Caringbah High School

Year 12 2026
Mathematics Extension 1
HSC Course
Assessment Task 4 Trial Examination

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- NESA-approved calculators may be used
- A reference sheet is provided
- In Questions 11-14, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 70

Section I **10 marks**
Attempt Questions 1-10
Mark your answers on the answer sheet provided. You may detach the sheet and write your name on it.

Section II **60 marks**
Attempt Questions 11-14
Write your answers in the answer booklets provided. Ensure your name or student number is clearly visible.

Student Number: _____ **Class:** _____

Marker's Use Only						
Section I	**Section II**				**Total**	
Q 1-10	**Q11**	**Q12**	**Q13**	**Q14**		
/10	/15	/15	/15	/15	/70	%

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

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1. What is the solution of the inequality $x(x-2)(x+2) < 0$?

(A) $(-2, 2)$

(B) $(-\infty, -2) \cup (2, \infty)$

(C) $(-2, 0) \cup (2, \infty)$

(D) $(-\infty, -2) \cup (0, 2)$

2. Consider the vectors $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$.

What is the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in terms of $\cos(\theta)$?

(A) $\cos(\theta) = \dfrac{17}{5\sqrt{13}}$

(B) $\cos(\theta) = \dfrac{-1}{5\sqrt{13}}$

(C) $\cos(\theta) = \dfrac{-1}{65}$

(D) $\cos(\theta) = 0$

3. The roots of $x^3 + x^2 - 2x = 0$ are α, β and γ.

What is the value of $\alpha^2 + \beta^2 + \gamma^2$?

(A) 2

(B) 3

(C) 4

(D) 5

4. Consider the points $A(1,2)$, $B(3,6)$ and $C(5,10)$.

Which of the following statements is true?

(A) The points are not collinear because $\overrightarrow{AC} = \frac{1}{2}\overrightarrow{AB}$

(B) The points are not collinear because $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are perpendicular

(C) The points are collinear because $\overrightarrow{AC} = 2\overrightarrow{AB}$

(D) The points are collinear because $\overrightarrow{AB} = 2\overrightarrow{AC}$

5. The discrete random variable X has a binomial with parameters n.

Its mean is twice its standard deviation. If $p = \frac{1}{4}$, what is the value of n?

(A) 6

(B) 12

(C) 18

(D) 24

6. If $\operatorname{cosec}(x) + \cot(x) = 2$, what is the value of $t = \tan\left(\frac{x}{2}\right)$?

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 2

(D) 4

7. The graph of an even polynomial function $P(x)$ passes through the point $(2, 0)$. What is the remainder when $P(x)$ is divided by $(x^2 - 4)$?

(A) 0

(B) –2

(C) 2

(D) –4

8. Which of the following is an expression for the integral below?

$$\int \frac{\cos(2x)}{(\cos(x) + \sin(x))^2}\, dx$$

(A) $\ln|\cos(x) + \sin(x)| + C$

(B) $\frac{-1}{\cos(x) + \sin(x)} + C$

(C) $\frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)} + C$

(D) $\frac{\sin(x) - \cos(x)}{\cos(x) + \sin(x)} + C$

9. At time t years after observation begins, the population size P of an animal colony is given by $P = 600 - 500e^{-0.1t}$.

What is the population size when it is increasing at a rate of 25 animals per year?

(A) 250

(B) 300

(C) 350

(D) 400

10. What is the value of the integral below?

$$\int_0^1 \frac{\tan^{-1} x}{1 + x^2}\, dx$$

(A) $\frac{\pi^2}{32}$

(B) $\frac{\pi^2}{48}$

(C) $\frac{\pi^2}{16}$

(D) $\frac{\pi^2}{8}$

End of Section I

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour 45 minutes for this section

Answer each question in the Section II Writing Booklet. Extra writing booklets are available.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Answer Question 11 on pages 2–6 of the Writing Booklet.

(a) Solve for x **3**

$$\frac{x^2+1}{x} \geq \frac{5}{2}$$

(b) Find the exact value of $\cos\left[\sin^{-1}\left(\frac{3}{8}\right)-\tan^{-1}\left(\frac{1}{4}\right)\right]$. **3**

You may leave your answer with an irrational denominator.

