Student Number: ____________________

James Ruse Agricultural High School

2025 Year 12 Trial Examination

Mathematics Extension 1

General Instructions

- Reading Time - 10 minutes
- Working Time - 120 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11-17, show relevant mathematical reasoning and/or calculations.

Total Marks: 70

Section I - 10 marks (pages 2-5)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II - 60 marks (pages 6-10)

- Attempt Questions 11-17
- Allow about 1 hour and 45 minutes for this section

Start your answers to each question on a separate sheet of paper.

Section I Multiple Choice

10 Marks

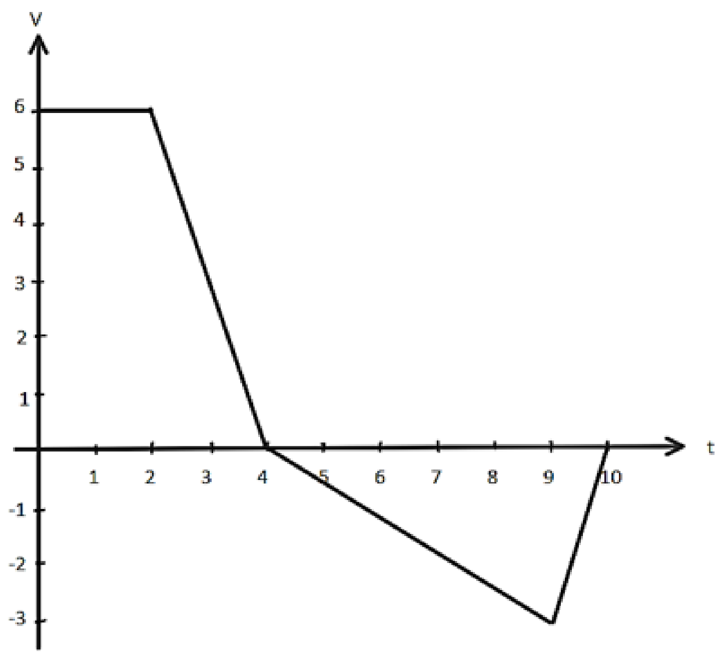
Attempt Questions 1 to 10

Allow approximately 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1 - 10.

Question 1 (1 mark)

The diagram below shows the velocity-time graph of an object that moves over a 10 second interval. For what percentage of the time is the speed of the object decreasing?



A. 30%

B. 60%

C. 70%

D. None of the above

Question 2 (1 mark)

Evaluate the integral $\displaystyle\int_0^{0.5} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$.

A. $\dfrac{\pi^2}{72}$

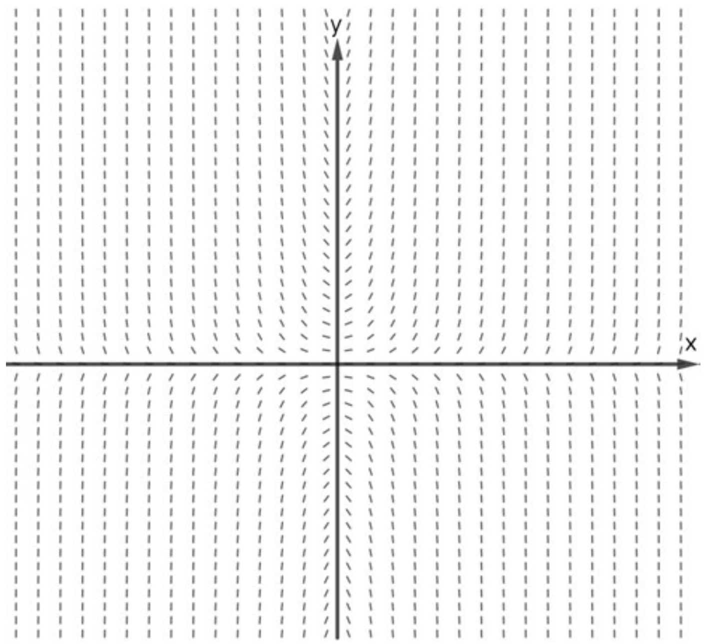
B. $\ln 6 - \ln \pi$

C. $\dfrac{\pi^2}{36}$

D. $\dfrac{\pi}{\sqrt{2}}$

Question 3 (1 mark)

The direction field of a differential equation is shown.



Which differential equation, does the direction field best represent?

A. $y' = \dfrac{x}{y}$

B. $y' = xy$

C. $y' = x^2y^2$

D. $y' = x + y$

Question 4 (1 mark)

If $\overrightarrow{OX} = \underset{\sim}{a} + 2\underset{\sim}{b}$ and $\overrightarrow{YX} = \underset{\sim}{a} - \underset{\sim}{b}$, which of the following is $\overrightarrow{OY}$?

A. $2\underset{\sim}{b}$

B. $3\underset{\sim}{b}$

C. $2\underset{\sim}{a} + \underset{\sim}{b}$

D. $2\underset{\sim}{a} + 3\underset{\sim}{b}$

Question 5 (1 mark)

$\sin(3x + x) - \sin(3x - x) =$

A. $-2\sin 3x \sin x$

B. $2\cos 3x \sin x$

C. $2\cos 3x \cos x$

D. $2\sin 3x \sin x$

Question 6 (1 mark)

Given that $\left[-\frac{\pi}{10}, \frac{\pi}{10}\right]$ is the restricted domain of the function $f(x) = \frac{\sin 5x}{3}$?
What is the domain of the corresponding inverse function?

A. Domain: $\left[-\frac{1}{3}, \frac{1}{3}\right]$

B. Domain: $[-1, 1]$

C. Domain: $\left[-\frac{1}{5}, \frac{1}{5}\right]$

D. Domain: $\left[-\frac{3}{5}, \frac{3}{5}\right]$

Question 7 (1 mark)

Which of the following is the cartesian equation for the given parametric equations?

$$x = \sin^{-1} t + 1$$

$$y = \frac{1}{\sqrt{1-t^2}}$$

A. $y = \dfrac{1}{1-\sin(x-1)}$

B. $y = \dfrac{1}{\sqrt{2\sin x - \sin^2 x}}$

C. $y = \dfrac{1}{\cos(x-1)}$

D. $y = \pm\dfrac{1}{\cos(x-1)}$

Question 8 (1 mark)

In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in five trials is:

A. $\dfrac{1}{32}$

B. $\dfrac{2}{32}$

C. $\dfrac{3}{32}$

D. $\dfrac{6}{32}$

Question 9 (1 mark)

Let α, β be the roots of the equation $x^2 - ax + b = 0$ and $A_n = \alpha^n + \beta^n$.
which of the following is equal to $A_{n+1} - aA_n + bA_{n-1}$?

A. $-a$

B. b

C. $a - b$

D. 0

Question 10 (1 mark)

Let X denote the number of times heads occur in n tosses of a fair coin, If $P(X = 4)$, $P(X = 5)$ and $P(X = 6)$ are in AP, then a possible value of n is :

A. 7

B. 10

C. 12

D. 15

Mathematics Extension 1

Section II 60 Marks

Attempt Questions 11-17

Allow about 1h and 45 minutes for this section.

