Merewether High School

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2025 **TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Extension 1

General Instructions

- 10 minutes reading time
- Working time – 2 hours
- Write using black pen.
- Calculators approved by NESA may be used.
- A reference sheet is provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations.
- Submit ALL work at the end of the examination.
- Diagrams are NOT drawn to scale.

Total Marks 70

Section I – 10 marks (pages 2 –7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section.

Section II – 60 marks (pages 8 – 16)

- Attempt Questions 11 – 14
- Allow about 105 minutes for this section.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple choice answer sheet Questions 1 – 10.

---

1 A Welfare Department at a school consists of 5 Male and 5 Female teachers. How many committees of 3 teachers can be formed which contain at least one female and one male?

A. 80

B. 100

C. 120

D. 200

2 Which group of three numbers could be the roots of the polynomial equation $x^3 + ax^2 - 41x + 42 = 0$?

A. 2, 3, 7

B. –1, –3, –14

C. 1,–6, 7

D. –1,– 2, 21

3 Which of the following is an expression for $\cos^4 2x - \sin^4 2x$?

A. $\cos 2x$

B. $\cos^2 2x$

C. $\cos 4x$

D. $\cos^2 4x$

4 Which of the following is the range of the function $y = 2\sin^{-1} x + \dfrac{\pi}{2}$

A. $-\pi \le y \le \pi$

B. $\dfrac{-\pi}{2} \le y \le \dfrac{\pi}{2}$

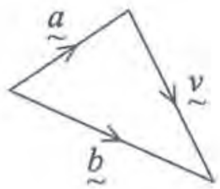
C. $\dfrac{-\pi}{2} \le y \le \dfrac{3\pi}{2}$

D. $-\pi \le y \le \dfrac{3\pi}{2}$

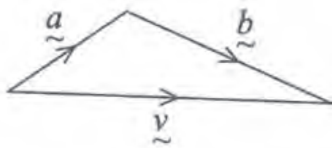
5 The vectors α are shown.

Which diagram below shows the vector $\underset{\sim}{v} = \underset{\sim}{b} - \underset{\sim}{a}$

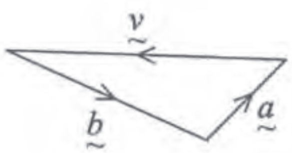
A.



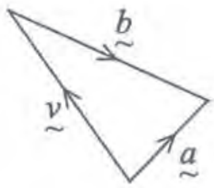
B.



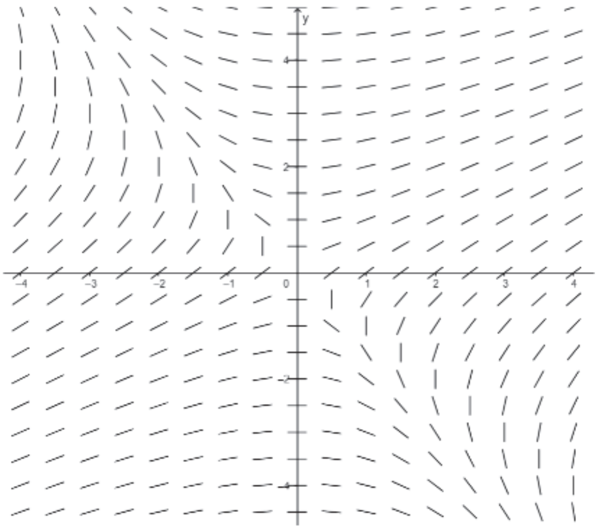
C.



D.



6 The slope field for the first order differential equation is shown below.



Which of the following could best represent the differential equation for the slope field?

A. $\dfrac{dy}{dx}=\dfrac{x+y}{x}$

B. $\dfrac{dy}{dx}=\dfrac{x}{x+y}$

C. $\dfrac{dy}{dx}=\dfrac{x}{y-2}$

D. $\dfrac{dy}{dx}=\dfrac{y}{x-2}$

7 Simplify the following $\dfrac{r \times {}^{n}C_{r}}{{}^{n}C_{r-1}}$.

A. $n+r$

B. $n+r+1$

C. $n-r-1$

D. $n-r+1$

8 The point $P(4, a)$ is on the graph of the parametric equations $x = \dfrac{t}{2}$, $y = 2\sqrt{t}$. There is another point F (2,0), such that $\left|\overrightarrow{PF}\right|$ is which of the following?

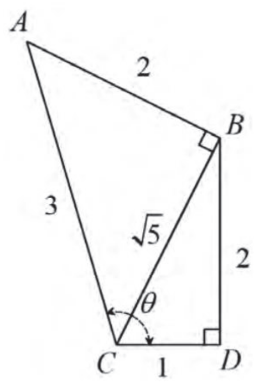
A. 4

B. 5

C. 6

D. 7

9. What is the value of $\sin\theta$ the unknown angle in the diagram below.



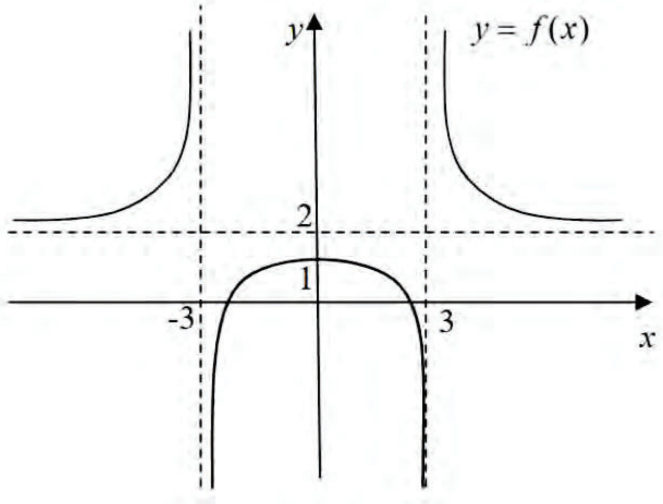
(A) $\dfrac{2}{3\sqrt{5}} + \dfrac{2}{3}$

(B) $\dfrac{2}{\sqrt{5}} + \dfrac{\sqrt{5}}{3}$

(C) $\dfrac{2}{\sqrt{5}} + \dfrac{2}{3}$

(D) $\dfrac{4}{3\sqrt{5}} + \dfrac{1}{3}$

10. Below is the function $y = f(x)$.



Which of the following is a possible equation for $y = f(x)$?

(A) $f(x) = \dfrac{2x^2 + 1}{x^2 - 9}$

(B) $f(x) = \dfrac{x^2 + 2x - 9}{x^2 - 9}$

(C) $f(x) = \dfrac{2x^2 - 9}{x^2 - 9}$

(D) $f(x) = 1 + \dfrac{x^2 + 9}{9 - x^2}$

End of Section 1

Section II
60 marks
Attempt questions 11 – 14
Allow about 105 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet

(a) Differentiate **2**

$$f(x) = \sin^{-1}(e^{2x})$$

(b) How many arrangements can be made from the word AMAZING if it must begin with a consonant and the vowels must be together. **2**

(c) Evaluate $\dfrac{x^2 + x - 6}{x} \geq 0$ **3**

(d) Find the angle between the vectors $\begin{pmatrix} -7 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ to the nearest degree. **2**

(e) Find $\displaystyle\int \frac{3\,dx}{2 + x^2}$ **1**

(f) Solve $\cos 2\theta = \cos^2\theta + \cos\theta + 1$ for $0 \leq \theta \leq 2\pi$ 2

(g) Use the substitution $u = \cos x$ to evaluate 3

$$\int_0^{\pi} \sin^3 x \, dx$$

Leave your answer in exact simplified form.

End of Question 11

Question 12 (15 marks) Use the Question 12 Writing Booklet.