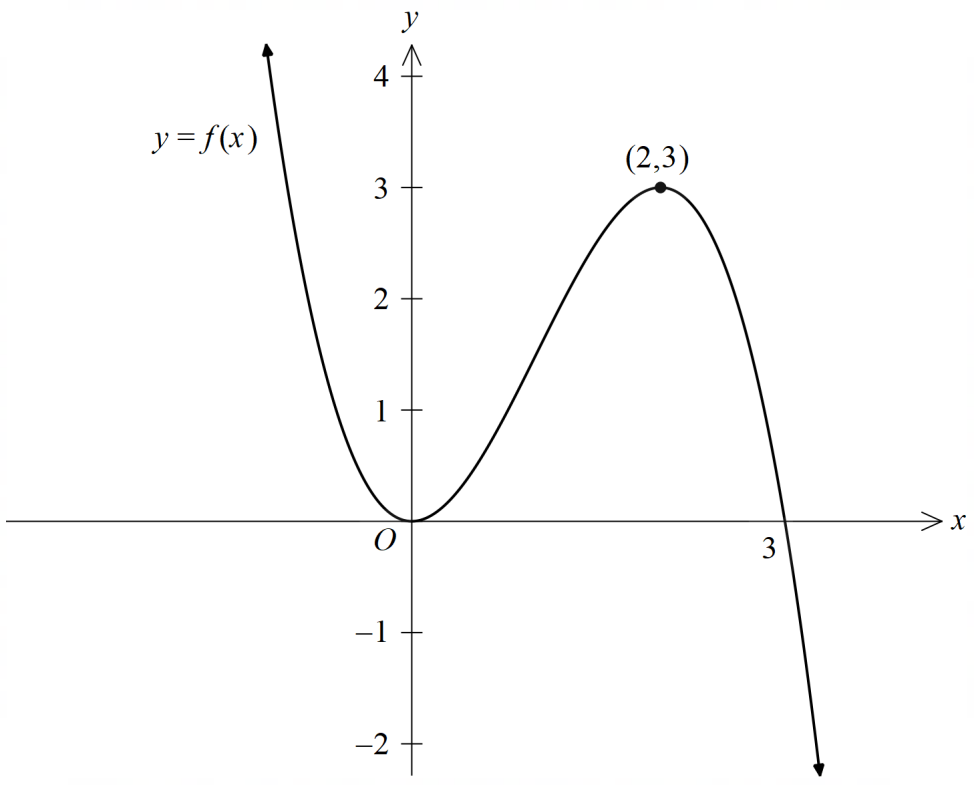
(c) The polynomial $P(x) = ax^4 - x^3 - 28x^2 + bx - 7$ has a remainder of $-2x-3$ when divided by $x^2 + 3x - 4$. Find the values of a and b. **3**

(d) **(i)** Write $\sin(x) + \sqrt{3}\cos(x)$ in the form $R\sin(x+\alpha)$, where $R > 0$, $0 \leq \alpha \leq \frac{\pi}{2}$ **2**

(ii) Hence, or otherwise, solve the equation $\sin(x) + \sqrt{3}\cos(x) = 1$, $0 \leq x \leq 2\pi$ **2**

Question 11 continues on the next page

(e) The graph of $y = f(x)$ is shown below. **2**



On the template provided on **page 6** of the Section II Writing Booklet, sketch the graph of $y = \sqrt{f(x)}$.

End of Question 11

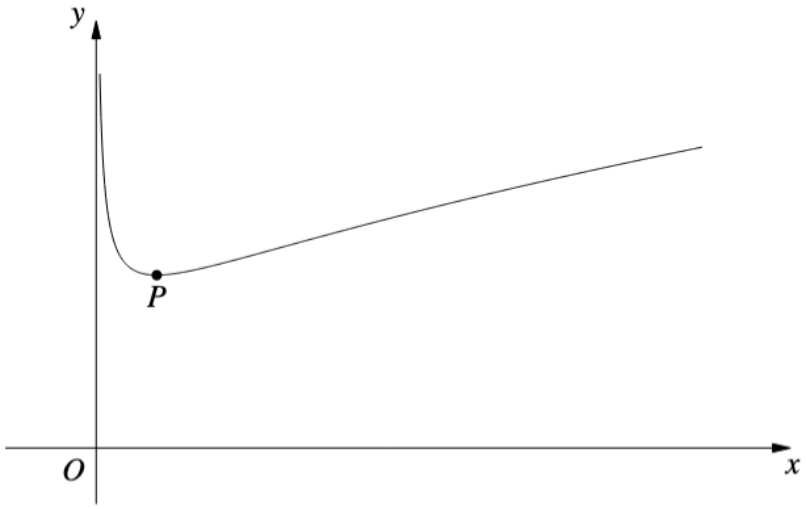
Question 12 (15 marks) Answer Question 12 on pages 8–14 of the Writing Booklet.

(a) A large cube of ice is melting. As it melts, its side length x millimetres changes at the rate of -0.8 millimetres per second. **2**

Find the rate at which the surface area of the cube of ice is changing when the side length of the cube is 10 millimetres.

(b) Let $f(x) = \dfrac{x+1}{\sqrt{x}}$.

The graph of $y = f(x)$ has a minimum turning point at P as shown.



(i) The domain of $f(x)$ is restricted to $x \geq a$ so that its inverse, $f^{-1}(x)$ is a function. Find the value of a. **2**

(ii) Find $f^{-1}(x)$. **3**

Question 12 continues on the next page

(c) A bag contains five cards numbered 2, 3, 4, 5, 6.

(i) Four cards are drawn at random from the bag, one at a time, with replacement. Find, in simplest fraction form, the probability that the sum of the four numbers chosen is odd. **2**

(ii) 150 cards are drawn at random from the bag, one at a time, with replacement. Use the standard normal approximation and the table on page 15 to find, correct to 2 decimal places, the probability that at least 49 of the 150 numbers chosen are odd. **3**

(d) Use the substitution $u = x - 1$, to evaluate: **3**

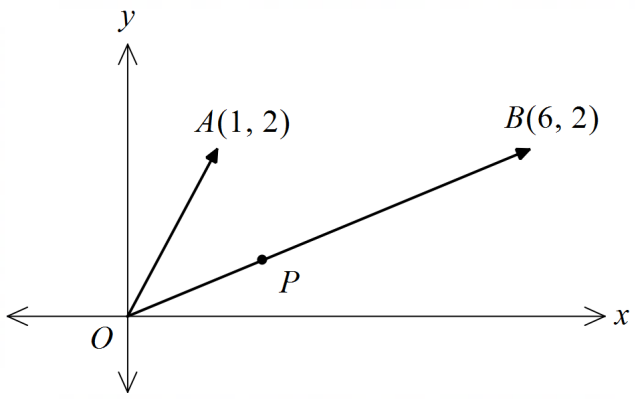
$$\int_{0.5}^{1.5} \frac{1}{\sqrt{2x - x^2}}\, dx$$

Give the answer in simplest exact form.

