VIRILE AGITUR

Name :________________________

Teacher : ________________________

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2025 | YEAR 12 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

MATHEMATICS EXTENSION 1

General Instructions:

- Reading Time – 10 minutes
- Working time – 2 hours
- Write using black or blue pen
- NESA approved calculators may be used
- A Reference sheet is provided at the back of this paper and separately on yellow paper (You may use either)

Total Marks: 70

Section I – 10 marks (pages 3-7)

- Attempt Questions 1 – 10
- Use the separate multiple-choice answer sheet provided
- Allow about 15 minutes for this section

Section II – 60 marks (pages 8-13)

- For Questions 11 – 14 write your solutions in the booklets provided
- Show all relevant mathematical reasoning and/or calculations
- Each of Questions 11 – 14 is of equal value
- Allow about 1 hour 40 minutes for this section

M/C	Q11	Q12	Q13	Q14	TOTAL
/10	/15	/15	/15	/15	/70

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 Find $\int \frac{1}{1+4x^2}\,dx$.

A. $\tan^{-1}2x + c$

B. $\frac{1}{2}\tan^{-1}2x + c$

C. $2\tan^{-1}2x + c$

D. $\frac{1}{2}\tan^{-1}\frac{x}{2} + c$

2 It is given that $\cos x = \frac{2}{3}$, where $0 < x < \frac{\pi}{2}$.

What is the value of $\cos 2x$?

A. $-\frac{1}{9}$

B. $-\frac{4}{3}$

C. $\frac{1}{9}$

D. $\frac{4}{3}$

3 A discrete random variable X is binomially distributed with a mean of 6 and a variance of 1.5.

What is the number of independent trials?

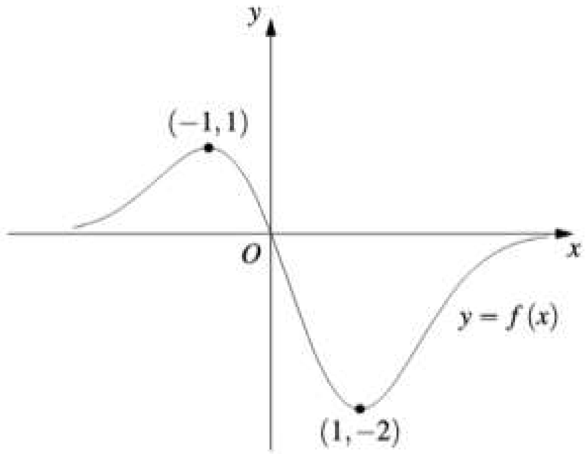
A. 5

B. 6

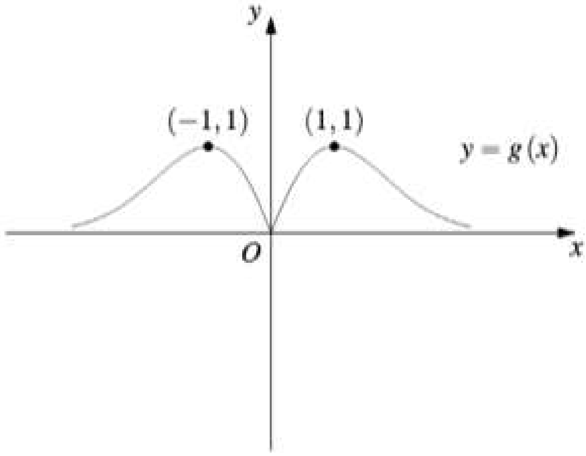
C. 8

D. 24

4 The graph of $y = f(x)$ is shown.



The graph of $y = f(x)$ was transformed to get the graph of $y = g(x)$ as shown below.



What transformation was applied?

A. $g(x) = f(|-x|)$

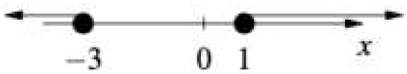
B. $g(x) = \sqrt{(f(-x)}$

C. $g(x) = f(-|x|)$

D. $[g(x)]^2 = -f(x)$

5 Which diagram best represents the solution set of $\frac{2}{3-x} \leq 1$?

A.



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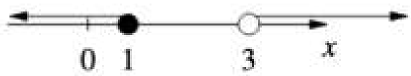
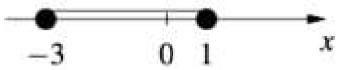
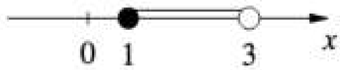


C.



D.



6 Which of the following is the simplest expression for $\dfrac{\cos 4\theta}{\sin 2\theta} + \dfrac{\sin 4\theta}{\cos 2\theta}$?

A. $\dfrac{2\cos 6\theta}{\cos 4\theta}$

B. $\dfrac{\sin 4\theta}{\cos 2\theta}$

C. $\sin 2\theta$

D. $\operatorname{cosec} 2\theta$

7 The line $y = 3x - 4$ is tangential to the graph of $y = f(x)$ at the point where $x = 1$. Assuming $y = f(x)$ is invertible, what is the equation of the normal to the graph of $y = f^{-1}(x)$ at the point where $x = -1$?