(a) Show that the exact value of $\sec 75°$ is: $\dfrac{2\sqrt{2}}{\sqrt{3}-1}$ 2

(b) Solve for x: $|3x+9| \geq 9$. 2

(c) Find the exact volume of solid of revolution when the curve $y = \sin x$ is rotated about the x axis between $x = \dfrac{-\pi}{2}$ and $x = \dfrac{\pi}{2}$. 3

(d) When a research botanist crossed a tall strain pea with a dwarf strain of pea, he found that $\dfrac{3}{4}$ of the offspring were tall and $\dfrac{1}{4}$ were dwarf.
Suppose six such offspring were selected at random. Find the probability that:

(i) All of the offspring were tall. 1

(ii) Only three of the offspring were tall. 1

(e) When the polynomial $P(x)$ is divided by $(x+1)(x-4)$ the quotient is $Q(x)$ and the remainder is $R(x)$.

(i) Given that $P(4) = -5$, show that $R(4) = -5$. 1

(ii) When $P(x)$ is divided by $(x+1)$, the remainder is 5. Find $R(x)$. 1

(f) Consider $y = \tan^{-1}\dfrac{1}{x}$ and $y = \tan^{-1} x$:

(i) show that $\dfrac{d}{dx}\left(\tan^{-1}x + \tan^{-1}\dfrac{1}{x}\right) = 0$ 2

(ii) hence or otherwise Sketch the curve $y = \left(\tan^{-1}x + \tan^{-1}\dfrac{1}{x}\right)$ 2

End of Question 12

Question 13 (15 marks) Use the Question 13 Writing Booklet

(a) Use Mathematical induction to prove that, for all positive integers, $n \geq 2$ that 3

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$$

(b) In a particular electronic circuit, the current I amperes is given by:

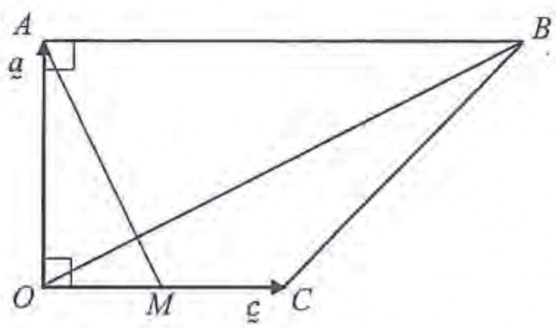
$$I = 4\sin\theta - 3\cos\theta$$

Where θ is an angle related to the voltage and $\theta > 0$.

(i) Express I in the form $I = R\sin(\theta - \alpha)$ where R>0 and $0 \leq \alpha \leq \frac{\pi}{2}$. 2

(ii) Find the value of θ for which the greatest value of I first occurs. Justify your answer. 2

(c) In the diagram, $OABC$ is a trapezium with $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OC} = \underset{\sim}{c}$, $\angle COA = \angle BAO = 90°$ and $OA = OC = \frac{1}{2}AB$. M is the midpoint of OC.



NOT TO SCALE

Use vector methods to show that $OB \perp AM$ **3**

(d) A colony of birds has an initial population size P_o . At time t the rate of change of the population size P is given by $\dfrac{dP}{dt} = Pe^{-0.5t}$.

(i) Show that $P = P_o e^{2-2e^{-0.5t}}$ 3

(ii) Find, in simplest form, the time taken for the population size to reach half its limiting value. 2

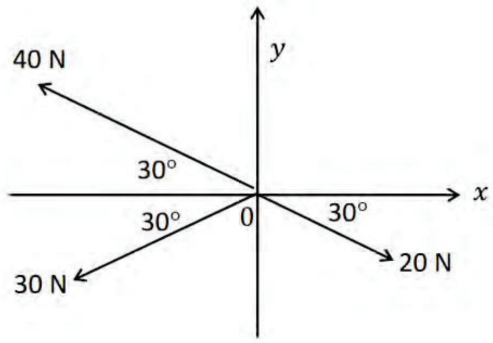
End of Question 13

Question 14 (15 marks) Use the Question 14 Writing Booklet

(a) The diagram shows three forces acting on an object in a PE lesson of tug of war.

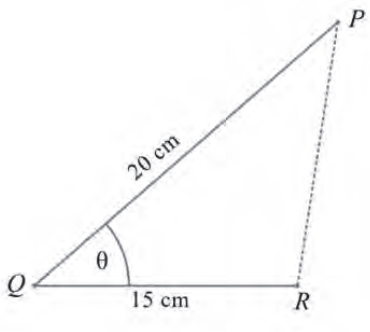
The teacher wants to find the magnitude of the resultant force in Newtons and its direction to the nearest degree so it can be displayed on a PowerPoint presentation.

Find the magnitude and direction of the resultant force. 2



(b) The triangle PQR, shown below, has two sides of fixed length: $PQ = 20$ cm and QR= 15 cm. $\angle PQR = \theta$ is increasing at a rate of $\frac{\pi}{6}$ radians/hour.

Find the rate at which the area of the triangle is changing at the time when $\theta = \frac{\pi}{3}$. 2

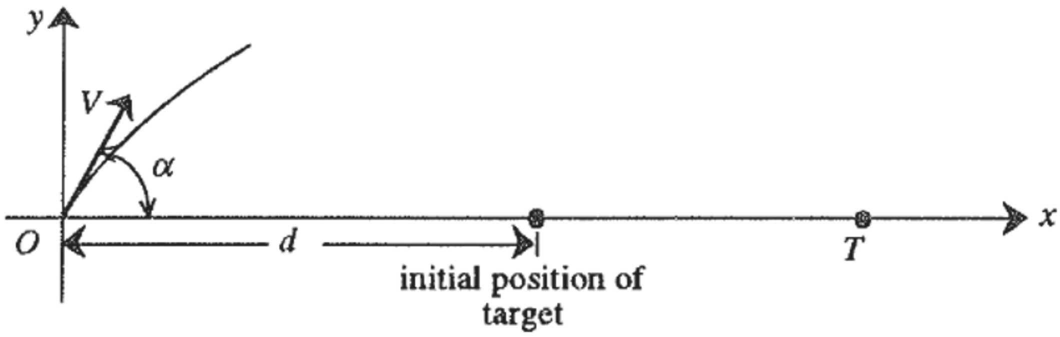


(c) Simplify $n\binom{n-1}{1} + n\binom{n-1}{2} + \ldots + n\binom{n-1}{n-2}$ 3

and hence solve:

$$n\binom{n-1}{1} + n\binom{n-1}{2} + \ldots + n\binom{n-1}{n-2} = 254n$$

(d) A projectile, of initial speed V m/s, is fired at an angle of elevation α from the origin O towards a target T, which is moving away from O along the x-axis.



You may assume that the projectile's trajectory is defined by:

$\underset{\sim}{r} = (Vt\cos\alpha)_{\underset{\sim}{i}} + (\frac{-gt^2}{2} + Vt\sin\alpha)_{\underset{\sim}{j}}$ where g is gravity and t is time in seconds.

(i) Show that the projectile is above the x axis for a total of $t = \dfrac{2V\sin\alpha}{g}$ seconds. 2

(ii) Show that the horizontal range of the projectile is $\dfrac{2V^2\sin\alpha\cos\alpha}{g}$ metres. 2

(iii) At the instant the projectile is fired, the target T is d metres from the origin O and is moving away at a constant speed of u m/s.