Question 11 (9 marks)

(a) i. Let $x = \alpha$ be a zero of a polynomial $P(x)$. If $P(\alpha) = P'(\alpha) = 0$, show that $x = \alpha$ is a zero of $P(x)$ of multiplicity at least 2. [2]

ii. The equation $ax^3 + bx^2 + c = 0$ has a double root, where a, b, and c are non-zero constants. Show that $27a^2c + 4b^3 = 0$. [3]

(b) The population $P(t)$ of bacteria in a dish is modelled by the logistic differential equation :

$$\frac{dP}{dt} = \frac{P}{6}\left(1 - \frac{P}{8000}\right)$$ where $P(0) = P_0$ and t in the time in hours.

i. Show that $\dfrac{Q}{P(Q-P)} = \dfrac{1}{P} + \dfrac{1}{Q-P}$. [1]

ii. If the initial population P_0 is 1000 bacteria, [3]

Show that $P(t) = \dfrac{8000\, e^{\frac{t}{6}}}{7 + e^{\frac{t}{6}}}$ by solving the differential equation.

Exam continues on the next page

Question 12 (9 marks)

(a) Differentiate $y = \ln(\tan^{-1} 2x)$. [2]

(b) i. Expand the binomial expansion of $(2 - x)^5$, simplifying all terms. [2]

ii. Hence, find the coefficient of y^3, in the expansion of $(2 - 3y + y^2)^5$. [2]

(c) Evaluate the integral $\displaystyle\int_0^{\sqrt{5}} \frac{2x^3}{\sqrt{x^2+4}}dx$ using the substitution $u = x^2 + 4$. [3]

Question 13 (8 marks)

A farmer noticed that some of the eggs laid by his hens had double yolks. he estimated the probability of this happening to be 0.05. Eggs are packed in boxes of 12.

i. Find the probability that in a box, the number of eggs with double yolks will be more than two. [2]

ii. A customer bought three boxes. Find the probability that only 2 of the boxes contained exactly 1 egg with a double yolk. [3]

iii. The farmer delivered 10 boxes to a local shop. Assume that normal approximation can be used, estimate the probability that the delivery contained at least 9 eggs with double yolks. [3]

Exam continues on the next page

Question 14 (8 marks)

(a) It is given that the angle θ satisfies the equation:

$$\sin\left(2\theta + \frac{\pi}{4}\right) = 3\cos\left(2\theta + \frac{\pi}{4}\right).$$

i. Show that $\tan 2\theta = \dfrac{1}{2}$. [2]

ii. Hence, solve for θ, $\theta \in [-2\pi, \pi]$. [3]

(b) Let $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ be non-zero vectors. [3]

Let α be the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$, and β be the angle between $\underset{\sim}{b}$ and $\underset{\sim}{c}$, where α and β are acute angles such that $\alpha + \beta \neq 90^o$.

Given that $proj_{\underset{\sim}{b}}\ \underset{\sim}{a} = \underset{\sim}{d}$, show that

$$proj_{\underset{\sim}{c}}\ \underset{\sim}{d} = \frac{\cos\alpha\cos\beta}{\cos(\alpha+\beta)}\ proj_{\underset{\sim}{c}}\ \underset{\sim}{a}.$$

Exam continues on the next page

Question 15 (9 marks)

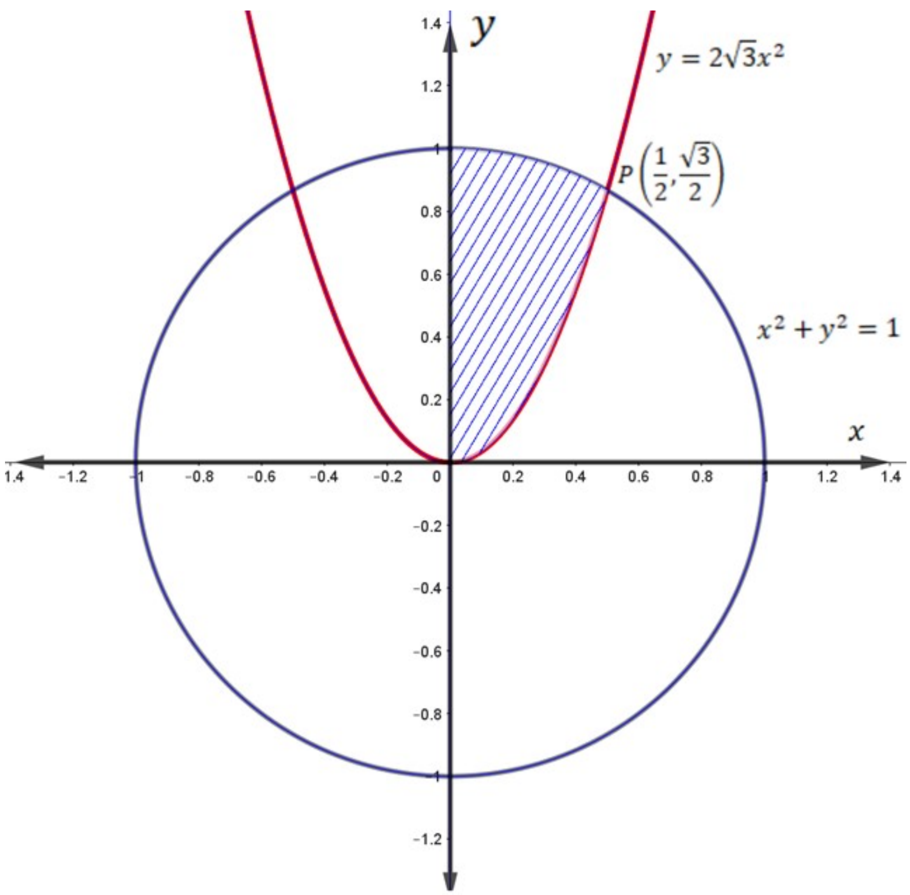
(a) The points R and S have the position vectors $\underset{\sim}{p} + 2\underset{\sim}{q}$ and $2\underset{\sim}{p} - 3\underset{\sim}{q}$ respectively.

i. If $ORST$ is a parallelogram, find $\overrightarrow{RT}$ in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$. [2]

ii. If $ORST$ is a square, show that $2|\underset{\sim}{p}|^2 = 29|\underset{\sim}{q}|^2$. [2]

(b) Show by mathematical induction that $7^{(2)^n} - 1$ is divisible by 2^{n+2} for all positive integers n. [3]

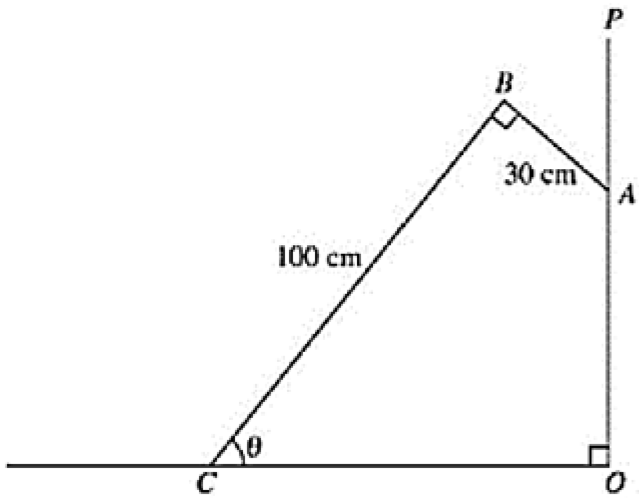
(c) The diagram below shows the graph of $x^2 + y^2 = 1$ and $y = 2\sqrt{3}x^2$. The two graphs intersect at $P\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$. Find the volume of the solid of revolution where the shaded area is rotated about the $x-$ axis. [2]