A. $y = -\frac{1}{3}x - \frac{2}{3}$

B. $y = \frac{1}{3}x + \frac{4}{3}$

C. $y = -3x - 2$

D. $y = -3x + 3$

8 In how many ways can the letters of the word DIFFERENT be arranged in a row with the Fs separated?

A. 20 160

B. 70 560

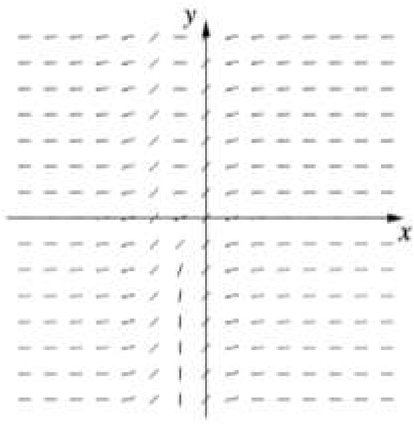
C. 80 640

D. 90 720

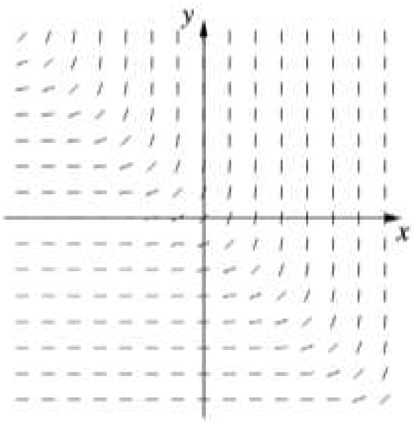
9 The general solution to a differential equation is $f(x) = \ln(e^x + c)$, where c is a constant.

Which of the following best represents the direction field for the differential equation?

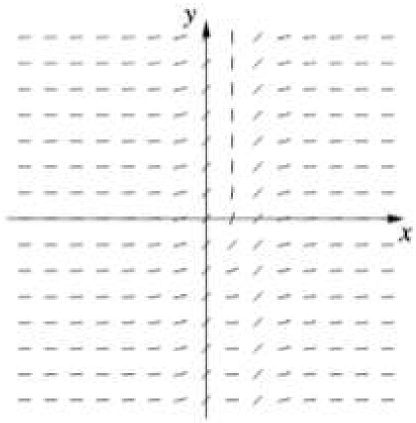
A.



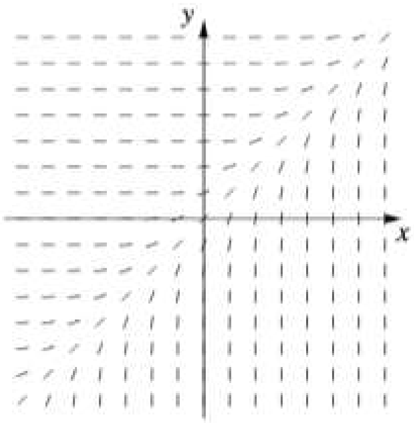
B.



C.

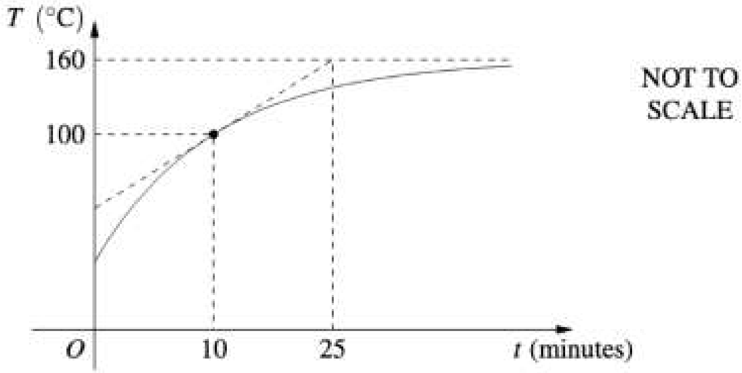


D.



10 The temperature of a pastry, T° C, is monitored after it is placed into an oven set to a temperature of 160° C. It is assumed that $T = 160 - Ae^{-kt}$, where A and k are constants, and t is the time in minutes after the pastry was placed in the oven.

A recording shows that the pastry reaches 100° C after $t = 10$ minutes and the linear rate of change in the temperature at that time would mean it reaches 160° C when $t = 25$ minutes, as shown.



What is the value of k?

A. $\frac{1}{4}$

B. $-\frac{1}{4}$

C. $\frac{1}{15}$

D. $-\frac{1}{15}$

End of Section I

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 40 minutes for this section

Answer each question in a new writing booklet. Extra writing booklets are available.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a NEW writing booklet.

(a) A mathematics class consists of 4 girls and 4 boys. A committee of 5 students must be formed.

(i) Find the number of different committees that can be formed. **1**

(ii) Find the probability that there are more girls than boys on the committee. **2**

(b) Let $f(x) = 2\sin x + 6\cos x + 13$, for $0 \le x \le 2\pi$.

(i) Write $2\sin x + 6\cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and α is acute. **2**

(ii) Hence, show that $\dfrac{1}{13+\sqrt{40}} \le \dfrac{1}{f(x)} \le \dfrac{1}{13-\sqrt{40}}$. **2**

(c) Find the coefficient of x^5 in the expansion of $(1+3x)^2(1+x)^7$. **3**

(d) Given that $\overrightarrow{OA} = \underset{\sim}{u} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ and $\overrightarrow{OB} = \underset{\sim}{v} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$ and $\underset{\sim}{w} = \overrightarrow{AB}$, find $proj_{\underset{\sim}{v}}\underset{\sim}{w}$ **2**

(e) Use the substitution $u = \sin^2 x$ to evaluate $\displaystyle\int_{\pi/4}^{\pi/3} \frac{\sin 2x}{1+\sin^2 x}\,dx$, leaving your final answer **3**

in the form $\ln\dfrac{a}{b}$.

End of Question 11

Question 12 (15 marks) Use a NEW writing booklet.

(a) To join the April Club, you must be born in the month of April. **1**
How many members will the club need to have to guarantee that five members have the same birthday?