End of Question 12

Question 13 (15 marks) Answer Question 13 on pages 16–20 of the Writing Booklet.

(a) The diagram shows the points $A(1, 2)$ and $B(6, 2)$. The point P is the closest point on the line OB to A.

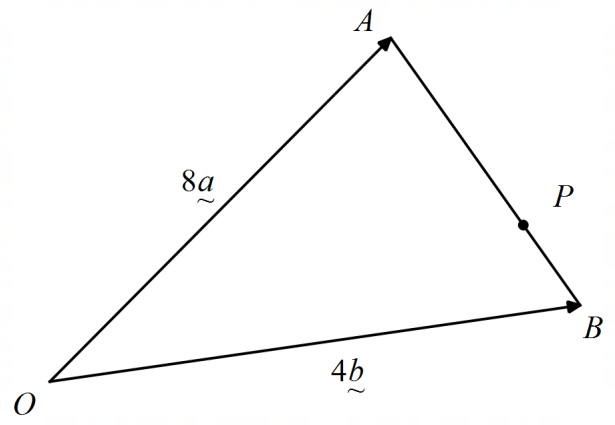


NOT TO SCALE

(i) Using the vector projection of $\overrightarrow{OA}$ onto $\overrightarrow{OB}$, or otherwise, find $\overrightarrow{OP}$ in the form $x\underset{\sim}{i} + y\underset{\sim}{j}$. **2**

(ii) Given $\overrightarrow{OB} = k\overrightarrow{OP}$, find the value of k. **1**

(b) OAB is a triangle where $\overrightarrow{OA} = 6\underset{\sim}{a}$, $\overrightarrow{OB} = 4\underset{\sim}{b}$ and P lies on AB such that $AP : PB = 3 : 1$. **3**



Show that $\overrightarrow{OP} = 2\underset{\sim}{a} + 3\underset{\sim}{b}$.

Question 13 continues on the next page

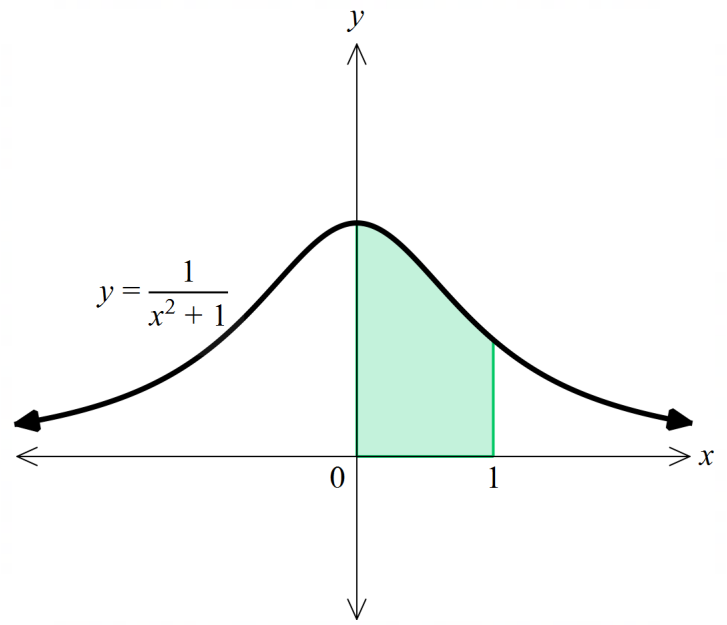
(c) **(i)** Show that **2**

$$\tan(4\theta) = \frac{4\tan(\theta) - 4\tan^3(\theta)}{1 - 6\tan^2(\theta) + \tan^4(\theta)}$$

(ii) Hence, find the values of x for $0 \leq \theta \leq \pi$ such that **3**

$$x^4 - 4x^3 - 6x^2 + 4x + 1 = 0$$

(d) The curve $y = \frac{1}{x^2 + 1}$ for $0 \leq x \leq 1$ is rotated about the x-axis. **4**



Find the volume of the solid of revolution formed using the substitution

$x = \tan\theta$

End of Question 13

Question 14 (15 marks) Answer Question 14 on pages 22–28 of the Writing Booklet.

(a) Ten people arrive to eat at a restaurant. The only seating available for them is at two circular tables, one that seats six people and another that seats four.