Exam continues on the next page

Question 16 (8 marks)

The figure shows a stage prop ABC used by a member of the theatre leaning against a vertical wall OP. It is given that $AB = 30$ cm,$BC = 100$ cm, $\angle ABC = \angle AOC = 90^o$ and $\angle BCO = \theta$, where θ is an acute angle.



i. Show that $OC = (100 \cos\theta + 30 \sin\theta)$ cm. [2]

Let D be foot of B on OC, let E be foot of A on BD.

ii. Express OC in terms of $R\cos(\theta - \alpha)$, where R is a positive constant and α is an acute angle. Show all necessary working. [2]

iii. State the maximum value of OC and the corresponding value of θ. [2]

iv. Find the value of θ for which $OC = 80$ cm. [2]

Exam continues on the next page

Question 17 (9 marks)

(a) Let $y = xu$.

i. Given that u is a function of x, find an expression for $\dfrac{dy}{dx}$. [1]

ii. Hence solve the initial value problem $\dfrac{dy}{dx} = \dfrac{y^2}{2x^2} + \dfrac{1}{2}$, given $y(1) = 2$. [3]
Express your answer with y as the subject in terms of x only.

(b) The position vectors of the points A, B and C are $\underset{\sim}{i}+\underset{\sim}{j}$, $2\underset{\sim}{i}+3\underset{\sim}{j}$ and $4\underset{\sim}{i}+2\underset{\sim}{j}$ respectively.

i. Find the position vector of the point P on $\overrightarrow{BC}$ such that $\overrightarrow{BP} = \dfrac{1}{3}\overrightarrow{BC}$. [2]

ii. The vertex D of a trapezium $ABCD$ is taken such that $\overrightarrow{BC}$ is parallel to $\overrightarrow{AD}$, and $\overrightarrow{PD}$ is perpendicular to $\overrightarrow{AC}$. Find the position vector of D. [3]

End of the Examination.

Question 1:

Speed decreasing when velocity heads towards 0, hence from $t = 2$ to $t = 4$, and then from $t = 9$ to $t = 10$.

Therefore 30%.

A.

Question 2:

$$\int_0^{0.5} \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx = \left[\frac{(\sin^{-1} x)^2}{2}\right]_0^{0.5}$$

$$= \left[\frac{\frac{\pi^2}{36}}{2} - 0\right]$$

$$= \frac{\pi^2}{72}$$

A

Question 3:

Gradient is positive in 1st and 3rd quadrant, and negative in 2nd and 4th quadrant, hence must be either A or B.

Along $y = 0$, the slope lines are horizontal and not vertical. Therefore answer is B.

B

Question 4:

$$\overrightarrow{OY} + \overrightarrow{YX} = \overrightarrow{OX}$$

$$\overrightarrow{OY} = \overrightarrow{OX} - \overrightarrow{YX}$$

$$= \left(\underset{\sim}{a} + 2\underset{\sim}{b}\right) - \left(\underset{\sim}{a} - \underset{\sim}{b}\right)$$

$$= 3\underset{\sim}{b}\,.$$

B

Question 5.

$$\begin{aligned}\sin(3x+x)-\sin(3x-x) &= (\sin 3x\cos x+\cos 3x\sin x)-(\sin 3x\cos x-\cos 3x\sin x)\\ &= 2\cos 3x\sin x\end{aligned}$$

B

Question 6.

When $x \in \left[-\frac{\pi}{10}, \frac{\pi}{10}\right]$

$5x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin 5x \in [-1, 1]$

$\frac{\sin 5x}{3} \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

Therefore domain of the inverse is given by also $\left[-\frac{1}{3}, \frac{1}{3}\right]$.

A

Question 7.

$\sin^{-1} t = x - 1$

$t = \sin(x-1)$ where $x - 1 \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$t^2 = \sin(x-1)$

$$\begin{aligned}y &= \frac{1}{\sqrt{1-\sin^2(x-1)}}\\ &= \frac{1}{\sqrt{\cos^2(x-1)}}\\ &= \frac{1}{|\cos(x-1)|}\\ &= \frac{1}{\cos(x-1)} \quad \left(\text{Since } x-1 \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]\right)\end{aligned}$$

C

Question 8

$$P(X=0)+P(X=1)=\left(\frac{1}{2}\right)^5+5\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1$$
$$=\frac{1}{32}+\frac{5}{32}$$
$$=\frac{6}{32}$$

D

Question 9

$$\alpha+\beta=a,\qquad \alpha\beta=b$$
$$A_{n+1}-aA_n+bA_{n-1}=(\alpha^{n+1}+\beta^{n+1})-a(\alpha^n+\beta^n)+b(\alpha^{n-1}+\beta^{n-1})$$
$$=(\alpha^{n+1}+\beta^{n+1})-(\alpha+\beta)(\alpha^n+\beta^n)+\alpha\beta(\alpha^{n-1}+\beta^{n-1})$$
$$=(\alpha^{n+1}+\beta^{n+1})-(\alpha^n+\beta^n+\alpha^n\beta+a\beta^n)+(\alpha^n\beta+a\beta^n)$$
$$=0$$

D

Question 10

$$P(X=5)-P(X=4)=P(X=6)-P(X=5)$$
$$2P(X=5)=P(X=6)+P(X=4)$$
$$\frac{2\times n!}{5!\,(n-5)!}=\frac{n!}{6!\,(n-6)!}+\frac{n!}{4!\,(n-4)!}$$
$$\frac{2\times 6(n-4)}{6!\,(n-4)!}=\frac{(n-5)(n-4)}{6!\,(n-4)!}+\frac{6\times 5}{6!\,(n-4)!}$$
$$12(n-4)=n^2-9n+20+30$$
$$n^2-21n+98=0$$
$$(n-14)(n-7)=0$$
$$n=14\text{ or }7$$

A

Solutions

Question 11

Marker's feedback

- Multiplicity 1 does *not* mean $P(x) = ax + b$.
- Remember to use $\int$ symbol when solving separable differential equations. Marks were not taken off but be wary.

  Don't write $\dfrac{8000}{P(8000-P)}dP = \dfrac{1}{6}dt$.
- Many students used only sum and product of roots methods in part (a)(ii), right after a question that *hints* at using roots of multiplicity instead. Be aware of easier methods that are hinted at by the question.
- When removing the absolute value symbols, you need to **justify** which case.
- Show/prove questions need to read like a good argument for why the statement is true. It is not a dump of ideas. Where there is a lot of algebraic steps, you need to guide the reader sometimes - especially if something derived earlier is used.

(a) (i) **Method 1**

Since $P(\alpha) = 0$, then $x - \alpha$ is a factor of $P(x)$. So $P(x)$ is of the shape

$$P(x) = (x-\alpha)A(x).$$

Differentiating gives $P'(x) = A(x) + (x-\alpha)A'(x)$.