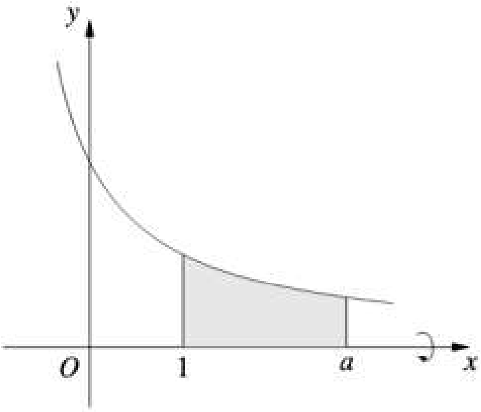
(b) Use mathematical induction to prove that $13 \times 6^n + 2$ is divisible by 5 for all integers $n \geq 1$. **3**

(c) It is known that an Olympic archer has an 80% chance of hitting the bullseye. **3**

What is the minimum number of shots that must be attempted for the probability of hitting a bullseye exactly 9 times to be higher than the probability of hitting the bullseye exactly 8 times?

(d) Using $t = \tan\frac{x}{2}$, solve the equation $5\sin x - 12\cos x = 5$ for $-\pi \leq x \leq \pi$, giving answers correct to two decimal places where necessary. **3**

(e) In the diagram below, the region enclosed by the curve $y = \sqrt{\dfrac{3}{2x+1}}$, the x-axis and the lines $x = 1$ and $x = a$ is shaded. When the shaded region is rotated about the x-axis, the solid generated has a volume of $\pi \ln\frac{64}{27}$ units^3. **3**



Find the exact value of a.

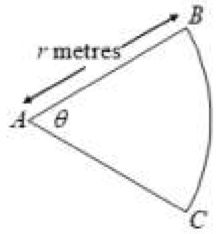
(f) When a polynomial $P(x)$ is divided $(x-2)(x+1)$ it gives a remainder of $x - b$. **2**
When this polynomial is divided by $x + 1$, it gives a remainder of 3.

What is the value of b?

End of Question 12

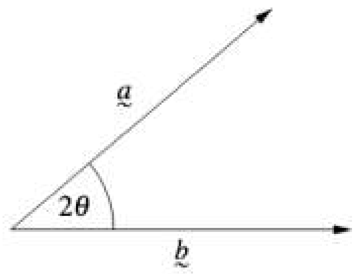
Question 13 (15 marks) Use a NEW writing booklet.

(a) The area of a sector ABC with a radius of r metres and an angle θ radians at the centre is increasing at a rate of 5 $\mathrm{m}^2 s^{-1}$ while θ remains constant. **2**



At what rate is the radius r increasing when the arc length of the sector is 20 m?

(b) In the diagram below, $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are unit vectors that intersect at an angle of 2θ. **3**



Express $\left|\underset{\sim}{a}+\underset{\sim}{b}\right|^2$ in terms of $\cos\theta$.

(c) If $f(x) = f^{-1}(x)$, then the function $f(x)$ is said to be *self-inverse* for all x in the domain. **3**

Let the function $f(x)$ be defined as $f(x)=\dfrac{3x-5}{x+k}$.

Find the value of k so that $f(x)$ is *self-inverse*.

(d) A curve is defined parametrically by the equations given below: **4**

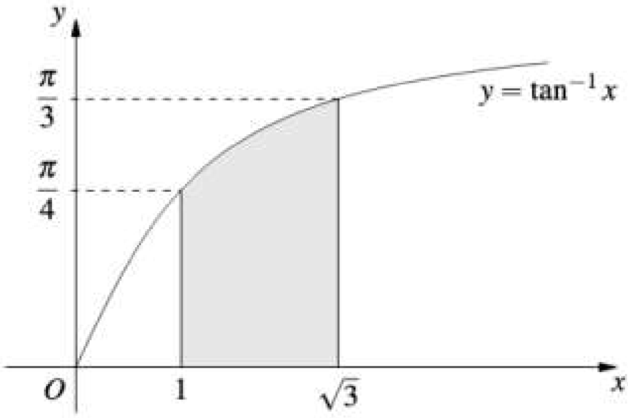
$$\begin{aligned} x &= \cos\left(\sin^{-1} t\right) \\ y &= \sin\left(\sin^{-1} t\right) \end{aligned}, \text{ where } 0 \le t \le 1.$$

Find the equation of the tangent to the curve at the point where $t = 0$.

Question 13 continues on the next page

Question 13 (continued)

(e) In the diagram below, the region enclosed by the graph of $y = \tan^{-1} x$, the x-axis and the lines $x = 1$ and $x = \sqrt{3}$ is shaded. **3**

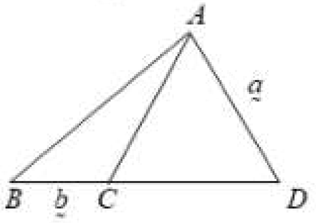


Find the area of the shaded region, leaving your answer in exact form.

End of Question 13

Question 14 (15 marks) Use a NEW writing booklet.

(a) The diagram shows a triangle, where $\overrightarrow{AD} = \underset{\sim}{a}$ and $\overrightarrow{BC} = \underset{\sim}{b}$. **3**



Not to Scale

Given that $5\overrightarrow{BD} = 6\overrightarrow{CD}$, prove that $\overrightarrow{AB} = \underset{\sim}{a} - 6\underset{\sim}{b}$.

(b) The proportion of incorrect answers in a standardised multiple-choice examination is $p = 0.12$. For quality control, a random sample of n examinations is selected, and the sample proportion of incorrect answers, $\hat{p}$ is calculated. **3**

It is assumed that $\hat{p}$ is approximately normally distributed with $\mu = p$ and $\sigma^2 = \frac{p(1-p)}{n}$.
If the proportion of incorrect answers exceeds 15%, the examination process is considered invalid.