Suppose that the projectile hits the target when fired at an angle of elevation α 2

Show that $u = V\cos\alpha - \dfrac{gd}{2V\sin\alpha}$

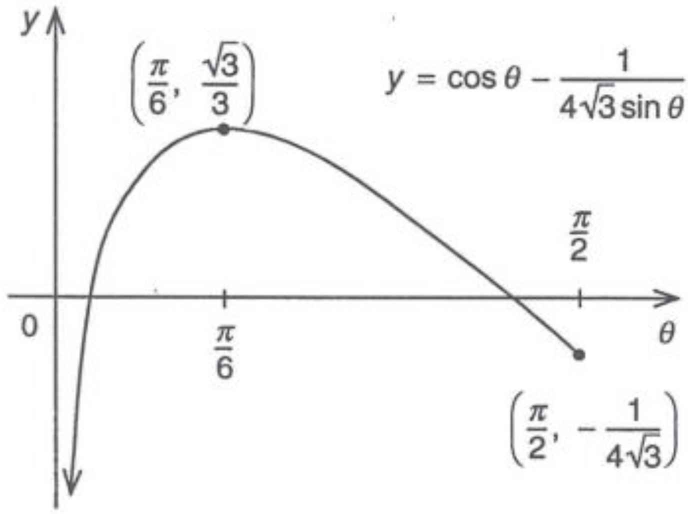
(d) Continues next page

In part (iv) assume that

$$gd = \frac{V^2}{2\sqrt{3}} \quad \text{and application of the given graph.}$$

(iv) By using (iii) and the graph below show that if $u > \dfrac{V}{\sqrt{3}}$ the target cannot be hit by the projectile irrespective of the angle α the projectile is fired. **2**

Since $f''(\theta) < 0$, the curve is concave down and has a maximum turning point at $\theta \doteqdot \frac{\pi}{6}$.



End of Question 14

End of Task

Merewether High School

Student Name/Number:____________________________

Class: ____________________________

YEAR 12 EXTENSION 1 MATHEMATICS
EXAMINATION 2025

ANSWER BOOKLET

Shade the most correct answer.

	A	B	C	D
1	⬭	⬭	⬭	⬭
2	⬭	⬭	⬭	⬭
3	⬭	⬭	⬭	⬭
4	⬭	⬭	⬭	⬭
5	⬭	⬭	⬭	⬭
6	⬭	⬭	⬭	⬭
7	⬭	⬭	⬭	⬭
8	⬭	⬭	⬭	⬭
9	⬭	⬭	⬭	⬭
10	⬭	⬭	⬭	⬭

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Total Marks 70

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- Attempt Questions 11 – 14
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Section I

10 marks
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Use the multiple choice answer sheet Questions 1 – 10.

---

1 A Welfare Department at a school consists of 5 Male and 5 Female teachers. How many committees of 3 teachers can be formed which contain at least one female and one male?

A. 80
B. 100
C. 120
D. 200

$= {}^{10}C_3 - {}^5C_3 \times {}^5C_3$

$= 120 - 20$

$= 100$

2 Which group of three numbers could be the roots of the polynomial equation $x^3 + ax^2 - 41x + 42 = 0$?

A. 2, 3, 7
B. –1, –3, –14
C. 1,–6, 7
D. –1,– 2, 21

$\alpha\beta\gamma = \frac{-d}{a}$

$\alpha\beta\gamma = -42$ $\therefore$ Not A or C

$1 \times -6 \times 7 = -42$

Roots two at a time

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$

$1 \times -6 + -6 \times 7 + 7 \times 1 = -41$

$-6 - 42 + 7 = -41$ ✓✓

3 Which of the following is an expression for $\cos^4 2x - \sin^4 2x$?

A. $\cos 2x$
B. $\cos^2 2x$
C. $\cos 4x$
D. $\cos^2 4x$

$= (\cos^2 2x - \sin^2 2x)(\cos^2 2x + \sin^2 2x)$

$= \cos 4x \times 1$

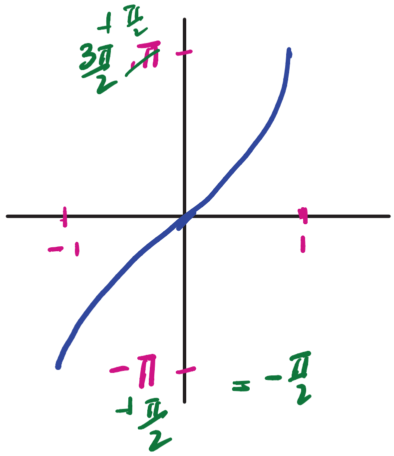
4 Which of the following is the range of the function $y = 2\sin^{-1} x + \frac{\pi}{2}$

A. $-\pi \le y \le \pi$

B. $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

C. $\frac{-\pi}{2} \le y \le \frac{3\pi}{2}$

D. $-\pi \le y \le \frac{3\pi}{2}$

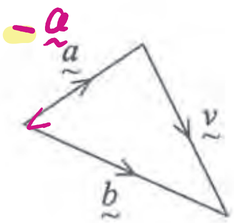


$y = 2\sin^{-1} x$

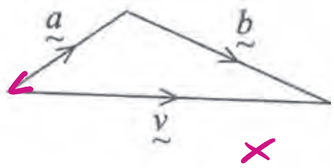
5 The vectors α are shown.

Which diagram below shows the vector $\underset{\sim}{v} = \underset{\sim}{b} - \underset{\sim}{a}$

A.

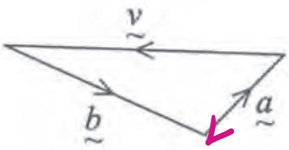


B.



×

C.



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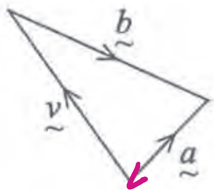
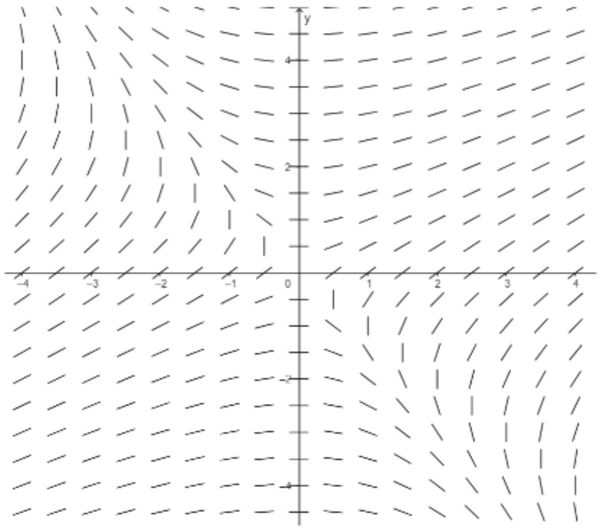
D.



×

6 The slope field for the first order differential equation is shown below.



Which of the following could best represent the differential equation for the slope field?

A. $\dfrac{dy}{dx} = \dfrac{x+y}{x}$

B. $\dfrac{dy}{dx} = \dfrac{x}{x+y}$ when $y=0$ $\frac{x}{x}=1$ ✓✓ 45°

C. $\dfrac{dy}{dx} = \dfrac{x}{y-2}$

D. $\dfrac{dy}{dx} = \dfrac{y}{x-2}$

7 Simplify the following $\frac{r \times {}^{n}C_{r}}{{}^{n}C_{r-1}}$.

Derry Cams

A. $n+r$

B. $n+r+1$

C. $n-r-1$

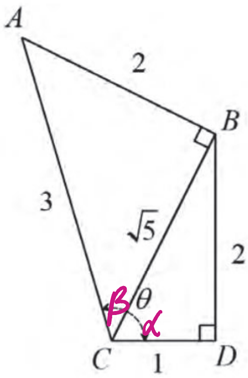
D. $n-r+1$

$$\frac{r\times n!}{(n-r)!\,r!} \div \frac{n!}{(n-(r-1))!\,(r-1)!}$$

$$\frac{r\times \cancel{n!}}{(n-r)!\,r!} \times \frac{(n-r+1)!\,(r-1)!}{\cancel{n!}}$$

$$\frac{\cancel{r}}{\cancel{(n-r)!}\,\cancel{r}\,\cancel{(r-1)!}} \times \frac{(n+r+1)\cancel{(n-r)!}\,\cancel{(r-1)!}}{}$$

8 What is the exact value of $\sin\theta$ in the diagram below?



$\sin\alpha = \frac{2}{\sqrt{5}}$ $\sin\beta = \frac{2}{3}$

$\cos\alpha = \frac{1}{\sqrt{5}}$ $\cos\beta = \frac{\sqrt{5}}{3}$

A. $\frac{2}{3\sqrt{5}}+\frac{2}{3}$

B. $\frac{2}{\sqrt{5}}+\frac{\sqrt{5}}{3}$

C. $\frac{2}{\sqrt{5}}+\frac{2}{3}$

D. $\frac{4}{3\sqrt{5}}+\frac{1}{3}$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{3} + \frac{1}{\sqrt{5}} \times \frac{2}{3}$$

$$= \frac{2}{3} + \frac{2}{3\sqrt{5}}$$

9 The point $P(4, a)$ $4\sqrt{2}$ is on the graph of the parametric equations $x = \frac{t}{2}$, $y = 2\sqrt{t}$. There is another point F (2,0), such that $|\overrightarrow{PF}|$ is which of the following?