(i) Using these tables, how many different seating arrangements are there for the ten people? **1**

(ii) Assuming that the seating arrangement is random, what is the probability that a particular couple will be seated at the same table? **2**

(b) Use Mathematical Induction to prove that for all positive integers $n \geq 1$, **4**

$$\frac{(x+0)!}{0!} + \frac{(x+1)!}{1!} + \frac{(x+2)!}{2!} + \ \ldots \ + \frac{(x+n-1)!}{(n-1)!} = \frac{(x+n)!}{(x+1)(n-1)!}$$

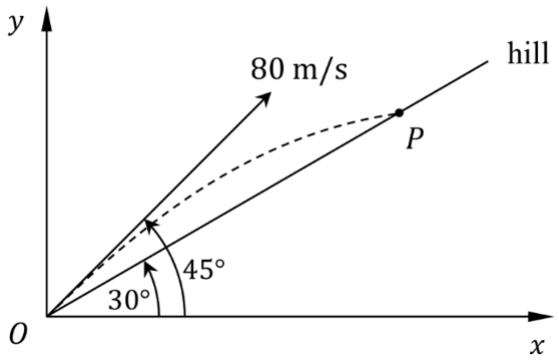
(c) By considering the expansion of $(1-x^2)^n$ for a positive even number n, **4**
show that

$$\binom{n}{0}^2 - \binom{n}{1}^2 + \binom{n}{2}^2 - \binom{n}{3}^2 + \ldots - \binom{n}{n-1}^2 + \binom{n}{n}^2 = \binom{n}{\frac{n}{2}} (-1)^{\frac{n}{2}}$$

Question 14 continues on the next page

(d) A golfer hits a golf ball at 80 metres per second from the bottom of a steep hill. The hill is at an angle of 30° to the horizontal, and the golf ball is hit at an angle of 45° to the horizontal. **4**

The golf ball is hit from O and lands at P, as shown in the diagram.



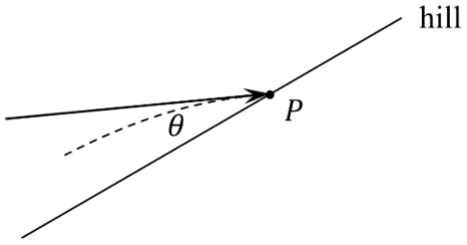
The flight path of the ball is given by

$$x = 40\sqrt{2}t$$
$$y = -5t^2 + 40\sqrt{2}t$$

(Do NOT prove these)

where t is the time in seconds since the golf ball was hit, and x and y are in metres.

The golf ball impacts the ground at an angle of θ to the hill, as shown below.



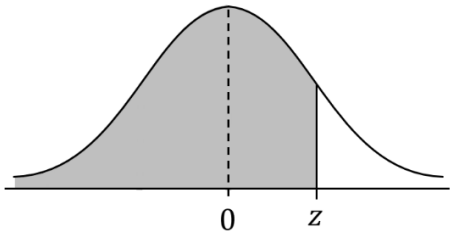
Find the size of θ, correct to the nearest degree.

End of Examination

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You may use the information below to answer Question 13 (c) (ii)

Table of values $P(Z < z)$ for the normal distribution $N(0, 1)$



Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

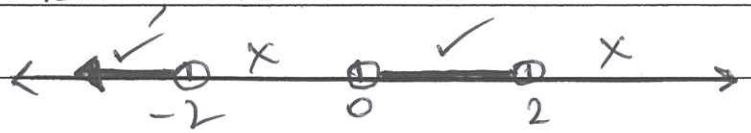
2026 Year 12 Mathematics Ext 1 Trial solutions

Q1	D
Q2	B
Q3	D
Q4	C
Q5	B
Q6	B
Q7	A
Q8	A
Q9	C
Q10	A

Q1 $x(x-2)(x+2) < 0$

Let $x(x-2)(x+2) = 0$

$x = 0, \pm 2$



$\therefore (-\infty, -2) \cup (0, 2)$ (D)

Q2 $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$\underset{\sim}{u} \cdot \underset{\sim}{v} = (2 \times 4) + (3 \times -3)$

$= -1$

$|\underset{\sim}{u}| = \sqrt{2^2 + 3^2}$, $|\underset{\sim}{v}| = \sqrt{4^2 + (-3)^2}$

$= \sqrt{13}$ $= 5$

$\therefore \cos\theta = \dfrac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|\underset{\sim}{u}| \times |\underset{\sim}{v}|}$

$= \dfrac{-1}{5\sqrt{13}}$ (B)

Q3 $x^3 + x^2 - 2x = 0$

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= \left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$

$= \left(-\frac{1}{1}\right)^2 - 2\left(\frac{-2}{1}\right)$

$= 5$ (D)

Do NOT write in this area.

Q4 $\vec{AB} = (3-1, 6-2)$, $\vec{AC} = (5-1, 10-2)$

$= (2, 4)$ $= (4, 8)$

Since $\vec{AC} = 2\vec{AB}$, the vectors are scalar multiples, meaning that A, B and C are collinear. (C)

Q5 $p = \frac{1}{4}$, $q = \frac{3}{4}$

$np = 2\sqrt{npq}$

$n \times \frac{1}{4} = 2\sqrt{n \times \frac{1}{4} \times \frac{3}{4}}$

$\frac{n}{4} = 2 \times \sqrt{\frac{3n}{16}}$

$\frac{n}{4} = 2 \times \frac{\sqrt{3n}}{4}$

$\frac{n}{4} = \frac{\sqrt{3n}}{2}$

$\frac{n^2}{16} = \frac{3n}{4}$

$\frac{n^2}{16} = \frac{12n}{16}$

$n^2 = 12n$

$n^2 - 12n = 0$

$n(n-12) = 0$

$n = 0, 12$

$\therefore n = 12$ since $n > 0$ (B)

Do NOT write in this area.

