Student Number

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Teacher:

Roseville College

VERITAS OMNIA VINCIT

2025	**TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 70

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6 – 10)

- Attempt Questions 11 – 14

Allow about 1 hour and 45 minutes for this section

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet provided for Questions 1-10.

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1 Let $P(x) = x^3 - x^2 + kx + 6$. Suppose $P(x)$ is divisible by $(x - 2)$.

Which of the following is a possible value of k?

A. $k = -5$

B. $k = 5$

C. $k = -9$

D. $k = 9$

2 A curve is given by $x = 4t + 1, y = t^2 - 1$.

Which derivative will find $\frac{dy}{dx}$?

A. $\dfrac{d^2y}{dt^2}$

B. $\dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

C. $\dfrac{\frac{dx}{dt}}{\frac{dy}{dt}}$

D. $\dfrac{d^2x}{dt^2}$

3 If $y = e^{2x}\cos^{-1}(3x)$, which of the following is the correct expression for $\frac{dy}{dx}$?

A. $e^{2x}\left[2\cos^{-1}(3x) + \dfrac{3}{\sqrt{1-9x^2}}\right]$

B. $2\,e^{2x}\cos^{-1}(3x) - \dfrac{3e^{2x}}{\sqrt{1-9x^2}}$

C. $e^{2x}\left[2\cos^{-1}(3x) - \dfrac{3}{\sqrt{1-3x^2}}\right]$

D. $2\,e^{2x}\cos^{-1}(3x) + \dfrac{3e^{2x}}{\sqrt{1-3x^2}}$

4 Consider the polynomial $Q(x) = x^3 + 3x^2 - x - 9$.

If $\alpha = 3$ is one root, then the sum of the other two roots is

A. -6

B. -3

C. 0

D. 3

5 Which of the following is the largest possible domain for $f(x) = \arcsin(x^2 - 1)$?

A. $x \le \sqrt{2}$

B. $x < \sqrt{2}$

C. $-\sqrt{2} \le x \le \sqrt{2}$

D. $-\sqrt{2} < x < \sqrt{2}$

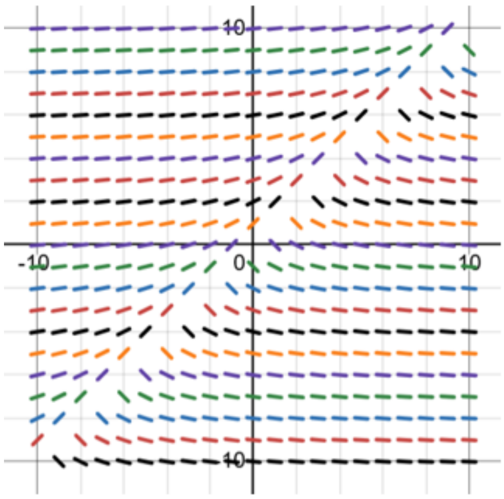
6 Which of the following differential equations matches the slope field shown?

A. $\dfrac{dy}{dx} = y - x$

B. $\dfrac{dy}{dx} = x - y$

C. $\dfrac{dy}{dx} = \dfrac{1}{y - x}$

D. $\dfrac{dy}{dx} = \dfrac{1}{x - y}$



7 Consider the following integral:

$$\int_1^e \frac{\ln x}{x}\,dx.$$

What value is obtained when you use the substitution $u = \ln x$ to evaluate the integral?

A. $-\dfrac{1}{2}$

B. $\ln 1$

C. $\dfrac{1}{2}$

D. 1

8 You are given distinct positive integers. You want to guarantee that two of them have a difference that is a multiple of 5.

What is the minimum number of distinct integers needed to guarantee that at least two of them have a difference that is a multiple of 5 ?