$x_1\ y_1$ $\quad x_2\ y_2$

A. 4

B. 5

C. 6

D. 7

$|\overrightarrow{PF}| = \sqrt{(4-2)^2 + (4\sqrt{2}-0)^2}$

$= \sqrt{4+32}$

$= \sqrt{36}$

$= 6$

$2x = t$ — ① $\quad y = 2\sqrt{t}$ — ②

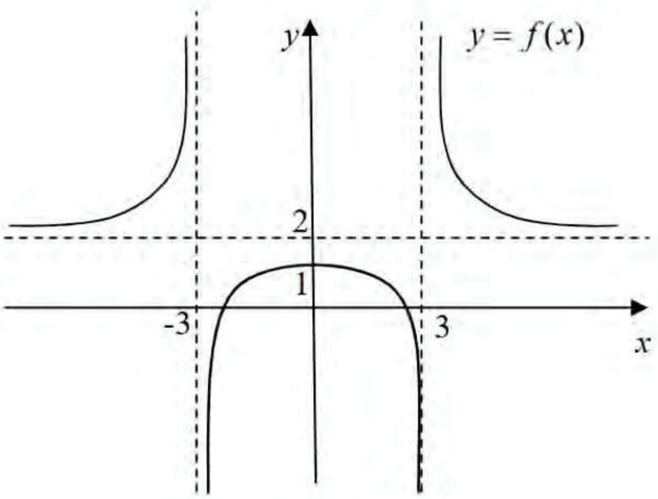
sub ① into ②

$y = 2\sqrt{2x}$

when $x = 4$

$y = 2\sqrt{8}$

10. Below is the function $y = f(x)$.



Which of the following is a possible equation for $y = f(x)$?

A. $f(x) = \dfrac{2x^2+1}{x^2-9}$

B. $f(x) = \dfrac{x^2+2x-9}{x^2-9}$

C. $f(x) = \dfrac{2x^2-9}{x^2-9}$

D. $f(x) = 1 + \dfrac{x^2+9}{9-x^2}$

by subbing $\underline{100}$

$(x-3)(x+3)$

$x \neq 3$ OR -3

End of Section 1

C $\therefore$ $2\times 3^2 - 9$

$y \neq 2$

Section II
60 marks
Attempt questions 11 – 14
Allow about 1 hour 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available. For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet.

(a) Differentiate 2

$$f(x) = \sin^{-1}(e^{2x}) .$$

(b) How many arrangements can be made from the word AMAZING if it must begin with a consonant and the vowels must be together? 2

(c) Evaluate $\dfrac{x^2 + x - 6}{x} \geq 0$. 3

(d) Find the angle between the vectors $\begin{pmatrix} -7 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ to the nearest degree. 2

(e) Find $\displaystyle\int \frac{3\,dx}{2 + x^2}$. 1

(f) Solve $\cos 2\theta = \cos^2\theta + \cos\theta + 1$ for $0 \leq \theta \leq 2\pi$. 2

(g) Use the substitution $u = \cos x$ to evaluate 3

$$\int_0^{\pi} \sin^3 x\, dx$$

Leave your answer in exact simplified form.

End of Question 11

Question 11 (15 marks) Use the Question 11 Writing Booklet

(a) Differentiate **2**

$$f(x) = \sin^{-1}(e^{2x})$$

$$f'(x) = \frac{2e^{2x}}{\sqrt{1-e^{4x}}}$$ 1 1

(b) How many arrangements can be made from the word AMAZING if it must begin with a consonant and the vowels must be together. **2**

7 letters

AMAZING

$\boxed{4} \times \boxed{A\ A\ I}$ $\boxed{}$ $\boxed{}$ $\boxed{}$

$\frac{3!}{2!}$

$4 \times 3 \times 4! = 288$

(c) Evaluate $\frac{x^2+x-6}{x} \geq 0$ **3**

let: $\frac{x^2+x-6}{x} = 0$ $x \neq 0$

$x^2+x-6=0$

$(x+3)(x-2)=0$

$x=-3$ $x=-2$ 1 mark

× ✓ × ✓

-3 0 2

Test $x=-4$ 1

$\frac{(-4)^2+(-4)-6}{-4} \geq 0$

$\frac{6}{-4} \not\geq 0$ ×

Test $x=-2$

$\frac{(-2)^2+(-2)-6}{-2} \geq 0$

$\frac{-4}{-2} \geq 0$ ✓

Test $x=1$

$\frac{1+1-6}{1} \geq 0$

$-4 \geq 0$ ×

Test $x=3$

$\frac{3^2+3-6}{3} \geq 0$

$2 \geq 0$ ✓, Tr

1 mark

$\therefore -3 \leq x < 0$ $x \geq 2$

(d) Find the angle between the vectors $\begin{pmatrix}-7\\1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}$ to the nearest degree. 2

$u.v = |u||v| \cos\theta$

$-7+-1 = 5\sqrt{2} \times \sqrt{2} \cos\theta$

$-8 = 10\cos\theta$

$\cos\theta = \frac{-8}{10}$

$\theta = 143.130...$

$\theta = 143^\circ$

$|u| = \sqrt{(-7)^2 (-1)^2}$

$= \sqrt{50} = 5\sqrt{2}$

$|v| = \sqrt{(1)^2 (-1)^2}$

$= \sqrt{2}$

(e) Find $\int \frac{3\,dx}{2+x^2}$ 1

$= 3\int \frac{1}{2+x^2}$

$= 3 \times \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$ ✓

$= \frac{3\sqrt{2}}{2} \tan^{-1}\left(\frac{\sqrt{2}x}{2}\right) + C$

(f) Use the substitution $u = \cos x$ to evaluate **3**

$$\int_0^{\pi} \sin^3 x \, dx$$

Leave your answer in exact simplified form.

(g) Solve $\cos 2\theta = \cos^2 \theta + \cos \theta + 1$ for $0 \leq \theta \leq 2\pi$ **2**

End of Question 11

(f) Solve $\cos 2\theta = \cos^2\theta + \cos\theta + 1$ for $0 \le \theta \le 2\pi$. 2

$\cos 2\theta = \cos^2\theta + \cos\theta + 1$

$\cos^2\theta - \sin^2\theta = \cos^2\theta + \cos\theta + 1$

$0 = \sin^2\theta + \cos\theta + 1$

$0 = (1 - \cos^2\theta) + \cos\theta + 1$

$0 = \cos^2\theta - \cos\theta - 2$ (1)

$0 = (\cos\theta - 2)(\cos\theta + 1)$

$\cancel{\cos\theta = 2}$ no solution $\quad \cos\theta = -1$ (1)

$\theta = \pi$

(g) Use the substitution $u = \cos x$ to evaluate 3

$$\int_0^{\pi} \sin^3 x \, dx$$

Leave your answer in exact simplified form.

$\int_0^{\pi} (\sin x)(\sin^2 x)\, dx$

$-\int_0^{\pi} -\sin x(1 - \cos^2 x)\, dx$ (1)

$-\int_1^{-1} (1 - u^2)\, du$

$-\left[u - \frac{u^3}{3}\right]_1^{-1}$

$-\left[\left(-1 + \frac{1}{3}\right) - \left(1 - \frac{1}{3}\right)\right]$

$= -\left[\frac{-2}{3} - \frac{2}{3}\right]$ (1)

$= \frac{4}{3}$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x \, dx$

at $x = \pi$

$u = \cos \pi$

$= -1$

at $x = 0$

$u = \cos 0$

$= 1$

(1)

Question 12 (15 marks) Use the Question 12 Writing Booklet.