Use the standard normal distribution table (provided at the end of Question 14) to determine the least sample size n required so that the probability of invalidating the examination process is less than 5%.

(c) Use the substitution $x = \sin\theta$, for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, to solve the differential equation **4**

$\frac{dy}{dx} = y\sqrt{1-x^2}$ where $-1 < x < 1$, given that $y = 2$ when $x = 0$.

(d) A particle is projected from the origin with initial speed v_0 m/s at an angle of θ to the horizontal. The velocity vector of the particle, $\underset{\sim}{v}(t)$, where t is the time after projection and g is the acceleration due to gravity, is given by:

$$\underset{\sim}{v}(t) = \begin{pmatrix} v_0\cos\theta \\ v_0\sin\theta - gt \end{pmatrix}$$ (Do NOT prove this.)

(i) The speed of the projectile is half of the initial speed when it is at a vertical height of H metres above the horizontal. **3**
Show that $H = \frac{3v_0^2}{8g}$.

(ii) If the speed of the projectile is half of its initial speed only once during its flight, find the value of θ. **2**

End of paper

Standard Normal Distribution Table

second decimal place

z	+.00	+.01	+.02	+.03	+.04	+.05	+.06	+.07	+.08	+.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916

Suggested Solution (s)	Comments

SOLUTIONS TO 2025 Y12 MATHS EXT 1 TRIAL

SECTION I

1. $\int \frac{1}{1+4x^2}\,dx$

$= \frac{1}{2}\int \frac{2}{1+(2x)^2}\,dx$

$= \frac{1}{2}\tan^{-1}2x + C$ $\boxed{B}$

2. $\cos 2x = 2\cos^2 x - 1$

$= 2\left(\frac{2}{3}\right)^2 - 1$

$= -\frac{1}{9}$ $\boxed{A}$

3. $np = 6 \quad \therefore \ p = \frac{6}{n}$

$np(1-p) = 1.5$

$n \times \frac{6}{n}\left(1 - \frac{6}{n}\right) = \frac{3}{2}$

$1 - \frac{6}{n} = \frac{1}{4}$

$\frac{6}{n} = \frac{3}{4}$

$\frac{n}{6} = \frac{4}{3}$

$n = 8$ $\boxed{C}$

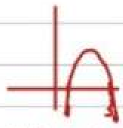
4. $g(x) = f(-|x|)$ $\boxed{C}$

5. $\frac{2}{3-x} \le 1$

$2(3-x) \le (3-x)^2$

$(3-x)(2-(3-x)) \le 0$

$(3-x)(x-1) \le 0$



$x \le 1,\ x > 3$ $\boxed{B}$

6. $\frac{\cos 2\theta \cos 4\theta + \sin 2\theta \sin 4\theta}{\sin 2\theta \cos 2\theta}$

$= \frac{\cos(4\theta - 2\theta)}{\sin 2\theta \cos 2\theta}$

$= \frac{\cancel{\cos 2\theta}}{\sin 2\theta \cancel{\cos 2\theta}}$

$= \operatorname{cosec} 2\theta$ $\boxed{D}$

7. $y = 3x - 4$

$f(1) = -1,\ f'(1) = 3$

$\therefore$ for $y = f^{-1}(x)$, $(-1, 1)$ satisfies

$\therefore \ m = \frac{1}{3}$

$\therefore m_{\perp} = -3$

$y - 1 = -3(x+1)$ $\boxed{C}$

$y = -3x - 2$

8. number of arrangements $= \frac{9!}{2! \times 2!}$

$= 90720$

If Fs are together : $\frac{8!}{2!}$

$= 20160$

$\therefore$ number of ways $= 90720 - 20160$

$= 70560$ $\boxed{B}$

9. $\frac{dy}{dx} = \frac{e^x}{e^x+c}$

$= \frac{e^x+c-c}{e^x+c}$

$= 1 - \frac{c}{e^x+c}$

as $x \to \infty$, $e^x + c \to \infty$

$\therefore \frac{dy}{dx} \to 1$ $\boxed{D}$

10. $T = 160 - Ae^{-kt}$

$\frac{dT}{dt} = kAe^{-kt}$

$= k(160-T)$

for tangent, $m = \frac{160-100}{25-10}$

$= 4$

$\therefore k(160-100) = 4$

$k = \frac{4}{60}$

$= \frac{1}{15}$ $\boxed{C}$

QUESTION 11

(a) (i) no of committees $= {}^8C_5$ ✓ ①
$= 56$

(ii) P(more girls than boys) ②

$= \frac{{}^4C_4 \times {}^4C_1 + {}^4C_3 \times {}^4C_2}{56}$ ✓

$= \frac{4 + 4\times 6}{56}$ ✓ 1 if answer only

$= \frac{1}{2}$

② (b) (i) $f(x) = 2\sin x + 6\cos x + 13$

$R\sin(x+\alpha) = R\sin x\cos\alpha + R\cos x\sin\alpha$

$\therefore R\cos\alpha = 2$

$R\sin\alpha = 6$

$\tan\alpha = 3$ ✓

$R = \sqrt{2^2+6^2}$

$= \sqrt{40}$

$= 2\sqrt{10}$ ✓

$\therefore 2\sin x + 6\cos x = 2\sqrt{10}\sin(x + \tan^{-1}3)$

② (ii) minimum value of $y = \frac{1}{f(x)}$ occurs when

$\sin(x+\tan^{-1}3) = 1$

$\therefore$ minimum $= \frac{1}{2\sqrt{10}+13}$ ✓

maximum value occurs when $\sin(x+\tan^{-1}3) = -1$

$\therefore$ maximum $= \frac{1}{13-2\sqrt{10}}$ ✓ since $2\sqrt{10} = \sqrt{40}$