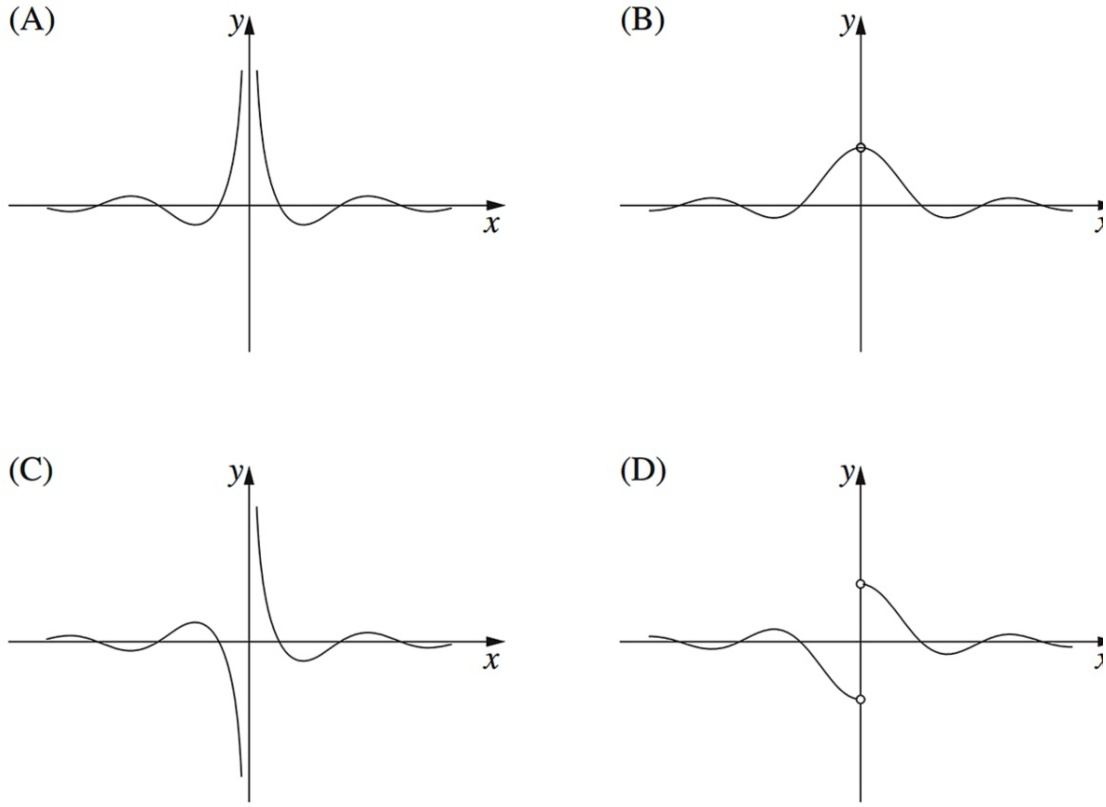
A. 5

B. 6

C. 10

D. 11

9 Which diagram best represents the graph $y = \dfrac{\sin x}{x}$?



10 In how many ways can you place five distinct objects into three distinct boxes, where each box must contain at least 1 object?

A. 120

B. 147

C. 150

D. 243

End of Section I

Section II

60 marks

Attempt Questions 11–14

Allow about 1 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) Let $\underset{\sim}{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $\underset{\sim}{v} = \begin{bmatrix} -1 \\ 5 \end{bmatrix}$.

(i) Find $\underset{\sim}{u} - 2\underset{\sim}{v}$. **1**

(ii) Find the angle θ between $\underset{\sim}{u}$ and $\underset{\sim}{v}$ where $0 \leq \theta \leq \frac{\pi}{2}$. **2**

(b) Solve the inequality $\dfrac{x+3}{x-1} > 1$. **2**

(c) Find the constant term in the expansion of $\left(x^2 + \frac{2}{x}\right)^6$. **2**

(d) The region bounded by $y = x$ and $y = x^2$ is rotated about the y-axis.