✓ Use of $P(\alpha) = 0$, factor theorem, and derivative.

Since $P'(\alpha) = 0$, then $A(\alpha) = 0$, so $x - \alpha$ is a root of $A(x)$, so $A(x)$ is of the shape

$$A(x) = (x-\alpha)B(x).$$

Therefore, $P(x) = (x-\alpha)^2 B(x)$, so α has multiplicity at least 2.

✓ Use of $P'(\alpha) = 0$ and factor theorem.

Method 2

Let α be a zero of multiplicity n, where $n \in \mathbb{N}$. We aim to show that $n \geq 2$.

Then the polynomial $P(x)$ is of the form $P(x) = (x-\alpha)^n Q(x)$ and $Q(\alpha) \neq 0$.

Differentiating gives $P'(x) = n(x-\alpha)^{n-1}Q(x) + (x-\alpha)^n Q'(x)$
$$= (x-\alpha)^{n-1}\big(nQ(x) + (x-\alpha)Q'(x)\big).$$

where $nQ(\alpha) + (x-\alpha)Q'(\alpha) \neq 0$.

Since $P'(\alpha) = 0$, then the factor $(x-\alpha)^{n-1} = 0$ as the other one is not zero. Since $P(x)$ is not identically zero, then $n-1 \geq 1$, i.e. $n \geq 2$.

(ii) Let α, α, β be roots of $ax^3 + bx^2 + c = 0$. Note also that none of the roots are zero, otherwise x will be a factor of the polynomial expression $P(x)$ on the left-hand side.

Differentiating gives $P'(x) = 3ax^2 + 2bx$. ✓Differentiating $P(x)$.

Since α is a double root, $P(\alpha) = P'(\alpha) = 0$, so

$$3a\alpha^2 + 2b\alpha = 0$$
$$a\alpha^3 + b\alpha^2 + c = 0.$$

Since $\alpha \neq 0$, the first equation reduces to $3a\alpha + 2b = 0$.

Making α the subject: $\alpha = -\dfrac{2b}{3a}$. ✓Significant progress towards solution.

Substitute this into the second equation:

$$a\left(-\frac{2b}{3a}\right)^3 + b\left(-\frac{2b}{3a}\right)^2 + c = 0$$
$$-\frac{8b^3}{27a^2} + \frac{4b^3}{9a^2} + c = 0$$
$$-8b^3 + 12b^3 + 27a^2c = 0$$
$$\therefore\ 27a^2c + 4b^3 = 0$$

✓Result proven.

(b) (i) $RHS = \dfrac{1}{P} + \dfrac{1}{Q-P}$
$$= \frac{Q-P+P}{P(Q-P)}$$
$$= \frac{Q}{P(Q-P)} = LHS$$

✓Result proven.

(ii) Write $\dfrac{dP}{dt} = \dfrac{P}{6}\left(\dfrac{8000-P}{8000}\right)$.

Divide factors containing P to the left-hand side, then integrate both sides:

$$\int_{1000}^{P} \frac{8000}{P(8000-P)}\, dP = \int_0^t \frac{1}{6}\, dt$$
$$\int_{1000}^{P} \left(\frac{1}{P} + \frac{1}{8000-P}\right) dP = \frac{t}{6}$$
$$\Big[\ln|P| - \ln|8000-P|\Big]_{1000}^{P} = \frac{t}{6}$$
$$\left[\ln\left|\frac{P}{8000-P}\right|\right]_{1000}^{P} = \frac{t}{6}$$
$$\ln\left|\frac{P}{8000-P}\right| - \ln\frac{1}{7} = \frac{t}{6}$$
$$\ln\left|\frac{7P}{8000-P}\right| = \frac{t}{6}$$
$$\left|\frac{7P}{8000-P}\right| = e^{t/6}.$$

✓Integrate correct expressions. Can do definite or indefinite.

✓Significant progress towards solution.

Since $0 < P < 8000$, the expression in the absolute values is positive.

✓Justify absolute value, and make P the subject with sufficient number of steps shown.

So,

$$\frac{7P}{8000-P} = e^{t/6}$$
$$7P = 8000e^{t/6} - Pe^{t/6}$$
$$7P + Pe^{t/6} = 8000e^{t/6}$$
$$\therefore\ P = \frac{8000e^{t/6}}{7+e^{t/6}}.$$

2025 - Trials - Ext 1 1/2

MATHEMATICS Extension 1: Question 12

Suggested Solutions	Marks Awarded	Marker's Comments

a) $y = \ln(\tan^{-1} 2x)$

Done well!

Let $y = \ln u$ $\quad u = \tan^{-1} 2x$

$\frac{dy}{du} = \frac{1}{u}$ $\quad \frac{du}{dx} = \frac{2}{1+(2x)^2}$

$y' = \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{1}{u} \times \frac{2}{1+4x^2}$ — (1)

$= \frac{2}{\tan^{-1}2x\,(1+4x^2)}$ — (1)

(b) i. Expand the binomial expansion of $(2-x)^5$, simplifying all terms. [2]

ii. Hence, find the coefficient of y^3, in the expansion of $(2-3y+y^2)^5$. [2]

bi) ${}^5C_0 2^5 x^0 - {}^5C_1 2^4 x^1 + {}^5C_2 2^3 x^2 - {}^5C_3 2^2 x^3$ — (1)

$+ {}^5C_4 2^1 x^4 - {}^5C_5 2^0 x^5$

$= 32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$ — (1) must simplify!

Some students didn't simplify

Method 1

ii) Let $x = (3y - y^2) \to$ (pt i)

$\therefore 32 - 80(3y-y^2) + 80(3y-y^2)^2 - 40(3y-y^2)^3$

$+ 10(3y-y^2)^4 - (3y-y^2)^5$ — (1) for using a method so find the cases

Cases for y^3

$\to 80(3y-y^2)^2 = -6 \times 80$

$\to -40(3y-y^2)^3 = 27 \times -40$

$= -1560$ — (1)

There were a handful of students who got the expansion wrong! and thus it was only case by case that had CFE!

Method 2

$(2-3y+y^2) = (2-y)(1-y)$

$\therefore (1-y)^5 = 1 - 5y + 10y^2 - 10y^3 + 5y^4 - y^5$

from pt i) $(2-y)^5 = 32 - 80y + 80y^2 - 40y^3 + 10y^4 - y^5$

Only a small number of students used Method 2.