Q6 $\operatorname{cosec}(x) + \cot(x) = 2$, $t = \tan\left(\frac{x}{2}\right)$

$$\frac{1+t^2}{2t} + \frac{1-t^2}{2t} = 2$$

$$\frac{1+t^2+1-t^2}{2t} = 2$$

$$\frac{2}{2t} = 2$$

$$\frac{1}{t} = 2$$

$$t = \frac{1}{2} \qquad (B)$$

Q7 $P(x)$ is an even function.

ie $P(2) = 0$, $P(-2) = 0$

$\therefore (x-2)$ and $(x+2)$ are components of $P(x)$

So, $(x-2)(x+2) = x^2 - 4$ are factors of $P(x)$.

$\therefore$ the remainder is zero. (A)

Q8 $$\int \frac{\cos(2x)}{(\cos(x)+\sin(x))^2}\, dx.$$

$$= \int \frac{\cos^2(x) - \sin^2(x)}{(\cos(x)+\sin(x))^2}\, dx$$

$$= \int \frac{(\cos(x)+\sin(x))(\cos(x)-\sin(x))}{(\cos(x)+\sin(x))^2}\, dx$$

$$= \int \frac{\cos(x)-\sin(x)}{\cos(x)+\sin(x)}\, dx$$

$$= \ln|\cos|x| + \sin|x|| + c \qquad (A)$$

Do NOT write in this area.

Q9 $P = 600 - 500e^{-0.1t}$

$\frac{dP}{dt} = 500e^{-0.1t}$

$= \frac{1}{10}(600 - P)$

$\frac{dP}{dt} = 25, \quad \frac{600 - P}{10} = 25$

$600 - P = 250$

$P = 350$ (C)

Q10 $\int_0^1 \frac{\tan^{-1}x}{1+x^2}\,dx$

$= \int_0^1 \left(\frac{1}{1+x^2}\right)\tan^{-1}x\,dx$

$= \frac{1}{2}\left[(\tan^{-1}x)^2\right]_0^1$

$= \frac{1}{2}\left[(\tan^{-1}(1))^2 - (\tan^{-1}(0))^2\right]$

$= \frac{1}{2}\left[\left(\frac{\pi}{4}\right)^2 - 0\right]$

$= \frac{1}{2}\left[\frac{\pi^2}{16}\right]$

$= \frac{\pi^2}{32}$ (A)

Do NOT write in this area.

Q11 (a) $\frac{x^2+1}{x} \geq \frac{5}{2}$, $x \neq 0$

$\frac{x^2+1}{x} = \frac{5}{2}$

$2x^2+2 = 5x$

$2x^2-5x+2=0$

$(2x-1)(x-2)=0$

$x = \frac{1}{2}, 2$ ✓

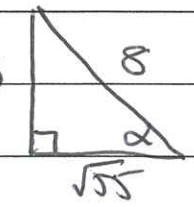


✓ ✓

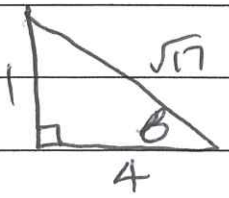
$\therefore 0 < x \leq \frac{1}{2}, \quad x \geq 2$ ✓

(b) $\cos\left[\sin^{-1}\left(\frac{3}{8}\right) - \tan^{-1}\left(\frac{1}{4}\right)\right]$

Let $\alpha = \sin^{-1}\left(\frac{3}{8}\right)$, so $\sin\alpha = \frac{3}{8}$



Let $\beta = \tan^{-1}\left(\frac{1}{4}\right)$, so $\tan\beta = \frac{1}{4}$ ✓



So, $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \frac{\sqrt{55}}{8} \times \frac{4}{\sqrt{17}} + \frac{3}{8} \times \frac{1}{\sqrt{17}}$ ✓

$= \frac{4\sqrt{55}+3}{8\sqrt{17}}$ ✓

Do NOT write in this area.

Q11 cont. (c) $P(x) = ax^4 - x^3 - 28x^2 + bx - 7$

$R(x) = -2x - 3$, $A(x) = x^2 + 3x - 4$

$= (x+4)(x-1)$

So $P(x) = (x+4)(x-1)Q(x) + R(x)$

ie $P(1) = R(1)$

sub $P(1) = a - 1 - 28 + b - 7$, $R(1) = -2 - 3$

$= a + b - 36$ $= -5$

$\therefore a + b - 36 = -5$

$a + b = 31 \ldots$ (1) ✓

Also, $P(-4) = R(-4)$

sub $P(-4) = a(-4)^4 - (-4)^3 - 28(-4)^2 + b(-4) - 7$

$= 256a + 64 - 448 - 4b - 7$

$= 256a - 4b - 391$

sub $R(-4) = -2(-4) - 3$

$= 5$

$\therefore 256a - 4b - 391 = 5$

$256a - 4b = 396$

$64a - b = 99 \ldots$ (2) ✓

Now solve simultaneously (1) & (2)

$64a - b = 99$

$+ \quad a + b = 31$

$65a = 130$

$a = 2$

$\therefore a = 2, b = 29$ ✓

Do NOT write in this area.