$\therefore \frac{1}{13+\sqrt{40}} \leq \frac{1}{f(x)} \leq \frac{1}{13-\sqrt{40}}$ as req'd (needs conclusion)

③ (c) $(1+3x)^2(1+x)^7$

$= (1+6x+9x^2)(1+x)^7$

$= (1+6x+9x^2)(1+7x+{}^7C_2x^2+\dots+x^7)$ ✓

Term in $x^5 = {}^7C_5x^5 + 6x.{}^7C_4x^4 + 9x^2.{}^7C_3x^3$ ✓

$\therefore$ coefficient of $x^5 = {}^7C_5 + 6\times{}^7C_4 + {}^7C_3\times 9$

$= 546$ ✓

② (d) $\vec{OA} = \underset{\sim}{u} = \begin{pmatrix}-3\\2\end{pmatrix}$ $\vec{OB} = \underset{\sim}{v} = \begin{pmatrix}5\\-3\end{pmatrix}$

$\underset{\sim}{w} = \vec{AB}$

$= \begin{pmatrix}5+3\\-3-2\end{pmatrix}$

$= \begin{pmatrix}8\\-5\end{pmatrix}$ ✓

$\text{proj}_{\underset{\sim}{v}}\underset{\sim}{w} = \dfrac{\underset{\sim}{w}\cdot\underset{\sim}{v}}{|\underset{\sim}{v}|^2}\underset{\sim}{v}$

$= \dfrac{5(8)-5(-3)}{5^2+(-3)^2}\begin{pmatrix}5\\-3\end{pmatrix}$

$= \dfrac{56}{34}\begin{pmatrix}5\\-3\end{pmatrix}$ ✓

Can be in component form

$\text{proj}_{\underset{\sim}{v}}\underset{\sim}{w} = \dfrac{56}{34}(5\underset{\sim}{i} - 3\underset{\sim}{j})$

(e) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin 2x}{1+\sin^2 x}\,dx$

let $u = \sin^2 x$

$\dfrac{du}{dx} = 2\sin x\cos x$

$= \sin 2x$

$= \displaystyle\int_{\frac{1}{2}}^{\frac{3}{4}} \frac{du}{1+u}$ ✓

when $x = \frac{\pi}{3}$, $u = (\sin\frac{\pi}{3})^2$

$= \frac{3}{4}$

$x = \frac{\pi}{4}$, $u = (\sin\frac{\pi}{4})^2$

$= \frac{1}{2}$

$= \Big[\ln|1+u|\Big]_{\frac{1}{2}}^{\frac{3}{4}}$ ✓

$= \ln|1+\frac{3}{4}| - \ln|1+\frac{1}{2}|$

$= \ln\frac{7}{4} - \ln\frac{3}{2}$

$= \ln(\frac{7}{4}\times\frac{2}{3})$

$= \ln\frac{7}{6}$ ✓

NB no mixing of variables

Suggested Solution (s)	Comments

QUESTION 12

① (a) 30 days in April

$\therefore$ by Pigeonhole Principle need $4 \times 30 + 1$

$= 121$ ✓

③ (b) RTP $13 \times 6^n + 2$ is divisible by 5 for $n \geq 1$

Step 1: Prove true for $n = 1$

If $n = 1$, $13 \times 6^n + 2$

$= 13 \times 6 + 2$

$= 80$

$= 5 \times 16$ ✓

$\therefore$ true for $n = 1$

must have divisible by 5 or 5×16

Step 2: Assuming true for $n = k$

ie. $13 + 6^k + 2 = 5m$, $m \in \mathbb{Z}^+$

prove true for $n = k+1$

ie. $13 + 6^{k+1} = 5N$, $N \in \mathbb{Z}^+$

Now, $13 \times 6^{k+1} + 2$

$= 13 \times 6^k \cdot 6 + 2$ ✓

$= 6(5m - 2) + 2$ by assumption

$= 30m - 12 + 2$

$= 30m - 10$

$= 5(6m - 2)$ ✓

$\therefore$ true for $n = k+1$

Step 3: Since true for $n = 1$ and true for $n = k+1$, assuming true for $n = k$ then by the Principle of Mathematical Induction, true for all $n \geq 1$, $n \in \mathbb{Z}^+$

3 (c) $P(X=8) = {}^nC_8\, 0.8^8\, 0.2^{n-8}$

$P(X=9) = {}^nC_9 (0.8)^9\, 0.2^{n-9}$ ✓

$\therefore \dfrac{n!}{9!(n-9)!} \times (0.8)^9 (0.2)^{n-9} > \dfrac{n!}{8!(n-8)!} \times (0.8)^8 (0.2)^{n-8}$

$\dfrac{n!}{9!(n-9)!} \times 0.8 > \dfrac{n!}{8!(n-8)!} \times 0.2$

$\dfrac{0.8}{9} > \dfrac{0.2}{n-8}$

$0.8(n-8) > 1.8$

$n-8 > \dfrac{9}{4}$ ✓

$n > 10.25$

i.e. minimum number is 11 ✓ CFPA

3 (d) $5\sin x - 12\cos x = 5$

let $t = \tan \dfrac{x}{2}$

$\therefore 5 \times \dfrac{2t}{1+t^2} - 12 \times \dfrac{1-t^2}{1+t^2} = 5$ ✓

$10t - 12(1-t^2) = 5(1+t^2)$

$10t - 12 + 12t^2 = 5 + 5t^2$

$7t^2 + 10t - 17 = 0$

$t = \dfrac{-10 \pm \sqrt{10^2 - 4(7)(-17)}}{2(7)}$

$= \dfrac{-10 \pm \sqrt{576}}{14}$

$= 1$ or $-\dfrac{17}{7}$ ✓

$\tan \dfrac{x}{2} = 1$ or $\tan \dfrac{x}{2} = -\dfrac{17}{7}$

$\dfrac{x}{2} = \dfrac{\pi}{4}, \dfrac{5\pi}{4}$ $\quad \dfrac{x}{2} = -\tan^{-1}\left(\dfrac{17}{7}\right),$ ✓ (both solutions correct or CFPA)