Cases for y^3

$32(-10y^3) = -320$

$-80y(10y^2) = -800$

$80y^2(-5y) = -400$

$-40y^3(1) = -40$

$= -1560$ — (1) showing cases for y^3

2025 - Trials - Ext 1

MATHEMATICS Extension 1: Question 12 — 2/2

Suggested Solutions	Marks Awarded	Marker's Comments
c) $\int_0^{\sqrt{5}} \frac{2x^3}{\sqrt{x^2+4}}\,dx \quad , u = x^2+4$ $u = x^2+4 \quad , \quad x^2 = u-4$ $\frac{du}{dx} = 2x \longrightarrow du = 2x\,dx$ $x = 0 \quad , \quad u = 4$ $x = \sqrt{5} \quad , \quad u = 9$ — 1 $\int_0^{\sqrt{5}} \frac{2x \times x^2}{\sqrt{x^2+4}}\,dx$ $= \int_0^{\sqrt{5}} \frac{x^2}{\sqrt{x^2+4}}\,2x\,dx$ $= \int_4^9 \frac{u-4}{\sqrt{u}}\,du$ $= \int_4^9 u^{\frac{1}{2}} - 4u^{-\frac{1}{2}}\,du$ — 1 $= \left[\frac{2}{3}u^{3/2} - 8u^{\frac{1}{2}}\right]_4^9$ $= \frac{14}{3}$ or $4\frac{2}{3}$ — 1	1 - showing the bounds for u 1 - correct integration 1 - answer	PLEASE !! Do not !! Have a mix of 'u' and 'x' It should be all in terms of 'x' only or all in terms of 'u' only

Question 13

i.

$\mathrm{Bin}(12, 0.05)$

Let the number of eggs with double yolks be represented by the random variable X.

$$
\begin{aligned}
P(X > 2) &= 1 - P(X \le 2) \\
&= 1 - \big(P(X = 0) + P(X = 1) + P(X = 2)\big) \\
&= 1 - \left(\binom{12}{0}(0.05)^0(0.95)^{12} + \binom{12}{1}(0.05)^1(0.95)^{11} + \binom{12}{2}(0.05)^2(0.95)^{10}\right) \\
&\approx 1 - 0.9804 \\
&= 0.0196 \\
&= 1.96\%
\end{aligned}
$$

1 mark for demonstrating an understanding of how to exercise binomial probability AND the need to use complement events in this question

1 mark for obtaining correct answer at the end

Note: Most common mistake here involves students forgetting that they need to evaluate $P(X = 2)$ as well. Students who didn't understand the need for $\binom{12}{r}$ functions get 0, for a complete failure to exercise binomial probability.

ii.

$$
\begin{aligned}
P(1 \text{ egg with double yolk in a box}) &= \binom{12}{1}(0.05)^1(0.95)^{11} \\
&\approx 0.3413
\end{aligned}
$$

Let X represent the number of boxes with exactly 1 egg with a double yolk

$$
\begin{aligned}
P(X = 2) &= \binom{3}{2}\left(\binom{12}{1}(0.05)^1(0.95)^{11}\right)^2\left(1 - \binom{12}{1}(0.05)^1(0.95)^{11}\right)^1 \\
&\approx 0.2302 \\
&= 23.02\%
\end{aligned}
$$

1 mark for correctly obtaining the probability of having 1 egg with double yolk in any given box.
1 mark for correctly applying binomial probability of there being 2 boxes.
1 mark for final answer

Note: Students who did not realise that they had to find the probability of getting 1 egg with double yolk first, forfeits 2 marks immediately, because it represents a "fatal" error in which the crux of the question not understood, and also became unable to find the complement of 1 egg with double yolk in any given box.

iii.

Method 1: Sample Proportion

$\mathrm{Bin}(120, 0.05)$

$\mu = p = 0.05$

$\sigma = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.05 \times 0.95}{120}} \approx 0.02$

$$\hat{p} = \frac{9}{120} \Rightarrow z = \frac{\frac{9}{120} - \frac{1}{20}}{\sqrt{\frac{0.05 \times 0.95}{120}}}$$
$$\approx 1.26$$

$$P\left(\hat{p} \geq \frac{9}{120}\right) = P(Z \geq 1.26)$$
$$\approx 1 - 0.8962$$
$$= 0.1038$$

Method 2: No Sample Proportion

$\mu = np = 0.05 \times 120 = 6$

$\sigma = \sqrt{npq} = \sqrt{120 \times 0.05 \times 0.95} \approx 2.39$

$$x = 9 \Rightarrow z = \frac{9 - 6}{\sqrt{5.7}}$$
$$\approx 1.26$$

$$P(X \geq 9) = P(Z \geq 1.26)$$
$$\approx 1 - 0.8962$$
$$= 0.1038$$

1 mark for working out the value of μ and σ.

1 mark for correctly finding the z score required for approximation.

1 mark for final answer

Note:

1. It is NOT necessary to use continuity correction, it is NOT in the syllabus! If you did use continuity correction, if you did it correctly you still get the full marks. However, if you used it incorrectly you lost a mark.
2. It is NOT necessary to use linear interpolation to obtain a probability value, also NOT in the syllabus!
3. Rounding is extremely important in this question, since the final mark is allocated on finding the correct value on the z-score table, markers are unable to assume whether your error was based on rounding or not knowing how to read from the table. Hence you MUST use the value of $z = 1.26$, (NOT $z = 1.25, 1.2, 1.3, 1$ etc...).

Student Number

Question 14

(a) (i) $\sin\left(2\theta + \frac{\pi}{4}\right) = 3\cos\left(2\theta + \frac{\pi}{4}\right)$

$\therefore \quad \dfrac{\sin(2\theta + \pi/4)}{\cos\left(2\theta + \frac{\pi}{4}\right)} = 3$

$\therefore \quad \tan\left(2\theta + \frac{\pi}{4}\right) = 3$

$\therefore \quad \dfrac{\tan 2\theta + \tan\frac{\pi}{4}}{1 - \tan 2\theta \tan\frac{\pi}{4}} = 3$ ✓

$\therefore \quad \dfrac{\tan 2\theta + 1}{1 - \tan 2\theta} = 3$

$\therefore \quad \tan 2\theta + 1 = 3 - 3\tan 2\theta$ ✓

$\therefore \quad 4\tan 2\theta = 2$

$\therefore \quad \tan 2\theta = \frac{1}{2}$

Comments: Generally well done, using the above method or Expand both sides.

- Use the formula sheet to avoid careless errors
- Using the double angle formula here, require the value of trig functions of $\pi/8$.
- Algebraic errors.

* Marks for other method are given for expanding correctly, then simplifying to obtain the required result.

Student Number ____________

(ii) $\tan 2\theta = \frac{1}{2}$ $\qquad \theta \in [-2\pi, \pi]$

$2\theta \in [-4\pi, 2\pi]$

$$2\theta = -4\pi + \tan^{-1}\left(\tfrac{1}{2}\right),\ -3\pi + \tan^{-1}\left(\tfrac{1}{2}\right),\ -2\pi + \tan^{-1}\left(\tfrac{1}{2}\right),$$
$$-\pi + \tan^{-1}\left(\tfrac{1}{2}\right),\ \tan^{-1}\left(\tfrac{1}{2}\right),\ \pi + \tan^{-1}\left(\tfrac{1}{2}\right)$$

$$\theta = -2\pi + \tfrac{1}{2}\tan^{-1}\left(\tfrac{1}{2}\right),\ -\tfrac{3\pi}{2} + \tfrac{1}{2}\tan^{-1}\left(\tfrac{1}{2}\right),\ -\pi + \tfrac{1}{2}\tan^{-1}\left(\tfrac{1}{2}\right),$$
$$-\tfrac{\pi}{2} + \tfrac{1}{2}\tan^{-1}\left(\tfrac{1}{2}\right),\ \tfrac{1}{2}\tan^{-1}\left(\tfrac{1}{2}\right),\ \tfrac{\pi}{2} + \tfrac{1}{2}\tan^{-1}\left(\tfrac{1}{2}\right)$$

OR

$$\theta = -6.05,\ -4.48,\ -2.91,\ -1.34,\ 0.232,\ 1.803$$

OR

$$\theta = -1.9262\pi,\ -1.426\pi,\ -0.9262\pi,\ -0.4238\pi,\ 0.0738\pi,$$
$$0.5738\pi$$

Comments:

* Domain in radians, solutions in radians
* Six solutions
* Solutions must be in the domain
* 3 marks for six correct answers
* 2 marks for at least 3 correct answers.
* 1 mark for less than 3 correct answers.