Trig

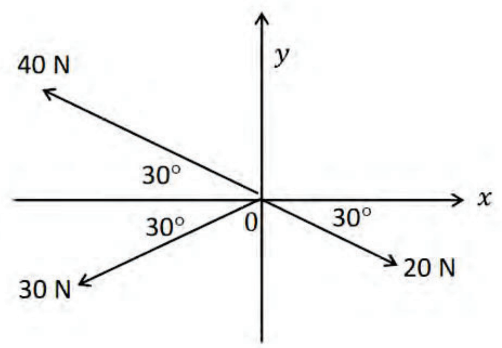
(a) Show that the exact value of $\sec 75°$ is: $\dfrac{2\sqrt{2}}{\sqrt{3}-1}$ 2

F

(b) Solve for x: $|3x+9| \geq 9$. 2

(c) The diagram shows three forces acting on an object in a PE lesson of tug of war.

Vectors



The teacher wants to find the magnitude of the resultant force in Newtons and its direction to the nearest degree so it can be displayed on a PowerPoint presentation.

Find the magnitude and direction of the resultant force. 2

D/C Stats

(d) When a research botanist crossed a tall strain of pea with a dwarf strain of pea, he found that $\frac{3}{4}$ of the offspring were tall and $\frac{1}{4}$ were dwarf.

Suppose six such offspring were selected at random. Find the probability that:

(i) All of the offspring were tall. 1

(ii) Only three of the offspring were tall. 1

Question 12 continues next page

Poly
Functions

(e) When the polynomial $P(x)$ is divided by $(x+1)(x-4)$ the quotient is $Q(x)$ and the remainder is $R(x)$.

(i) Given that $P(4)=-5$, show that $R(4)=-5$. 1

(ii) When $P(x)$ is divided by $(x+1)$, the remainder is 5. Find $R(x)$. 2

Poly
Inverse

(f) Consider $y=\tan^{-1}\frac{1}{x}$ and $y=\tan^{-1}x$:

(i) Show that $\frac{d}{dx}\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right)=0$. 2

(ii) Hence or otherwise sketch the curve $y=\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right)$. 2

End of Question 12

(a) Show that the exact value of $\sec 75°$ is: $\dfrac{2\sqrt{2}}{\sqrt{3}-1}$ 2

$$\cos 75 = \cos(30+45)$$
$$= \cos 30 \cos 45 - \sin 30 \sin 45$$
$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}} \quad (1)$$
$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$
$$\therefore \sec 75 = \frac{1}{\cos 75}$$
$$= \frac{2\sqrt{2}}{\sqrt{3}-1} \quad (1)$$

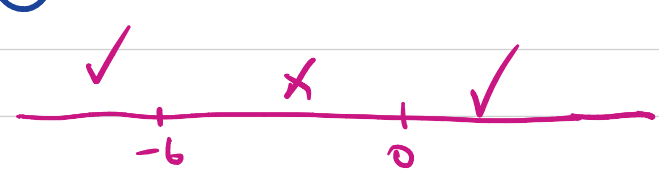
(b) Solve for x: $|3x+9| \geq 9$. 2

b) let $|3x+9| = 9$

$3x+9=-9$ $\qquad$ $3x+9=9$

$3x=-18$ $\qquad$ $3x=0$

(1) $x=-6$ $\qquad$ $x=0$



$\therefore x \leq -6, \; x \geq 0$ (1)

Test $x=-7$

$|3x-7+9| \geq 9$

$12 \geq 9$ ✓ True

Test $x=-1$

$|3x-1+9| \geq 9$

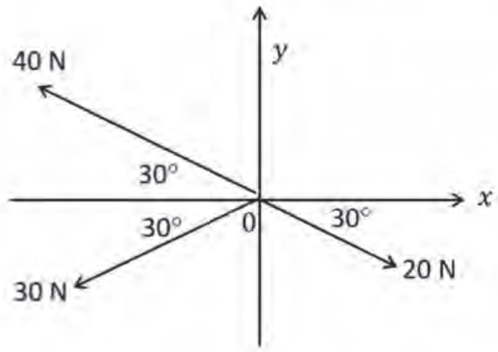
$6 \geq 9$ x false

test $x=1$

$12 \geq 9$ ✓ true.

include test

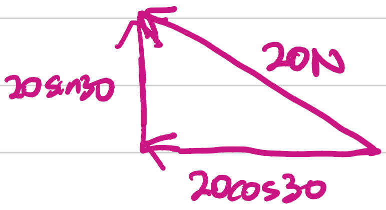
(c) The diagram shows three forces acting on an object in a PE lesson of tug of war.

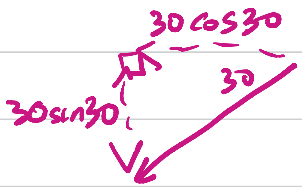


The teacher wants to find the magnitude of the resultant force in Newtons and its direction to the nearest degree so it can be displayed on a PowerPoint presentation.

Find the magnitude and direction of the resultant force. 2

$40 - 20 = 20$





Vertical $= 20 \times \frac{1}{2} - 30 \times \frac{1}{2}$

$= 10 - 15$

$= -5N$

HORIZONTAL

$= -20\cos 30 - 30\cos 30$

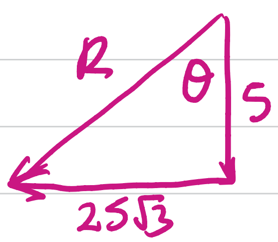
$= -20 \times \frac{\sqrt{3}}{2} - 30 \times \frac{\sqrt{3}}{2}$

$= -10\sqrt{3} - 15\sqrt{3}$

$= -25\sqrt{3}$

1

Resultant Vector



$R^2 = (25\sqrt{3})^2 + (5)^2$

$= 1900$

$R = 10\sqrt{19}$

1

$\tan\theta = \frac{25\sqrt{3}}{5}$

$\theta = \tan^{-1}(5\sqrt{3})$

$= 83.41$

$\therefore$ Bearing 263°

$=$ 187° from positive x-axis

(d) When a research botanist crossed a tall strain of pea with a dwarf strain of pea, he found that $\frac{3}{4}$ of the offspring were tall and $\frac{1}{4}$ were dwarf.

Suppose six such offspring were selected at random. Find the probability that:

(i) All of the offspring were tall. 1

(ii) Only three of the offspring were tall. 1

i) $= \left(\frac{3}{4}\right)^6$

ii) $P(3\text{ successes}) = \binom{n}{r} p^r (1-p)^{n-r}$

$= \binom{6}{3}\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^3$

$= {}^6C_3\left(\frac{3}{4}\right)^3 \times \left(\frac{1}{4}\right)^{6-3}$

$= \frac{135}{1024}$

(e) When the polynomial $P(x)$ is divided by $(x+1)(x-4)$ the quotient is $Q(x)$ and the remainder is $R(x)$.