(i) Determine the points of intersection and sketch the region. **2**

(ii) Find the volume of the resulting solid of revolution. **3**

(e) Use mathematical induction to prove that $4^{2n} - 1$ is divisible by 3 for all integers $n \geq 1$. **3**

Question 12 (15 marks)

(a) Solve the differential equation, $\dfrac{dy}{dx} = (2x+1)y$ given that when $x = 0,\ y = 4$. **3**

(b) Integrate $\displaystyle\int \cos^4 x\, dx$. **4**

(c) By using the substitution $u = \sec 2x$ evaluate $\displaystyle\int_0^{\frac{\pi}{6}} \sec 2x \tan 2x\, dx$. **3**

(d) A conical tank has its radius and height in the ratio of 1: 2 respectively. Water is entering the tank at a rate of 1200 L/min.

The volume V is given by $\frac{1}{3}\pi r^2 h$ where r is the radius in centimetres and h is the height in centimetres.

Let the water depth be h with the radius r at the surface.

(i) Show that $r = ½h$. **1**

(ii) Find $\dfrac{dh}{dt}$ at the time when $h = 80$ cm. **4**

Question 13 (15 marks)

(a) A curve is defined parametrically by $x = \cos t$, $y = \cos^2 t$. **3**

Find the exact area enclosed by this curve, for $0 \leq t \leq \frac{\pi}{2}$, above the x-axis.

(b) There are seven distinct people who are to be seated around a round table. Of these seven people, Alice and Bob must sit next to each other, while Charlie and Dean must not sit next to each other. **3**

In how many ways could these seven people be seated with the above conditions?

(c) Let $\underset{\sim}{u} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}$ and $\underset{\sim}{v} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$. **3**

By considering the projection of $\underset{\sim}{u}$ on $\underset{\sim}{v}$, or otherwise, write $\underset{\sim}{u}$ as the sum of two perpendicular components, one parallel to $\underset{\sim}{v}$ and the other perpendicular to $\underset{\sim}{v}$.

(d) Solve $\sin 3x + \sin x = 0$ for $0 \leq x \leq 2\pi$. **3**

(e) Consider a parallelogram $OABC$ where the point P is the midpoint of AB. **3**

The point Q lies on the diagonal AC so that $AQ : QC = 1 : 2$.

Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$.

Prove that O, P, Q are collinear.

Question 14 (15 marks)

(a) A population $P(t)$ in thousands satisfies the following equation:

$$\frac{dP}{dt} = 0.03P(8-P)$$

where t is measured in years and $P \geq 0$.

(i) Show $\dfrac{1}{P(8-P)} = \dfrac{1}{8}\left(\dfrac{1}{P} + \dfrac{1}{8-P}\right)$. **1**

(ii) Given the initial condition $P(0) = 2$, solve for $P(t)$ in terms of t. **3**

(iii) Hence, find the limiting value of $P(t)$ for large values of t. **1**

(b) Two projectiles are launched simultaneously at $t = 0$.

Projectile A is launched towards the right, from ground level.

Projectile A has an initial velocity of $\mathbf{u}_A = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$.

Projectile B is launched towards the left, from a ledge 15 m above the ground and 30 m horizontally to the right of where Projectile A is launched.

Projectile B has an initial velocity of $\mathbf{u}_{\boldsymbol{B}} = \begin{bmatrix} -5 \\ 2.5 \end{bmatrix}$.

Let acceleration due to gravity be $9.8\ \mathrm{m/s^2}$ and assume there is no air resistance.

(i) Write down the position vectors $\mathbf{r}_A(t)$ and $\mathbf{r}_B(t)$ for each projectile using the given initial data. **3**

(ii) Find the time when the two projectiles collide and the coordinates of the collision. **2**

(iii) Find the speed of Projectile A at the point of collision. **1**

Question 14 continues on page 10

Question 14 (continued)

(c) Show that ${}^{n}\mathbf{C}_{r} = {}^{n}\mathbf{C}_{n-r}$ **1**

(d) Find the exact value of $\sin\left(\cos^{-1}\frac{2}{3} + \sin^{-1}\frac{1}{4}\right)$. **3**

End of Paper