Student Number ____________

(b) $\underset{\sim}{d} = \text{proj}_{\underset{\sim}{b}} \underset{\sim}{a} = \dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \underset{\sim}{b}$

$$= \frac{|\underset{\sim}{a}|\,|\underset{\sim}{b}| \cos\alpha}{|\underset{\sim}{b}|^2} \underset{\sim}{b}$$

$$= \frac{|\underset{\sim}{a}| \cos\alpha}{|\underset{\sim}{b}|} \underset{\sim}{b}$$

$$\text{proj}_{\underset{\sim}{c}} \underset{\sim}{d} = \frac{\underset{\sim}{d} \cdot \underset{\sim}{c}}{\underset{\sim}{c} \cdot \underset{\sim}{c}} \underset{\sim}{c}$$

$$= \frac{\frac{|\underset{\sim}{a}| \cos\alpha}{|\underset{\sim}{b}|} \underset{\sim}{b} \cdot \underset{\sim}{c}}{\underset{\sim}{c} \cdot \underset{\sim}{c}} \underset{\sim}{c} \quad \checkmark$$

$$= \frac{|\underset{\sim}{a}| \cos\alpha}{|\underset{\sim}{b}|} \, \frac{|\underset{\sim}{b}|\,|\underset{\sim}{c}| \cos\beta}{|\underset{\sim}{c}|^2} \underset{\sim}{c} \quad \checkmark$$

$$= \frac{|\underset{\sim}{a}|}{|\underset{\sim}{c}|} \cos\alpha \cos\beta \, \underset{\sim}{c}$$

$$= \frac{\cos\alpha \cos\beta}{\cos(\alpha+\beta)} \, \frac{|\underset{\sim}{a}|\,|\underset{\sim}{c}| \cos(\alpha+\beta)}{|\underset{\sim}{c}|^2} \underset{\sim}{c}$$

$$= \frac{\cos\alpha \cos\beta}{\cos(\alpha+\beta)} \, \frac{\underset{\sim}{a} \cdot \underset{\sim}{c}}{\underset{\sim}{c} \cdot \underset{\sim}{c}} \underset{\sim}{c} \quad \checkmark$$

$$= \frac{\cos\alpha \cos\beta}{\cos(\alpha+\beta)} \, \text{proj}_{\underset{\sim}{c}} \underset{\sim}{a}$$

Student Number ______________

Alternative method:

$$RHS = \frac{\cos\alpha\cos\beta}{\cos(\alpha+\beta)}\,\text{proj}_{\underset{\sim}{c}}\,\underset{\sim}{a}$$

$$= \frac{\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}\;\dfrac{\underset{\sim}{b}\cdot\underset{\sim}{c}}{|\underset{\sim}{b}||\underset{\sim}{c}|}}{\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{c}}{|\underset{\sim}{a}||\underset{\sim}{c}|}}\;\frac{\underset{\sim}{a}\cdot\underset{\sim}{c}}{\underset{\sim}{c}\cdot\underset{\sim}{c}}\,\underset{\sim}{c} \quad \checkmark$$

$$= \frac{(\underset{\sim}{a}\cdot\underset{\sim}{b})(\underset{\sim}{b}\cdot\underset{\sim}{c})}{|\underset{\sim}{b}|^2|\underset{\sim}{c}|^2}\,\underset{\sim}{c}$$

$$= \frac{\left(\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{b}}{|\underset{\sim}{b}|^2}\right)\underset{\sim}{b}\cdot\underset{\sim}{c}}{|\underset{\sim}{c}|^2}\,\underset{\sim}{c} \quad \checkmark$$

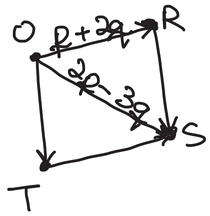
$$= \frac{\underset{\sim}{d}\cdot\underset{\sim}{c}}{|\underset{\sim}{c}|^2}\,\underset{\sim}{c} \quad \checkmark$$

$$= \text{proj}_{\underset{\sim}{c}}\,\underset{\sim}{d}$$

Comments: Majority of students did it correctly using similar methods.

- Major issue of not using vector notation
- the dot product in some cases like minus sign
- need to explain that the angle between $\underset{\sim}{b}$ and $\underset{\sim}{c}$ is the same as the angle between $\underset{\sim}{d}$ and $\underset{\sim}{c}$ since $\underset{\sim}{d}$ is the projection of $\underset{\sim}{a}$ into $\underset{\sim}{b}$.
- Starting from the RHS is a bit tricky because you need to pull out the expression of $\underset{\sim}{d}$ and use the corresponding dot product.
- Starting from RHS, students create new formula

MATHEMATICS Extension 1 : Question..15...

Suggested Solutions	Marks	Marker's Comments
i)  $\vec{OR}+\vec{RS}=\vec{OS}$ $\vec{RS}=\vec{OS}-\vec{OR}$ $=2\underset{\sim}{p}-3\underset{\sim}{q}-(\underset{\sim}{p}+2\underset{\sim}{q})$ $=\underset{\sim}{p}-5\underset{\sim}{q}$	1	Some students put T in the wrong place making ORTS rather than ORST.
$\vec{RT}=\vec{RS}+\vec{ST}$ $=\vec{RS}-\vec{OR}$ $=\underset{\sim}{p}-5\underset{\sim}{q}-(\underset{\sim}{p}+2\underset{\sim}{q})$ $=-7\underset{\sim}{q}$	1	
ii) $\vec{OS}\cdot\vec{RT}=0$ (diagonals in square are perpendicular to each other) $(2\underset{\sim}{p}-3\underset{\sim}{q})\cdot(-7\underset{\sim}{q})=0$ $-14\underset{\sim}{p}\cdot\underset{\sim}{q}+21\underset{\sim}{q}\cdot\underset{\sim}{q}=0$ $14\underset{\sim}{p}\cdot\underset{\sim}{q}=21\lvert\underset{\sim}{q}\rvert^2$ $2\underset{\sim}{p}\cdot\underset{\sim}{q}=3\lvert\underset{\sim}{q}\rvert^2$ – ① $\lvert\vec{OS}\rvert=\lvert\vec{RT}\rvert$ (diagonals are equal) $(2\underset{\sim}{p}-3\underset{\sim}{q})\cdot(2\underset{\sim}{p}-3\underset{\sim}{q})=(-7\underset{\sim}{q})\cdot(-7\underset{\sim}{q})$ $4\underset{\sim}{p}\cdot\underset{\sim}{p}-12\underset{\sim}{p}\cdot\underset{\sim}{q}+9\underset{\sim}{q}\cdot\underset{\sim}{q}=49\underset{\sim}{q}\cdot\underset{\sim}{q}$ $4\lvert\underset{\sim}{p}\rvert^2-12\underset{\sim}{p}\cdot\underset{\sim}{q}-40\lvert\underset{\sim}{q}\rvert^2=0$ – ② sub. ① into ② $4\lvert\underset{\sim}{p}\rvert^2-6(3\lvert\underset{\sim}{q}\rvert^2)-40\lvert\underset{\sim}{q}\rvert^2=0$ $4\lvert\underset{\sim}{p}\rvert^2=58\lvert\underset{\sim}{q}\rvert^2$ $2\lvert\underset{\sim}{p}\rvert^2=29\lvert\underset{\sim}{q}\rvert^2$		① for having one equation: either $14\underset{\sim}{p}\cdot\underset{\sim}{q}=21\lvert\underset{\sim}{q}\rvert^2$ OR $4\lvert\underset{\sim}{p}\rvert^2-12\underset{\sim}{p}\cdot\underset{\sim}{q}-40\lvert\underset{\sim}{q}\rvert^2=0$ ① the remaining working. Students need to have correct notation. cannot write: $\underset{\sim}{p}\underset{\sim}{q}$, $\underset{\sim}{p}^2$ must have: $\underset{\sim}{p}\cdot\underset{\sim}{q}$, $\lvert\underset{\sim}{p}\rvert^2$ dot absolute values