$x = \dfrac{\pi}{2}$ $\quad x = -2\tan^{-1}\left(\dfrac{17}{7}\right),$

$= -2.3603\ldots$

$\therefore x = \dfrac{\pi}{2}, -2.36$ (to 2dp)

③ e) $V = \pi \int y^2\,dx \qquad y = \sqrt{\frac{3}{2x+1}}$

$\therefore \pi \ln \frac{64}{27} = \pi \int_1^a \frac{3}{2x+1}\,dx$ ✓

$= \frac{3\pi}{2} \int_1^a \frac{2}{2x+1}\,dx$

$\therefore \ln \frac{64}{27} = \frac{3}{2}\left[\ln|2x+1|\right]_1^a$ ✓

$\frac{2}{3} \ln \frac{64}{27} = \ln(2a+1) - \ln(2+1)$

$\ln\left(\frac{64}{27}\right)^{\frac{2}{3}} = \ln \frac{2a+1}{3}$

$\left(\frac{4}{3}\right)^2 = \frac{2a+1}{3}$

$a = \frac{3\left(\frac{16}{9}\right) - 1}{2}$

$a = \frac{13}{6}$ ✓

① f) $P(x) = (x-2)(x+1)Q(x) + x - b$ ✓

and $P(-1) = 3$

$\therefore P(-1) = (-1-2)(-1+1)Q(x) + x - b$

$\therefore 3 = -1 - b$ ✓

$b = -4$

QUESTION 13

(2) (a) $\frac{dA}{dt} = 5$

$A = \frac{\theta r^2}{2}$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$

$5 = \theta r \times \frac{dr}{dt}$ ✓

$l = \theta r$

$20 = \theta r$

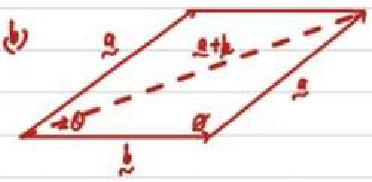
when $l = 20$,

$5 = 20 \times \frac{dr}{dt}$ ✓

$\therefore \frac{dr}{dt} = 0.25$

$\therefore$ radius increasing at rate of 0.25 m/s ✓

(3) (b)



$|\underset{\sim}{a}+\underset{\sim}{b}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 - 2|\underset{\sim}{a}||\underset{\sim}{b}|\cos\phi$ ✓

$= 1^2 + 1^2 - 2(1)(1)\cos(\pi - 2\theta)$

$= 2 + 2\cos 2\theta$ ✓

$= 2 + 2(2\cos^2\theta - 1)$

$= 4\cos^2\theta$ ✓

OR $|\underset{\sim}{a}+\underset{\sim}{b}|^2 = (\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}+\underset{\sim}{b})$

$= \underset{\sim}{a}\cdot\underset{\sim}{a} + 2\underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{b}$

$= |\underset{\sim}{a}|^2 + 2\underset{\sim}{a}\cdot\underset{\sim}{b} + |\underset{\sim}{b}|^2$

$= |\underset{\sim}{a}|^2 + 2|\underset{\sim}{a}||\underset{\sim}{b}|\cos 2\theta + |\underset{\sim}{b}|^2$

$= 1 + 2\cos 2\theta + 1$

$= 2(1+\cos 2\theta)$ since $\underset{\sim}{a}, \underset{\sim}{b}$ are unit vector

$= 2 \cdot 2\cos^2\theta$

$= 4\cos^2\theta$

3 (c) $f(x) = f^{-1}(x)$ self-inverse

$f(x) = \frac{3x-5}{x+k}$

Let $y = \frac{3x-5}{x+k}$

for inverse: $x = \frac{3y-5}{y+k}$

$x(y+k) = 3y-5$

$xy + kx = 3y-5$

$y(x-3) = -kx-5$

$\therefore f^{-1}(x) = \frac{-kx-5}{x-3}$

$= \frac{5+kx}{3-x}$ ✓

for $f(x)$ to self-inverse

$\frac{3x-5}{x+k} = \frac{kx+5}{3-x}$ ✓

$(3x-5)(3-x) = (kx+5)(x+k)$

$-3x^2+14x-15 = kx^2 + x(k^2+5)+5k$

Comparing powers of x

$\Rightarrow k = -3$ ✓

(check $k^2+5 = (-3)^2+5 = 14$

$5k = 5(-3) = -15$)

4 (d) $x = \cos(\sin^{-1} t)$

$y = \sin(\sin^{-1} t) \qquad 0 \le t \le 1$

$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= \cos(\sin^{-1} t) \cdot \frac{1}{\sqrt{1-t^2}} \times \frac{1}{-\sin(\sin^{-1} t) \cdot \frac{1}{\sqrt{1-t^2}}}$ ✓

$= \frac{\cos(\sin^{-1} t)}{\sqrt{1-t^2}} \times \frac{\sqrt{1-t^2}}{-\sin(\sin^{-1} t)}$