(i) Given that $P(4)=-5$, show that $R(4)=-5$. 1

(ii) When $P(x)$ is divided by $(x+1)$, the remainder is 5. Find $R(x)$. 2

$P(x) = (x+1)(x-4)\,Q(x) + R(x)$

i) $P(4) = (4+1)(4-4)\,Q(4) + R(4)$

$(-5) = 0 + R(4)$

ii) $= \dfrac{\cancel{(x+1)}(x-4)\,Q(x)}{\cancel{(x+1)}} + \dfrac{R(x)}{(x+1)} = 5$

$\Rightarrow R(x) = \text{constant} = 5$

$P(4) = 0 + R(4) = -5$

$P(-1) = 0 + R(-1) = 5$

$\therefore R(x) = ax + b$

$4a + b = -5$

$-a + b = 5$

$5a = -10$

$a = -2$

$b = 3$

$\therefore R(x) = -2x + 3$

(f) Consider $y=\tan^{-1}\frac{1}{x}$ and $y=\tan^{-1}x$:

(i) Show that $\frac{d}{dx}\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right)=0$. 2

(ii) Hence or otherwise sketch the curve $y=\left(\tan^{-1}x+\tan^{-1}\frac{1}{x}\right)$. 2

$$\frac{d}{dx}\left(\tan^{-1}x+\tan^{-1}\left(\frac{1}{x}\right)\right)$$

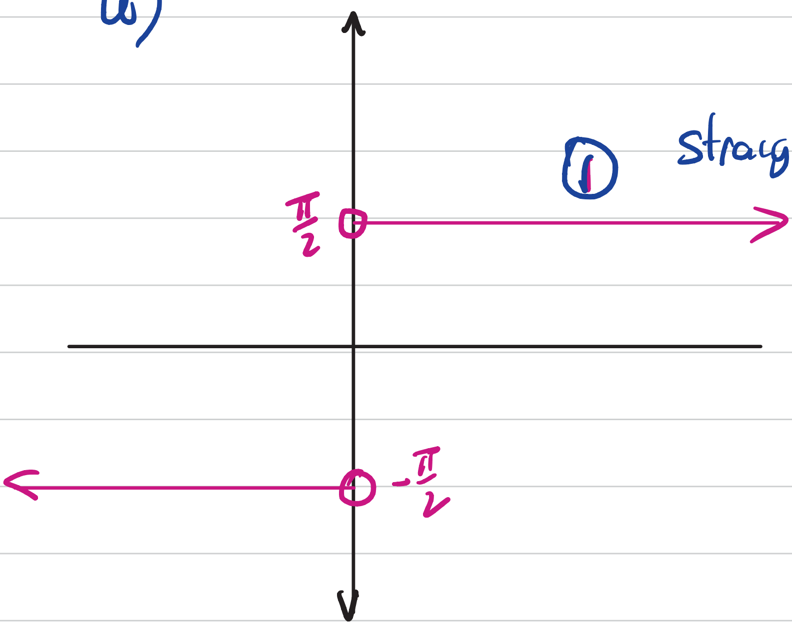
$$=\frac{1}{1+x^2}+\frac{-\frac{1}{x^2}}{1+\left(\frac{1}{x}\right)^2} \quad 1$$

$$=\frac{1}{1+x^2}-\frac{1}{x^2\left(1+\frac{1}{x^2}\right)}$$

$$=\frac{1}{1+x^2}-\frac{1}{1+x^2} \quad 1$$

$$=0$$

ii)



1 straight line $\frac{\pi}{2}$, $-\frac{\pi}{2}$

1 Open circle at $\frac{\pi}{2}$

Question 13 (15 marks) Use the Question 13 Writing Booklet

(a) Use Mathematical induction to prove that, for all positive integers, $n \geq 2$ that 3

Ind

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\ldots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$$

(b) In a particular electronic circuit, the current I amperes is given by:

$$I = 4\sin\theta - 3\cos\theta$$

Where θ is an angle related to the voltage and $\theta > 0$.

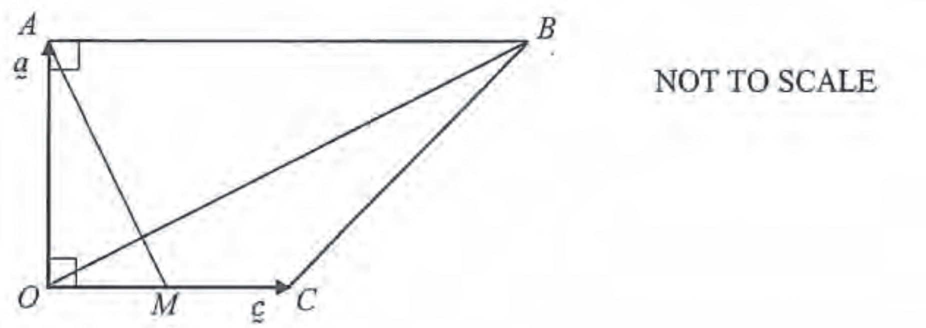
Trig

(i) Express I in the form $I = R\sin(\theta-\alpha)$ where R>0 and $0 \leq \alpha \leq \frac{\pi}{2}$. 2

(ii) Find the value of θ for which the greatest value of I first occurs. Justify your answer. 2

(c) In the diagram, $OABC$ is a trapezium with $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OC} = \underset{\sim}{c}$, $\angle COA = \angle BAO = 90°$ and $OA = OC = \frac{1}{2}AB$. M is the midpoint of OC.

Vectors



Use vector methods to show that $OB \perp AM$ **3**

(a) Use Mathematical induction to prove that, for all positive integers, $n \geq 2$ that 3

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\cdots\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$$

Prove true for $n=2$

$LHS = 1-\frac{1}{2^2}$ $\quad$ $RHS = \frac{2+1}{2\times 2}$

$= 1-\frac{1}{4}$

$= \frac{3}{4}$ $\quad$ $= \frac{3}{4}$ $\quad$ $\therefore LHS = RHS$ $\quad$ $\therefore$ true $n=2$

Assume true $n=k$

1

$$\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{k^2}\right)=\frac{k+1}{2k} \quad *$$

Prove true for $n=k+1$

$$\left[\left(1-\frac{1}{2^2}\right)\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{4^2}\right)\left(1-\frac{1}{k^2}\right)\right]\left(1-\frac{1}{(k+1)^2}\right)=\frac{(k+1)+1}{2(k+1)}$$

$$=\frac{k+2}{2(k+1)}$$

$LHS = \left(\frac{k+1}{2k}\right)\left(1-\frac{1}{(k+1)^2}\right)$ using * $\quad$ 1

$$=\left(\frac{k+1}{2k}\right)\left(\frac{(k+1)^2-1}{(k+1)^2}\right)$$

$$=\left(\frac{k+1}{2k}\right)\left(\frac{k^2+2k}{(k+1)^2}\right)$$

$$=\frac{\cancel{k}\cancel{(k+1)}(k+2)}{2\cancel{k}\cancel{(k+1)}(k+1)}$$

$$=\frac{k+2}{2(k+1)}$$

$= RHS$ $\quad$ 1

$\therefore$ Since true for $n=1$ and true for $n=k+1$, it is true for $n=k$ and all integers ≥ 2.

(b) In a particular electronic circuit, the current I amperes is given by:

$$I = 4\sin\theta - 3\cos\theta$$

Where θ is an angle related to the voltage and $\theta > 0$.

(i) Express I in the form $I = R\sin(\theta - \alpha)$ where R>0 and $0 \le \alpha \le \frac{\pi}{2}$. **2**

(ii) Find the value of θ for which the greatest value of I first occurs. Justify your answer. **2**

$I = 4\sin\theta - 3\cos\theta$

$R\sin(\theta - \alpha) = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$4\sin\theta - 3\cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$4\sin\theta = R\sin\theta\cos\alpha$

$R\cos\alpha = 4$ 1 $R\sin\alpha = 3$

$R^2\cos^2\alpha = 16$ — (1) $R^2\sin^2\alpha = 9$ — (2)

(1)+(2)

$R^2 = 25$

$R = 5$

$5\cos\alpha = 4$ $\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

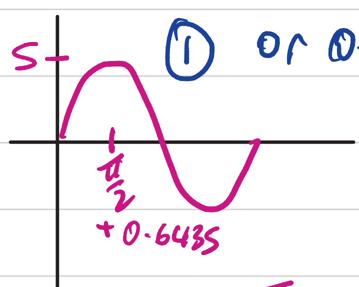
$\therefore \tan\alpha = \frac{\sin\alpha}{\cos\alpha}$