MATHEMATICS Extension 1 : Question 15

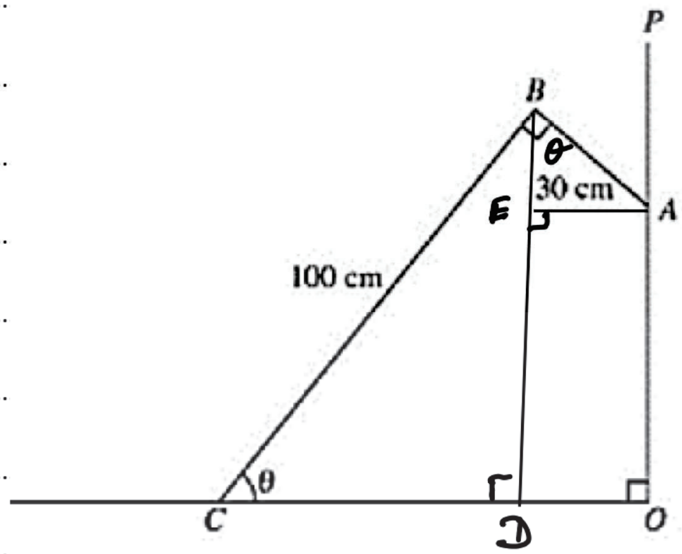
Suggested Solutions	Marks	Marker's Comments
This method is wrong even though you get the same answer:		
$\|\vec{RT}\| = \|\vec{OS}\|$		0 marks awarded.
$\|-7q\|^2 = \|2p - 3q\|^2$		
$49\|q\|^2 = 4\|p\|^2 - 9\|q\|^2$ cannot just square each vector.		
$4\|p\|^2 = 58\|q\|^2$		
$2\|p\|^2 = 29\|q\|^2$		
b) Prove true for $n=1$		
$7^{2^1} - 1 = 7^2 - 1 = 48$ $2^{1+2} = 2^3 = 8$		
$\therefore (7^{2^1} - 1) \div 8 = 48 \div 8 = 6$	1	base case
$\therefore 7^{2^1} - 1$ is divisible by 2^{1+2}. $\therefore$ true for $n=1$		need to show that you obtain an integer when dividing.
Assume true for $n=k$, $k \in \mathbb{Z}^+$ ie. $7^{2^k} - 1 = (2^{k+2})M$, $M \in \mathbb{Z}$		
Prove true for $n=k+1$ ie. $7^{2^{k+1}} - 1 = (2^{k+3})N$, $N \in \mathbb{Z}$		
$7^{2^{k+1}} - 1 = 7^{2^k \times 2} - 1 = (7^{2^k})^2 - 1$		
$= (2^{k+2}M - 1)^2 - 1$ by assumption	1	recognising the square and using the assumption.

$7^2 \times 7^{2^k}$ leads nowhere. It is incorrect.

MATHEMATICS Extension 1 : Question 15

Suggested Solutions	Marks	Marker's Comments
$= 2^{2k+4}M^2 + 2(2^{k+2}M) + 1 - 1$		
$= 2^{k+3}(2^{k+1}M^2 + M)$	1	stating $2^{k+1}M^2+M$ is an integer students must also have a conclusion.
$\therefore 7^{2^{k+1}} - 1$ is divisible by 2^{k+3} as $2^{k+1}M^2 + M \in \mathbb{Z}$		
$\therefore$ true for $n = k+1$		
$\therefore$ the statement is true by the principle of mathematical induction.		A lot of algebraic mistakes were made!
c) $x^2 + y^2 = 1$ $y^2 = 1 - x^2$ $y = \sqrt{1+x^2}$ as $y \geq 0$		
$\therefore V = \pi \int_0^{\frac{1}{2}} (\sqrt{1+x^2})^2 - (2\sqrt{3}x^2)^2 \, dx$	1	correct set up rotating around x-axis $\Rightarrow$ use dx not dy.
$= \pi \int_0^{\frac{1}{2}} 1 + x^2 - 12x^4 \, dx$		
$= \pi \left[x - \frac{x^3}{3} - \frac{12}{5}x^5 \right]_0^{\frac{1}{2}}$		Question was done well.
$= \pi \left(\frac{1}{2} - \frac{(\frac{1}{2})^3}{3} - \frac{12}{5}(\frac{1}{2})^5 - 0 \right)$		
$= \frac{23\pi}{60} u^3$	1	
$= 1.20427\ldots$		
$= 1.204 u^3$ (3dp)		

Q16



* Draw a diagram
* show all working

i) $\angle ABD = \theta$

In ΔABE

$\sin\theta = \frac{AE}{30}$ $\quad \therefore AE = 30\sin\theta$

In ΔCBD

$\cos\theta = \frac{CD}{100}$ $\quad \therefore CD = 100\cos\theta$

$OC = CD + DO$

$= CD + AE = 100\cos\theta + 30\sin\theta$

ii) $R\cos(\theta - \alpha) = 100\cos\theta + 30\sin\theta$

$R\cos\theta\cos\alpha + R\sin\theta\sin\alpha = 100\cos\theta + 30\sin\theta$

$\therefore \quad R\cos\alpha = 100 \quad$ —— ①

$R\sin\alpha = 30 \quad$ —— ②

$①^2 + ②^2 \quad R^2 = 100^2 + 30^2$

$R = \sqrt{10900} = 10\sqrt{109}$ since $R>0$

$\tan\alpha = \frac{3}{10} \Rightarrow \alpha = 16^\circ 42'$

$\therefore 100\cos\theta + 30\sin\theta = 10\sqrt{109}\cos(\theta - 16^\circ 42')$

iii) $OC = 10\sqrt{109}\cos(\theta - 16^\circ 42')$

max OC occurs when $\cos(\theta - 16^\circ 42') = 1$

$\therefore$ Max $OC = 10\sqrt{109}$ cm ✓ with justification for max

$\cos(\theta - 16^\circ 42') = 1$

$\therefore \theta = 16^\circ 42'$ since θ is acute angle.

iv) $10\sqrt{109}\cos(\theta - 16^\circ 42') = 80$

$\cos(\theta - 16^\circ 42') = \frac{8}{\sqrt{109}}$

$\theta - 16^\circ 42' = 39^\circ 59'$ since θ is acute angle.