Q11 cont. (d)

(i) $\sin x + \sqrt{3}\cos x = R\sin(x+\alpha)$

$r = \sqrt{(\sqrt{3})^2 + 1^2}$

$r = 2$

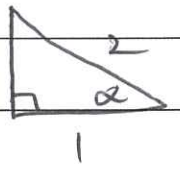
$\therefore \sin x + \sqrt{3}\cos x = 2\sin(x+\alpha)$

$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \sin(x+\alpha)$

$\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x = \sin x\cos\alpha + \sin\alpha\cos x$. ✓

Equating:

$\cos\alpha = \frac{1}{2}$, $\sin\alpha = \frac{\sqrt{3}}{2}$



$\therefore \alpha = 60°$

$\therefore \sin x + \sqrt{3}\cos x = 2\sin(x+60°)$

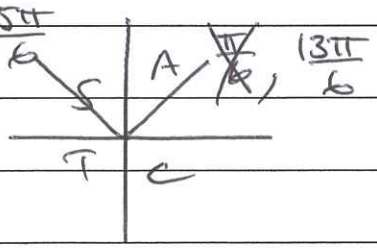
$= 2\sin(x+\frac{\pi}{3})$ ✓

(ii) $\sin x + \sqrt{3}\cos x = 1$, $0 \le x \le 2\pi$

$2\sin(x+\frac{\pi}{3}) = 1$, $\frac{\pi}{3} \le x+\frac{\pi}{3} \le \frac{7\pi}{3}$

$\sin(x+\frac{\pi}{3}) = \frac{1}{2}$ ✓

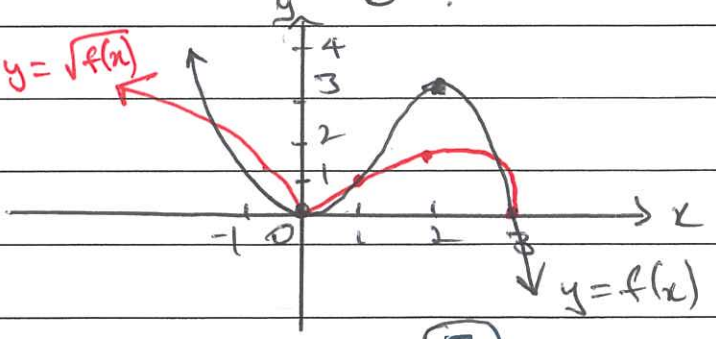
For acute $\angle$, $x+\frac{\pi}{3} = \frac{\pi}{6}$.



$\therefore x+\frac{\pi}{3} = \frac{5\pi}{6}, \frac{13\pi}{6}$

$x = \frac{\pi}{2}, \frac{11\pi}{6}$. ✓

(e) $y = \sqrt{f(x)}$



1 = detail

1 = correct shape.

Do NOT write in this area.

Q12 (a) $\frac{dx}{dt} = -0.8\ mm/sec$

$A = 6x^2$

$\frac{dA}{dx} = 12x$ ✓

So, $\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt}$

$= 12x \times -0.8$

$= -9.6x\ mm^2/sec.$ ✓

when $x = 10$, $\frac{dA}{dt} = -9.6 \times 10 = -96 mm^2/sec.$

∴ when the side length of the cube is 10cm the SA is decreasing at a rate of $96 mm^2/sec.$

b(i) $f(x) = \frac{x+1}{\sqrt{x}}$, min. t.p. at P.

To restrict the domain so the inverse exists, we look for the domain over which $f(x)$ is strictly increasing.

$f(x) = \frac{x+1}{x^{1/2}} = x^{1/2} + x^{-\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

$= \frac{1}{2} \times \frac{1}{\sqrt{x}} - \frac{1}{2} \times \frac{1}{x\sqrt{x}}$

$= \frac{1}{2\sqrt{x}} - \frac{1}{2x\sqrt{x}}$

$= \frac{x-1}{2x\sqrt{x}}$ ✓

PTO

Do NOT write in this area.

Q12(b)(i) cont.

$f'(x)=0$, t.p.

$\frac{x-1}{2x\sqrt{x}}=0$, $2x\sqrt{x}\neq 0$

$x-1=0$

$x=1$

For $x>1$, $f'(x)>0$, so increasing.

$\therefore f(x)$ is strictly increasing for $x\geq 1$

$\therefore a=1$ ✓

(ii) Let $y=f^{-1}(x)$, then $x=f(y)$, where $y\geq 1$

$x=\sqrt{y}+\frac{1}{\sqrt{y}}$

$x^2=\left(\sqrt{y}+\frac{1}{\sqrt{y}}\right)^2$

$x^2=y+2+\frac{1}{y}$

$x^2y=y^2+2y+1$

$y^2-x^2y+2y+1=0.$

$y^2-(x^2+2)y+1=0$ ✓

Now quad. formula!

$y=\frac{(x^2-2)\pm\sqrt{(x^2-2)^2-4(1)(1)}}{2(1)}$

$y=\frac{(x^2-2)\pm\sqrt{x^4-4x^2+4-4}}{2}$

$=\frac{(x^2-2)\pm\sqrt{x^4-4x^2}}{2}$ ✓

since $y\geq 1$, we take the positive case.

$f^{-1}(x)=\frac{x^2-2+\sqrt{x^4-4x^2}}{2}$

$=\frac{x^2-2+\sqrt{x^2(x^2-4)}}{2}$

$=\frac{x^2-2+x\sqrt{x^2-4}}{2}$ ✓

Do NOT write in this area.