$= \frac{\cos(\sin^{-1} t)}{-\sin(\sin^{-1} t)}$ ✓

when $t = 0$

$m = \frac{\cos(\sin^{-1} 0)}{-\sin(\sin^{-1} 0)}$

$= \frac{1}{0}$ ✓

i.e. undefined

when $t=0$, $x = \cos(\sin^{-1} 0)$

$= \cos 0$

$= 1$ ✓

$\therefore$ eqn of tangent: $x = 1$

or $\cos^2(\sin^{-1} t) + \sin^2(\sin^{-1} t) = 1$

$x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$y = \sqrt{1-x^2}$ since $0 \le t \le 1$

$\frac{dy}{dx} = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot -2x$

$= \frac{-x}{\sqrt{1-x^2}}$

when $t = 0$, $x = \cos(\sin^{-1} 0)$

$= \cos 0$

$= 1$

$\frac{dy}{dx} = \frac{-1}{\sqrt{1-1}}$

which is undefined

$\therefore$ tangent is vertical

$x = 1$

or

$x = \cos(\sin^{-1} t) \qquad y = \sin(\sin^{-1} t)$

$\cos^{-1} x = \sin^{-1} t \qquad = \sin(\cos^{-1} x)$

$\frac{dy}{dx} = \cos(\cos^{-1} x) \cdot \frac{-1}{\sqrt{1-x^2}}$

$= -\frac{\cos(\cos^{-1} x)}{\sqrt{1-x^2}}$

$= \frac{-x}{\sqrt{1-x^2}}$ etc.

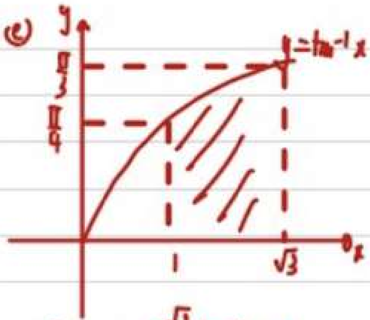
when $t = 0$, $x = 1$, $\frac{dy}{dx} = \frac{-1}{\sqrt{1-1}}$

which is undefined

$\therefore$ tangent vertical

$\therefore x = 1$

3 (c)



$$\text{Area} = \int_1^{\sqrt{3}} \tan^{-1}x\,dx$$

$$= \frac{\pi\sqrt{3}}{3} - \frac{\pi}{4}\times 1 - \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x\,dx \quad ✓$$

$$= \frac{\pi\sqrt{3}}{3} - \frac{\pi}{4} - \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin x}{\cos x}\,dx$$

$$= \frac{\pi\sqrt{3}}{3} - \frac{\pi}{4} + \left[\ln|\cos x|\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} \quad ✓$$

$$= \frac{\pi\sqrt{3}}{3} - \frac{\pi}{4} + \ln\left|\cos\frac{\pi}{3}\right| - \ln\left|\cos\frac{\pi}{4}\right|$$

$$= \frac{4\pi\sqrt{3}-3\pi}{12} + \ln\frac{\frac{1}{2}}{\frac{1}{\sqrt{2}}}$$

$$= \frac{4\pi\sqrt{3}-3\pi}{12} + \ln\frac{\sqrt{2}}{2} \quad ✓$$

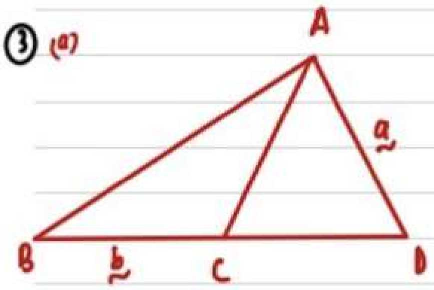
$$= \frac{4\pi\sqrt{3}-3\pi}{12} + \ln 2^{-\frac{1}{2}}$$

$$= \frac{4\pi\sqrt{3}-3\pi}{12} - \frac{1}{2}\ln 2$$

$$= \frac{4\pi\sqrt{3}-3\pi-6\ln 2}{12}$$

QUESTION 14

③ (a)



$5\overrightarrow{BD} = 6\overrightarrow{CD}$

$\therefore \overrightarrow{CD} = \frac{5}{6}\overrightarrow{BD}$

$= \frac{5}{6}(\underset{\sim}{b} + \overrightarrow{CD})$ ✓ or similar

$6\overrightarrow{CD} = 5\underset{\sim}{b} + 5\overrightarrow{CD}$

$\therefore \overrightarrow{CD} = 5\underset{\sim}{b}$ ✓

$\overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DB}$

$= \underset{\sim}{a} - (5\underset{\sim}{b} + \underset{\sim}{b})$ ✓

$= \underset{\sim}{a} - 6\underset{\sim}{b}$ as req'd

3 (b)

$p = 0.12$

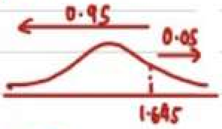
exam process invalid if $\hat{p} > 0.15$

want $P(\hat{p} > 0.15) < 0.05$

$$P\left(Z > \frac{0.15 - 0.12}{\sqrt{\frac{0.12 \times 0.88}{n}}}\right) < 0.05 \checkmark$$

from standard Normal Distribution Table,
prob of 0.95 corresponds to:

$$P\left(Z \leq \frac{1.64 + 1.65}{2}\right)$$
$$= P(Z \leq 1.645) \checkmark$$



$$\therefore 1.645 = \frac{(0.15 - 0.12) \times \sqrt{n}}{\sqrt{0.1056}}$$
$$n = \left(\frac{1.645 \times \sqrt{0.1056}}{0.03}\right)^2$$
$$= 317.50\ldots \checkmark$$