$\theta = 56^\circ 41'$

* θ must be expressed in degree as in the qustion
* Many steudents incorrectly left θ in radian

Y12 Ext 1 Trial solutions for Q17

Suggested Solutions

Question 17

a i, Given $y = xu$; find $\frac{dy}{dx}$

$y = xu$; using chain rule

if $y = uv$

$y' = u'v + uv'$

$\therefore \frac{dy}{dx} = u + x\frac{du}{dx}$

$\frac{dy}{dx} = u + xu'$

Marker's Feedback

Poorly done! Most students struggled to apply chain rule.

Page ______

Suggested Solutions

Question 17 a, ii,

Solve $y' = \frac{y^2}{2x^2} + \frac{1}{2}$ given $y(1) = 2$

$y' = \frac{y^2}{2x^2} + \frac{1}{2}$

Since $y' = u + xu'$ and $y = xu$

$u + xu' = \frac{(xu)^2}{2x^2} + \frac{1}{2}$

$u + xu' = \frac{u^2}{2} + \frac{1}{2}$

$xu' = \frac{u^2}{2} + \frac{1}{2} - u$

$xu' = \frac{1}{2}(u^2 - 2u + 1)$

$xu' = \frac{1}{2}(u-1)^2$

$x\frac{du}{dx} = \frac{1}{2}(u-1)^2$

$\int \frac{1}{(u-1)^2}\,du = \frac{1}{2}\int \frac{1}{x}\,dx$ 1m

$\int (u-1)^{-2}\,du = \frac{1}{2}\ln|x| + c$

$\frac{-1}{(u-1)} = \frac{1}{2}\ln|x| + c$

$\frac{1}{1-u} = \frac{1}{2}\ln|x| + c$

but $u = \frac{y}{x}$

$\frac{1}{1-\frac{y}{x}} = \frac{1}{2}\ln|x| + c$

Marker's Feedback

Suggested Solutions

Question 17a ii, continued

$$1-\frac{y}{x}=\frac{1}{\frac{1}{2}\ln|x|+c}$$

$$1-\frac{1}{\frac{1}{2}\ln|x|+c}=\frac{y}{x}$$

$$y=x-\frac{x}{\frac{1}{2}\ln|x|+c} \quad \text{— 1m}$$

Given $y(1)=2$

$$2=1-\frac{1}{\frac{1}{2}\ln|1|+c}$$

$$2=1-\frac{1}{c}$$

$$c=-1$$

$$\therefore y=x-\frac{x}{\frac{1}{2}\ln|x|-1} \quad \text{— 1m}$$

or

$$y=x\left(1-\frac{1}{\frac{1}{2}\ln|x|-1}\right) \text{ or } y=x\left(1-\frac{2}{\ln|x|-2}\right) \text{ or } y=\frac{x(4-\ln x)}{2-\ln x}$$

Marker's Feedback

All students who used part i, and substituted $y=xu$ in $y'=\frac{y^2}{2x^2}+\frac{1}{2}$ have got full marks. other than a few calculation mistakes.

Suggested Solutions

Question 17 b

i, Given $\overrightarrow{BP} = \frac{1}{3}\overrightarrow{BC}$

$\overrightarrow{BP} = \overrightarrow{OP} - \overrightarrow{OB}$

$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$\therefore$

$\overrightarrow{OP} - \overrightarrow{OB} = \frac{1}{3}(\overrightarrow{OC} - \overrightarrow{OB})$ — 1m

$\overrightarrow{OP} = \frac{1}{3}(\overrightarrow{OC}) - \frac{1}{3}(\overrightarrow{OB}) + \overrightarrow{OB}$

$\overrightarrow{OP} = \frac{1}{3}\overrightarrow{OC} + \frac{2}{3}\overrightarrow{OB}$

$\overrightarrow{OP} = \frac{1}{3}(4\underset{\sim}{i} + 2\underset{\sim}{j}) + \frac{2}{3}(2\underset{\sim}{i} + 3\underset{\sim}{j})$

$= \frac{8}{3}\underset{\sim}{i} + \frac{8}{3}\underset{\sim}{j}$. — 1m

Marker's Feedback

Most students have shown very good understanding in this part.
A few calculation mistakes were also observed.

Suggested Solutions

Question 17 b (ii)

$\overrightarrow{BC} \parallel \overrightarrow{AD}$ $\quad$ $\overrightarrow{PD} \perp \overrightarrow{AC}$ $\quad$ Find $\overrightarrow{OD}$

$\overrightarrow{BC} = 2\underset{\sim}{i} - \underset{\sim}{j}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$ and $\overrightarrow{AD} = K\overrightarrow{BC}$ $(\because \overrightarrow{BC} \parallel \overrightarrow{AD})$

$\overrightarrow{OD} - \overrightarrow{OA} = K\overrightarrow{BC}$

$\overrightarrow{OD} = \overrightarrow{OA} + K\overrightarrow{BC}$

$\overrightarrow{OD} = (\underset{\sim}{i} + \underset{\sim}{j}) + K(2\underset{\sim}{i} - \underset{\sim}{j})$

$\overrightarrow{OD} = (1+2K)\underset{\sim}{i} + (1-K)\underset{\sim}{j}$ — 1m

Also $\overrightarrow{OP} = \frac{8}{3}(\underset{\sim}{i} + \underset{\sim}{j})$ & $\overrightarrow{AC} = 3\underset{\sim}{i} + \underset{\sim}{j}$

$\overrightarrow{PD} \perp \overrightarrow{AC}$

$\therefore (\overrightarrow{OD} - \overrightarrow{OP}) \cdot \overrightarrow{AC} = 0$

$$\begin{pmatrix} \frac{6K-5}{3} \\ \frac{-3K-5}{3} \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \end{pmatrix} = 0$$

Marker's Feedback

Suggested Solutions

Question

$$6k-5-\frac{3k+5}{3}=0$$

$$18k-15-3k-5=0$$

$$k=\frac{4}{3} \quad \text{——} \quad 1m$$

$$\therefore \vec{OD}=(1+2k)\underset{\sim}{i}+(1-k)\underset{\sim}{j}$$

$$=\left(1+2\left(\frac{4}{3}\right)\right)\underset{\sim}{i}+\left(1-\left(\frac{4}{3}\right)\right)\underset{\sim}{j}$$

$$=\frac{11}{3}\underset{\sim}{i}-\frac{1}{3}\underset{\sim}{j} \quad \text{——} \quad 1m$$

Marker's Feedback

Most students have shown good understanding of application of parallel and ⊥ vectors.

Watch out for calculation mistakes.