$= \frac{3}{5} \div \frac{4}{5}$

$= \frac{3}{4}$ 1

$\alpha \approx 0.6435\ldots$

$\therefore I = 5\sin(\theta - 0.6435\ldots)$



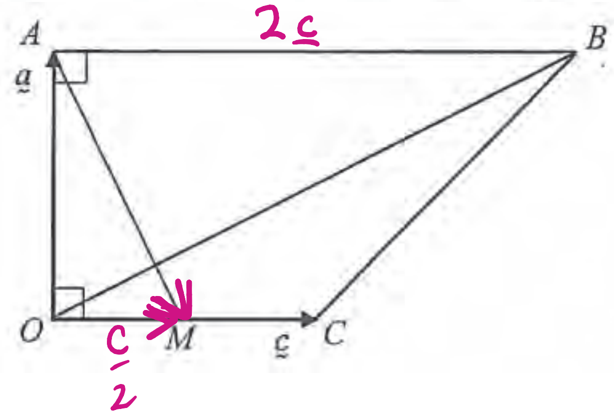
1 or otherwise

$\therefore \theta = \frac{\pi}{2} + 0.6435\ldots$ 1

$= 2.21$ (2dp)

(c) In the diagram, $OABC$ is a trapezium with $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OC} = \underset{\sim}{c}$,

$\angle COA = \angle BAO = 90°$ and $OA = OC = \frac{1}{2}AB$. M is the midpoint of OC.



NOT TO SCALE

Use vector methods to show that $OB \perp AM$ **3**

~~End of Question 13~~

$\overrightarrow{AM} = -\underset{\sim}{a} + \frac{\underset{\sim}{c}}{2}$ $\quad \overrightarrow{OB} = \underset{\sim}{a} + 2\underset{\sim}{c}$ ①

$\overrightarrow{AM}.\overrightarrow{OB} = -\underset{\sim}{a}.\underset{\sim}{a} - 2\underset{\sim}{a}.\underset{\sim}{c} + \frac{1}{2}\underset{\sim}{a}.\underset{\sim}{c} + \underset{\sim}{c}.\underset{\sim}{c}$

$= |\underset{\sim}{c}|^2 - |\underset{\sim}{a}|^2 - \frac{3}{2}\underset{\sim}{a}.\underset{\sim}{c}$ ①

$|\underset{\sim}{a}| = |\underset{\sim}{c}|$ As $OA = OC$.

$\therefore \overrightarrow{AM}.\overrightarrow{OB} = -\frac{3}{2}\underset{\sim}{a}.\underset{\sim}{c}$ ① we know $\underset{\sim}{a}.\underset{\sim}{c} = 0$

$= 0$

$\therefore \perp$

(d) A colony of birds has an initial population size P_o . At time t the rate of change of the population size P is given by $\dfrac{dP}{dt} = Pe^{-0.5t}$.

DE

(i) By solving the differential equation above, show that $P = P_o e^{2-2e^{-0.5t}}$ 3

(ii) Find, in simplest form, the time taken for the population size to reach half its limiting value. 2

End of Question 13

Answer

$$\frac{dP}{dt} = Pe^{-0.5t}$$

$$\int_{P_0}^{P} \frac{dP}{P} = \int_0^t e^{-0.5t}\,dt \quad 1$$

$$\left[\ln P\right]_{P_0}^{P} = -2\left[e^{-0.5t}\right]_0^t \quad 1$$

$$\ln P - \ln P_0 = -2\left[e^{-0.5t} - e^0\right)$$

$$\ln \frac{P}{P_0} = -2(e^{-0.5t} - 1)$$

$$\frac{P}{P_0} = e^{2(1-e^{-0.5t})} \quad 1$$

$$P = P_0 e^{2-2e^{-0.5t}}$$

ii) $\therefore$ lim as $t \to \infty$ $= P_0 e^{2-2\times 0}$

$= P_0 e^2$ 1

$P = \frac{1}{2} P_0 e^2$ when

$$e^{-2e^{-0.5t}} = \frac{1}{2}$$

$$e^{-2e^{-0.5t}} = 2^{-1}$$

$$e^{2e^{-0.5t}} = 2 \quad 1$$

$$2e^{-0.5t} = \ln 2$$

$$e^{-0.5t} = \tfrac{1}{2}\ln 2$$

$$e^{0.5t} = \frac{2}{\ln 2}$$

$$0.5t = \ln\left(\frac{2}{\ln 2}\right)$$

$$t = 2\ln\left(\frac{2}{\ln 2}\right) \quad 1$$

1 mark where they have simplified and applied appropriate logs

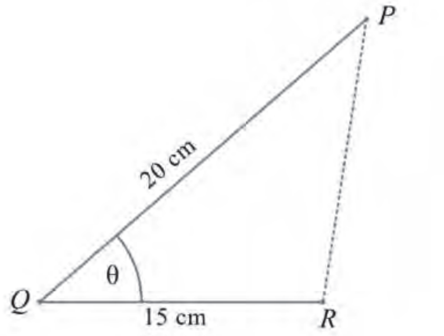
F
Intes

Question 14 (15 marks) Use the Question 14 Writing Booklet.

(a) Find the exact volume of the solid of revolution formed when the curve $y = \sin x$ is rotated about the x-axis between $x = \frac{-\pi}{2}$ and $x = \frac{\pi}{2}$. 3

(b) The triangle PQR, shown below, has two sides of fixed length: $PQ = 20$ cm and $QR = 15$ cm. $\angle PQR = \theta$ is increasing at a rate of $\frac{\pi}{6}$ radians/hour.

Find the rate at which the area of the triangle is changing at the time when $\theta = \frac{\pi}{3}$. 2



(c) Simplify $n\binom{n-1}{1} + n\binom{n-1}{2} + \ldots + n\binom{n-1}{n-2}$ 3

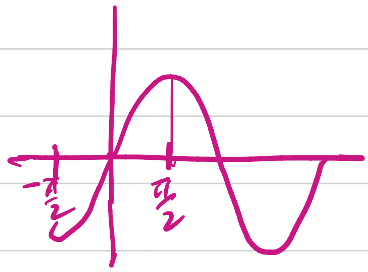
P/C

and hence solve:

$$n\binom{n-1}{1} + n\binom{n-1}{2} + \ldots + n\binom{n-1}{n-2} = 254n$$

(a) Find the exact volume of the solid of revolution formed when the curve $y = \sin x$ is rotated about the x-axis between $x = \frac{-\pi}{2}$ and $x = \frac{\pi}{2}$. 3

$$2\pi\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$$



$$= 2\pi\int \tfrac{1}{2}[1-\cos 2x]\,dx$$ 1

$$= 2\pi\left[\tfrac{1}{2}x - \frac{\sin 2x}{4}\right]_0^{\frac{\pi}{2}}$$ 1

0

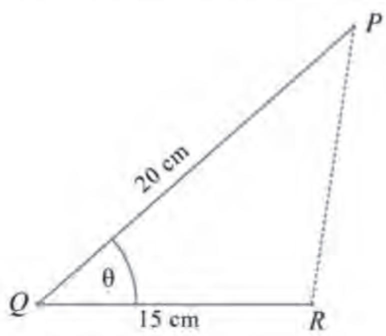
$$= 2\pi\left(\left[\frac{\pi}{4} - \cancel{\frac{\sin\pi}{4}}\right] - \left[0 - \cancel{\sin 0}\right]\right)$$

$$= \frac{2\pi^2}{4}$$

$$= \frac{\pi^2}{2}$$ 1 or equivalent.

(b) The triangle PQR, shown below, has two sides of fixed length: $PQ = 20$ cm and $QR = 15$ cm. $\angle PQR = \theta$ is increasing at a rate of $\frac{\pi}{6}$ radians/hour.