Q12(c)

(i) Let the random variable X be the no. of odd cards chosen. Then X is binomial with $n=4, p=\frac{2}{5}, q=\frac{3}{5}$

$P(\text{sum is odd}) = P(X=1)+P(X=3)$

$= {}^4C_1\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^3 + {}^4C_3\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^1$ ✓

$= \left(4 \times \frac{2}{5} \times \frac{27}{125}\right) + \left(4 \times \frac{8}{125} \times \frac{3}{5}\right)$

$= \frac{216}{625} + \frac{96}{625}$

$= \frac{312}{625}$ ✓

(ii) $n=150, p=\frac{2}{5}, q=\frac{3}{5}$

mean $\Rightarrow np = 150 \times \frac{2}{5} = 60$

s.d. $\Rightarrow \sigma = \sqrt{npq} = \sqrt{150 \times \frac{2}{5} \times \frac{3}{5}} = 6$ ✓

$\therefore P(X \geq 49) \doteq P\left(Z \geq \frac{49-60}{6}\right)$ ✓

$= P(Z \geq -1.8\dot{3})$

$= P(Z \leq 1.8\dot{3})$, since symmetrical

$= 0.9664$ ✓

Do NOT write in this area.

Q12(d) $\int_{0.5}^{1.5} \frac{1}{\sqrt{2x-x^2}}\,dx$

$u \geq x-1$

Also $2x-x^2 = 2(u+1)-(u+1)^2$

$\frac{du}{dx} = 1$

$= 2u+2-(u^2+2u+1)$

$du = dx$

$= 2u+2-u^2-2u-1$

$= 1-u^2$

when $x=1.5$, $u=0.5$

when $x=0.5$, $u=-0.5$

So $\int_{0.5}^{1.5} \frac{1}{\sqrt{2x-x^2}}\,dx$

$= \int_{-0.5}^{0.5} \frac{1}{\sqrt{1-u^2}}\,du$ ✓

$= \left[\sin^{-1} u\right]_{-0.5}^{0.5}$ ✓

$= \sin^{-1}(0.5) - (\sin^{-1}(-0.5))$

$= \frac{\pi}{6} - \left(\frac{-\pi}{6}\right)$

$= \frac{\pi}{3}$ ✓

Q13(a) A(1,2) B(6,2)

(i) $\vec{OP} = \text{proj}_{\vec{OB}} \vec{OA} \times \vec{OB}$

$= \frac{1\times 6+2\times 2}{6^2+2^2} \times (6\underset{\sim}{i}+2\underset{\sim}{j})$ ✓

$= \frac{1}{4}(6\underset{\sim}{i}+2\underset{\sim}{j})$

$= \frac{3}{2}\underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}$ ✓

(ii) $\begin{pmatrix}6\\2\end{pmatrix} = k\begin{pmatrix}3/2\\1/2\end{pmatrix}$

$\therefore k=4$ by inspection. ✓

Do NOT write in this area.

Q13(b) $\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

$= \overrightarrow{OA} + \frac{3}{4}\overrightarrow{AB}$ ✓

$= \overrightarrow{OA} + \frac{3}{4}(\overrightarrow{OB} - \overrightarrow{OA})$

$= \overrightarrow{OA} + \frac{3}{4}\overrightarrow{OB} - \frac{3}{4}\overrightarrow{OA}$

$= \frac{1}{4}\overrightarrow{OA} + \frac{3}{4}\overrightarrow{OB}$

$= \frac{1}{4}(8\underset{\sim}{a}) + \frac{3}{4}(4\underset{\sim}{b})$ ✓

$= 2\underset{\sim}{a} + 3\underset{\sim}{b}$ ✓

(c)(i) $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$

LHS: $\tan 4\theta = \tan 2(2\theta)$

$= \dfrac{2\tan(2\theta)}{1 - \tan^2(2\theta)}$

$= \dfrac{2\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)}{1 - \left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)^2}$ ✓

$= \dfrac{4\tan\theta}{1-\tan^2\theta} \times \dfrac{(1-\tan^2\theta)^2}{(1-\tan^2\theta)^2 - 4\tan^2\theta}$

$= \dfrac{4\tan\theta}{1-\tan^2\theta} \times \dfrac{(1-\tan^2\theta)^2}{1 - 2\tan^2\theta + \tan^4\theta - 4\tan^2\theta}$

$= \dfrac{4\tan\theta(1-\tan^2\theta)}{1 - 6\tan^2\theta + \tan^4\theta}$

$= \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$ ✓

= RHS

Do NOT write in this area.

Q13(c)(ii) $x^4 - 4x^3 - 6x^2 + 4x + 1 = 0$

$4x^3 - 4x = x^4 - 6x^2 + 1$

$\frac{4x^3 - 4x}{x^4 - 6x^2 + 1} = 1$

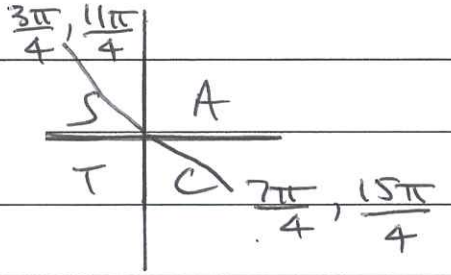
Let $x = \tan\theta$,

$\frac{4\tan^3\theta - 4\tan\theta}{\tan^4\theta - 6\tan^2\theta + 1} = 1$

$-\frac{(4\tan\theta - 4\tan^3\theta)}{\tan^4\theta - 6\tan^2\theta + 1} = 1$

$\frac{4\tan\theta - 4\tan^3\theta}{\tan^4\theta - 6\tan^2\theta + 1} = -1$

$\therefore \tan 4\theta = -1$ ✓



$0 \le \theta \le \pi$

$0 \le 4\theta \le 4\pi$

$\therefore 4\theta = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}$

$\theta = \frac{3\pi}{16}, \frac{7\pi}{16}, \frac{11\pi}{16}, \frac{15\pi}{16}$ ✓

$\therefore x = \tan\frac{3\pi}{16}, \tan\frac{7\pi}{16}, \tan\frac{11\pi}{16}, \tan\frac{15\pi}{16}$ ✓

Do NOT write in this area.

