0^{\frac{\pi}{4}} \cos^2\theta d\theta$ correctly. Some students couldn't show $\pi\int_0^3 y^2 dx = 48\pi\int_0^{\frac{\pi}{4}} \cos^2\theta d\theta$ by substitution. Some students forgot to subtract the volume of the cone from the revolution created by $\pi\int_0^3 y^2 dx$. Some students mistakenly subtracted the area of the triangle.

Solutions	Marker's Comments
Question 14 (a) $$(x+1)\frac{dy}{dx}=xy$$ $$\left(\frac{1}{y}\right)\frac{dy}{dx}=\frac{x}{(x+1)}$$ $$\int\frac{1}{y}dy=\int\frac{x}{(x+1)}dx$$ ① $$\int\frac{1}{y}dy=\int\frac{x+1-1}{x+1}dx$$ $$\int\frac{1}{y}dy=\int\frac{x+1}{x+1}-\frac{1}{x+1}dx$$ $$\int\frac{1}{y}dy=\int 1-\frac{1}{x+1}dx$$ ① $$\ln\lvert y\rvert = x-\ln\lvert x+1\rvert + C,\ \ x>-1$$ $$\lvert y\rvert = e^{x-\ln\lvert x+1\rvert + C},\ \ x\neq -1$$ $$y=Ae^{x-\ln\lvert x+1\rvert},\ \text{ where } A=\pm e^{C}$$ At (0, 1), $(1)=Ae^{(0)-\ln\lvert(0)+1\rvert},\ x\neq -1$ $$\therefore A=1$$ ① $$\therefore y=e^{x-\ln\lvert x+1\rvert}$$ $$y=\frac{e^{x}}{e^{\ln\lvert x+1\rvert}}$$ $$y=\frac{e^{x}}{\lvert x+1\rvert},\ \ x>-1$$ ①	Poorly done. Many students did not conclude their statements with $x\neq -1$. This topic is based on your integration skill. A few students had differentiated and missed the whole purpose of the exercise.
(b) $\underset{\sim}{u}=\begin{bmatrix}4\\2\end{bmatrix}$ and $\underset{\sim}{v}=\begin{bmatrix}7\\4\end{bmatrix}$. Let the perpendicular required vector be $\underset{\sim}{p}$. Let vector projection of $\underset{\sim}{v}$ onto $\underset{\sim}{u}$ be $\underset{\sim}{s}=\frac{\underset{\sim}{v}\bullet\underset{\sim}{u}}{\lvert\underset{\sim}{u}\rvert}\hat{\underset{\sim}{u}}$ $$\underset{\sim}{s}=\frac{28+8}{\left(\sqrt{4^2+2^2}\right)^2}\begin{bmatrix}4\\2\end{bmatrix}$$ $$\underset{\sim}{s}=\frac{36}{20}\begin{bmatrix}4\\2\end{bmatrix}$$ $$\underset{\sim}{s}=\frac{9}{5}\begin{bmatrix}4\\2\end{bmatrix}$$ $$\underset{\sim}{s}=\frac{18}{5}\begin{bmatrix}2\\1\end{bmatrix}$$ $$\therefore \underset{\sim}{p}=\underset{\sim}{v}-\underset{\sim}{s}$$	Poorly done. Some students did not know how to find the vector that is being projected onto another vector. Many students used dot product trying to find the perpendicular vector and failed.

$=\begin{bmatrix}7\\4\end{bmatrix}-\frac{18}{5}\begin{bmatrix}2\\1\end{bmatrix}$ $=\begin{bmatrix}-\frac{1}{5}\\ \frac{2}{5}\end{bmatrix}$	
(c)(i) $\underset{\sim}{r}(t)=Vt\cos\theta\underset{\sim}{i}+\left(Vt\sin\theta-\frac{gt^2}{2}\right)\underset{\sim}{j}.$ $\begin{bmatrix}R\\0\end{bmatrix}=\begin{bmatrix}Vt\cos\theta\\ Vt\sin\theta-\frac{gt^2}{2}\end{bmatrix}$ Equating the $\underset{\sim}{i}$ and $\underset{\sim}{j}$ components: $Vt\cos\theta=R$ — ① and $Vt\sin\theta-\frac{gt^2}{2}=0$ — ② $\therefore t=\frac{R}{V\cos\theta}$ — ①’ Sub ①’ into ②, $V\left(\frac{R}{V\cos\theta}\right)\sin\theta-\frac{g}{2}\left(\frac{R}{V\cos\theta}\right)^2=0$ $R\tan\theta-\frac{gR^2}{2V^2\cos^2\theta}=0$ $R\tan\theta-\frac{gR^2}{2V^2}\sec^2\theta=0$ $R\tan\theta-\frac{gR^2}{2V^2}\left(\tan^2\theta+1\right)=0$ $\tan\theta-\frac{gR}{2V^2}\left(\tan^2\theta+1\right)=0$ $\tan\theta=\frac{gR}{2V^2}\left(\tan^2\theta+1\right)$ $\left(\frac{2V^2}{gR}\right)\tan\theta=\tan^2\theta+1$ $\therefore \tan^2\theta-\left(\frac{2V^2}{gR}\right)\tan\theta+1=0$ (as required)	Poorly done. About 21% of the cohort did not know how to find the Cartesian equation of the trajectory. 10% cohort did not attempt this question.
(ii) $\tan^2\theta-\left(\frac{2V^2}{gR}\right)\tan\theta+1=0$ with two real roots θ_1 and θ_2, $\therefore\Delta>0$ $b^2-4ac>0$ $\left(-\frac{2V^2}{gR}\right)^2-4(1)(1)>0$	Mostly done well by students who understood the existence of two real roots means $\Delta>0$ from the quadratic equation. 23% did poorly. 22% cohort did not attempt this question.

$$\frac{4V^4}{g^2R^2}-4>0$$ $$\frac{V^4}{g^2R^2}-1>0$$ $$\frac{V^4}{g^2R^2}>1$$ $$\frac{V^2}{gR}>1$$ $$\therefore R<\frac{V^2}{g}$$	
(iii) Since $\theta_1+\theta_2=90^\circ$ Then $\theta_1=90^\circ-\theta_2$ i.e. $\sin\theta_1=\sin\left(90^\circ-\theta_2\right)$ $\sin\theta_1=\cos\theta_2$ Using $t\left(V\sin\theta-\frac{gt}{2}\right)=0$ $t=0$ and $V\sin\theta-\frac{gt}{2}=0$ $\frac{gt}{2}=V\sin\theta$ $\therefore t=\frac{2V\sin\theta}{g}$ i.e. time of flight. $\therefore t_1=\frac{2V\sin\theta_1}{g}$ and $t_2=\frac{2V\sin\theta_2}{g}$ $t_2=\frac{2V\cos\theta_1}{g}$ $\therefore t_1^2+t_2^2=\left(\frac{2V\sin\theta_1}{g}\right)^2+\left(\frac{2V\cos\theta_1}{g}\right)^2$ $=\frac{4V^2\sin^2\theta_1}{g^2}+\frac{4V^2\cos^2\theta_1}{g^2}$ $=\frac{4V^2\left(\sin^2\theta_1+\cos^2\theta_1\right)}{g^2}$ since $\sin^2\theta_1+\cos^2\theta_1=1$ $=\frac{4V^2(1)}{g^2}$ $\therefore t_1^2+t_2^2=\frac{4V^2}{g^2}$ (as required)	Poorly done by overall. Students who used the time of flight from vertical component worked this question out easily. Students that used the time from horizontal component had trouble getting to the answer. 27% attempted poorly. 31% cohort did not attempt this question.