Find the rate at which the area of the triangle is changing at the time when $\theta = \frac{\pi}{3}$. 2



$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2} \times 20 \times 15 \sin\theta$

$= 150\sin\theta$

$\frac{d\theta}{dt} = \frac{\pi}{6}$

$\frac{dA}{d\theta} = 150\cos\theta$ 1

$\frac{dA}{dt} = \frac{dA}{d\theta} \times \frac{d\theta}{dt}$

$= 150(\cos\theta) \times \frac{\pi}{6}$ at $\theta = \frac{\pi}{3}$

$= \frac{150\pi}{6} \times \cos\frac{\pi}{3}$ 1

$= \frac{150\pi}{6} \times \frac{1}{2}$

$= \frac{150\pi}{12} = \frac{25\pi}{2}$ rad/sec

(c) Simplify $n\binom{n-1}{1}+n\binom{n-1}{2}+\ldots+n\binom{n-1}{n-2}$ 3

and hence solve:

$$n\binom{n-1}{1}+n\binom{n-1}{2}+\ldots+n\binom{n-1}{n-2}=254n$$

i) Simplify $n\binom{n-1}{1}+n\binom{n-1}{2}+\ldots+n\binom{n-1}{n-2}$

$= n\left[\binom{n-1}{1}+\binom{n-1}{2}+\ldots\binom{n-1}{n-2}\right]$

Consider the expansion of $(x+1)^{n-1}$

$(1+x)^{n-1}=\binom{n-1}{0}+\binom{n-1}{1}x+\binom{n-1}{2}x^2+\ldots\binom{n-1}{n-2}x^{n-2}+\binom{n-1}{n-1}x^{n-1}$

for $x=1$

$2^{n-1}=1+\binom{n-1}{1}+\binom{n-1}{2}+\ldots\binom{n-1}{n-2}+1$ collecting terms

$\binom{n-1}{1}+\binom{n-1}{2}+\ldots\binom{n-1}{n-2}=2^{n-1}-2$

$=2(2^{n-2}-1)$

$\therefore n\left[\binom{n-1}{1}+\binom{n-1}{2}+\ldots+\binom{n-1}{n-2}\right]=2n(2^{n-2}-1)$

ii) $2n(2^{n-2}-1)$

Hence or otherwise solve

$n\binom{n-1}{1}\ldots n\binom{n-1}{n-2}=254n$ (1)

$2n(2^{n-2}-1)=254n$

$2^{n-2}-1=127$

$2^{n-2}=128$

OR $2^{n-2}=2^7$

$n-2=7$ (1)

$n=9$

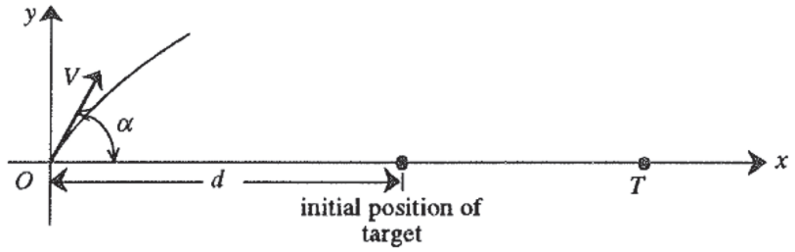
$\ln 2^{n-2}=\ln 128$

$(n-2)\ln 2=\frac{\ln 128}{\ln 2}$

$n-2=\frac{\ln 128}{\ln 2}$

$n=\frac{\ln 128}{\ln 2}+2=9$

(d) A projectile, of initial speed V m/s, is fired at an angle of elevation α from the origin O towards a target T, which is moving away from O along the x-axis.



Projectile

You may assume that the projectile's trajectory is defined by:

$\underset{\sim}{r} = (Vt\cos\alpha)\underset{\sim}{i} + (\frac{-gt^2}{2} + Vt\sin\alpha)\underset{\sim}{j}$ where g is gravity and t is time in seconds.

(i) Show that the projectile is above the x-axis for a total of $t = \dfrac{2V\sin\alpha}{g}$ seconds. 2

(ii) Show that the horizontal range of the projectile is $\dfrac{2V^2\sin\alpha\cos\alpha}{g}$ metres. 1

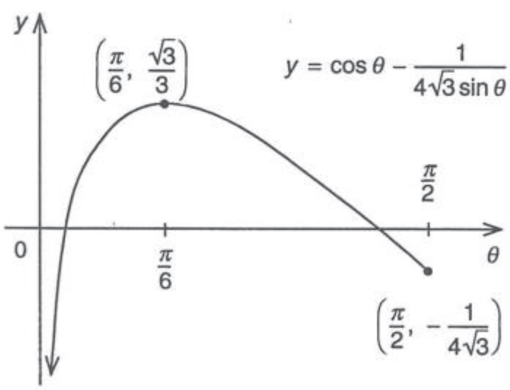
(iii) At the instant the projectile is fired, the target T is d metres from the origin O and is moving away at a constant speed of u m/s.

Suppose that the projectile hits the target when fired at an angle of elevation α

Show that $u = V\cos\alpha - \dfrac{gd}{2V\sin\alpha}$. 2

(iv) By using (iii) and the graph below show that if $u > \dfrac{V}{\sqrt{3}}$, the target cannot be hit by the projectile irrespective of the angle α at which the projectile is fired.

You may assume that : $gd = \dfrac{V^2}{2\sqrt{3}}$. 2



End of Task

b) $\underset{\sim}{r} = (Vt\cos\alpha)\underset{\sim}{i} + \left(\frac{-gt^2}{2} + Vt\sin\alpha\right)\underset{\sim}{j}$

d) i)

hits ground

$$\frac{-gt^2}{2} + Vt\sin\alpha = 0$$

$$t\left(\frac{-gt}{2} + V\sin\alpha\right) = 0$$

$$t = 0 \text{ or } \frac{gt}{2} = V\sin\alpha$$

$$gt = 2V\sin\alpha$$

$$t = \frac{2V\sin\alpha}{g}$$

ii) at $t = \frac{2V\sin\alpha}{g}$ sub $\Rightarrow$

$$\underset{\sim}{x} = Vt\cos\alpha\, i$$

$$= \frac{V\cos\alpha}{1} \cdot \frac{2V\sin\alpha}{g}$$

$$= \frac{2V^2\sin\alpha\cos\alpha}{g} = \frac{V^2\sin 2\alpha}{g}$$

$$\text{Target} = d + u \cdot \frac{2v\sin\alpha}{g}$$

$$\text{Projectile} = \frac{2v^2\sin\alpha\cos\alpha}{g}$$

at projectile range (x-axis)

$$u \cdot \frac{2v\sin\alpha}{g} = \frac{2v^2\sin\alpha\cos\alpha}{g} - d.$$

$$u\, 2v\sin\alpha = 2v^2\sin\alpha\cos\alpha - dg.$$

$$u = v\cos\alpha - \frac{dg}{2v\sin\alpha}.$$

iv) <u>Projectile</u>

$$u = v\cos\alpha - \frac{v^2}{4\sqrt{3}\,v\sin\alpha}$$

$$u = v\cos\alpha - \frac{v}{4\sqrt{3}\sin\alpha}$$

$$u = v\left(\cos\alpha - \frac{1}{4\sqrt{3}\sin\alpha}\right)$$

$$\frac{\cancel{v}}{\sqrt{3}} > \cancel{v}\left(\cos\alpha - \frac{1}{4\sqrt{3}\sin\alpha}\right)$$

$$\frac{\sqrt{3}}{\sqrt{3}} \times \frac{1}{\sqrt{3}} > \left(\cos\alpha - \frac{1}{4\sqrt{3}\sin\alpha}\right) \text{ graph.}$$

$$\frac{\sqrt{3}}{3} > \cos\alpha - \frac{1}{4\sqrt{3}\sin\alpha}.)$$

not possible as is ~~always~~ on graph

Merewether High School

Student Name: ANSWERS.

Class:

YEAR 12 EXTENSION 1 MATHEMATICS
EXAMINATION 2025

ANSWER BOOKLET

Shade the most correct answer.

	A	B	C	D
1	○	●	○	○
2	○	○	●	○
3	○	○	●	○
4	○	○	●	○
5	●	○	○	○
6	○	●	○	○
7	○	○	○	●
8	●	○	○	○
9	○	○	●	○
10	○	○	●	○