$\therefore$ sample of 318 req'd (needs final sample size)

range 317-320 accepted
no penalty for
interpolation issues

④ (c) $\frac{dy}{dx} = y\sqrt{1-x^2}$

$\int \frac{dy}{y} = \int \sqrt{1-x^2}\,dx$ let $x = \sin\theta$ $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\frac{dx}{d\theta} = \cos\theta$

$= \int \sqrt{1-\sin^2\theta}.\cos\theta\,d\theta$ ✓

$= \int \cos^2\theta\,d\theta$

$= \frac{1}{2}\int \cos 2\theta + 1\,d\theta$ ✓

$\ln|y| = \frac{1}{2}\left(\frac{\sin 2\theta}{2} + \theta\right) + C$

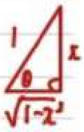
when $x = 0$, $y = 2$

$\therefore 0 = \sin\theta$

$\therefore \theta = 0$

$\ln 2 = \frac{\sin 0}{4} + \frac{0}{2} + C$ ✓

$\therefore C = \ln 2$



$\therefore \ln|y| = \frac{\sin 2\theta}{4} + \frac{\theta}{2} + \ln 2$

$= \frac{2\sin\theta\cos\theta}{4} + \frac{\theta}{2} + \ln 2$

$= \frac{x\sqrt{1-x^2} + \sin^{-1}x}{2} + \ln 2$ ✓

$\therefore |y| = e^{\frac{x\sqrt{1-x^2}+\sin^{-1}x}{2} + \ln 2}$ given it passes thru $(0,2)$

$\therefore y = e^{\frac{x\sqrt{1-x^2}+\sin^{-1}x}{2}}.e^{\ln 2}$

$= 2e^{\frac{x\sqrt{1-x^2}+\sin^{-1}x}{2}}$

d) $\underset{\sim}{v}(t) = \begin{pmatrix} v_0 \cos\theta \\ v_0 \sin\theta - gt \end{pmatrix}$

3 i) $v = \frac{1}{2}v_0$ when $x=0, y=H$

$\therefore \frac{1}{2}v_0 = \sqrt{(v_0\cos\theta)^2 + (v_0\sin\theta - gt)^2}$

$\frac{1}{4}v_0^2 = v_0^2\cos^2\theta + v_0^2\sin^2\theta - 2v_0 gt\sin\theta + g^2t^2$

$= v_0^2 - 2v_0 gt\sin\theta + g^2t^2$

$\therefore g^2t^2 - (2v_0 g\sin\theta)t + \frac{3}{4}v_0^2 = 0$ ✓

$\therefore 4g^2t^2 - (8v_0 g\sin\theta)t + 3v_0^2 = 0$ — ①

for vertical component of trajectory,

$r_j = v_0 t\sin\theta - \frac{gt^2}{2} + c$

when $t=0$, $r_j = 0$ $\therefore c = 0$ ✓

$\therefore r_j = v_0 t\sin\theta - \frac{gt^2}{2}$ — ②

Rearranging ① $\Rightarrow t^2 = \frac{(8v_0 g\sin\theta)t - 3v_0^2}{4g^2}$

sub into ② $y = v_0 t\sin\theta - \frac{g}{2}\left(\frac{(8v_0 g\sin\theta)t - 3v_0^2}{4g^2}\right)$

$= v_0 t\sin\theta - \frac{1}{8g}((8v_0 g\sin\theta)t - 3v_0^2)$

$= v_0 t\sin\theta - v_0 t\sin\theta + \frac{3v_0^2}{8g}$ ✓

$= \frac{3v_0^2}{8g}$ as req'd

3 ii) Now, from ①

$t = \frac{8v_0 g \pm \sqrt{(-8v_0 g\sin\theta)^2 - 4.4g^2.3v_0^2}}{2.4g^2}$ ✓

for one sol'n, $\Delta = 0$

$\therefore 64v_0^2g^2\sin^2\theta - 48g^2v_0^2 = 0$

$4\sin^2\theta - 3 = 0$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

but $0 \leq \theta \leq 90°$ ✓

$\therefore \theta = 60°$

4 (c) $\frac{dy}{dx} = y\sqrt{1-x^2}$

$\int \frac{dy}{y} = \int \sqrt{1-x^2}\,dx$ Let $x = \sin\theta$ $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$\frac{dx}{d\theta} = \cos\theta$

$= \int \sqrt{1-\sin^2\theta}\,.\cos\theta\,d\theta$ ✓

$= \int \cos^2\theta\,d\theta$

$= \frac{1}{2}\int \cos 2\theta + 1\,d\theta$ ✓

$\ln|y| = \frac{1}{2}\left(\frac{\sin 2\theta}{2} + \theta\right) + c$

when $x = 0$, $y = 2$

$\therefore 0 = \sin\theta$

$\therefore \theta = 0$

$\ln 2 = \frac{\sin 0}{4} + \frac{0}{2} + c$ ✓

$\therefore c = \ln 2$

$\therefore \ln|y| = \frac{\sin 2\theta}{4} + \frac{\theta}{2} + \ln 2$

$= \frac{2\sin\theta\cos\theta}{4} + \frac{\theta}{2} + \ln 2$

$= \frac{x\sqrt{1-x^2} + \sin^{-1}x}{2} + \ln 2$ ✓

$\therefore |y| = e^{\frac{x\sqrt{1-x^2}+\sin^{-1}x}{2} + \ln 2}$ given it passes thru $(0,2)$

$\therefore y = e^{\frac{x\sqrt{1-x^2}+\sin^{-1}x}{2}} . e^{\ln 2}$

$= 2e^{\frac{x\sqrt{1-x^2}+\sin^{-1}x}{2}}$