Q13(d) $y = \frac{1}{x^2+1}$, $y^2 = \frac{1}{(x^2+1)^2}$

$x = \tan\theta$

$\frac{dx}{d\theta} = \sec^2\theta$.

when $x=1$, $\theta = \frac{\pi}{4}$

when $x=0$, $\theta = 0$.

$$V = \pi\int_0^1 y^2\,dx$$

$$= \pi\int_0^1 \frac{1}{(x^2+1)^2}\,dx.$$

$$= \pi\int_0^{\frac{\pi}{4}} \frac{\sec^2\theta}{(\tan^2\theta+1)^2}\,d\theta. \checkmark$$

$$= \pi\int_0^{\frac{\pi}{4}} \frac{\sec^2\theta}{(\sec^2\theta)^2}\,d\theta.$$

$$= \pi\int_0^{\frac{\pi}{4}} \frac{\sec^2\theta}{\sec^4\theta}\,d\theta$$

$$= \pi\int_0^{\frac{\pi}{4}} \frac{1}{\sec^2\theta}\,d\theta$$

$$= \pi\int_0^{\frac{\pi}{4}} \cos^2\theta\,d\theta \checkmark$$

$$= \pi\int_0^{\frac{\pi}{4}} \frac{1+\cos 2\theta}{2}\,d\theta.$$

$$= \frac{\pi}{2}\int_0^{\frac{\pi}{4}} (1+\cos 2\theta)\,d\theta$$

$$= \frac{\pi}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/4} \checkmark$$

$$= \frac{\pi}{2}\left[\frac{\pi}{4} + \frac{\sin\frac{\pi}{2}}{2} - 0\right]$$

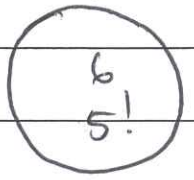
$$= \frac{\pi}{2}\left[\frac{\pi}{4} + \frac{1}{2}\right]$$

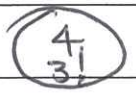
$$= \left(\frac{\pi^2}{8} + \frac{\pi}{4}\right) u^3 \checkmark$$

Do NOT write in this area.

Q14(a)(i), 10 people in total.

6 people can be chosen in $^{10}C_6$ ways





$^{10}C_6 \times 5! = 1$ mark

No. of arrangements $= {}^{10}C_6 \times 5! \times 3!$

$= 151\,200$ ✓

(ii) If couple are around large table:

no. of ways around BOTH tables

$= ({}^{8}C_4 \times 5!) \times 3!$

If couple are around small table:

no. of ways around BOTH tables

$= ({}^{8}C_2 \times 3!) \times 5!$

✓ BOTH

So, probability required:

$$= \frac{{}^{8}C_4 \times 5! \times 3! + {}^{8}C_2 \times 3! \times 5!}{151\,200}$$

$$= \frac{7}{15}$$ ✓

OR

$$\frac{6}{10} \times \frac{5}{9} + \frac{4}{10} \times \frac{3}{9} = \frac{7}{15}$$

Do NOT write in this area.

Q14(b) $\frac{(x+0)!}{0!} + \frac{(x+1)!}{1!} + \ldots + \frac{(x+n-1)!}{(n-1)!} = \frac{(x+n)!}{(x+1)(n-1)!}$, $n \geq 1$

Let $n = 1, 2, 3, \ldots$ be the sequence of propositions.

$$\frac{(x+0)!}{0!} + \frac{(x+1)!}{1!} + \frac{(x+2)!}{2!} + \ldots + \frac{(x+n-1)!}{(n-1)!} = \frac{(x+n)!}{(x+1)(n-1)!}$$

Let $n=1 \Rightarrow$ LHS: $\frac{(x+1-1)!}{(1-1)!} = \frac{(x+0)!}{0!} = x!$

RHS: $\frac{(x+1)!}{(x+1)(1-1)!} = \frac{(x+1)x!}{(x+1)(0!)} = x!$

$\therefore n=1$ is true ✓

Assume $n=k$,

$$\frac{(x+0)!}{0!} + \frac{(x+1)!}{1!} + \frac{(x+2)!}{2!} + \ldots + \frac{(x+k-1)!}{(k-1)!} = \frac{(x+k)!}{(x+1)(k-1)!}$$

Prove $n=k+1$,

$$\frac{(x+0)!}{0!} + \frac{(x+1)!}{1!} + \frac{(x+2)!}{2!} + \ldots + \frac{(x+k-1)!}{(k-1)!} + \frac{(x+(k+1)-1)!}{((k+1)-1)!} = \frac{(x+k+1)!}{(x+1)(k+1-1)!}$$

ie $\frac{(x+0)!}{0!} + \frac{(x+1)!}{1!} + \frac{(x+2)!}{2!} + \ldots + \frac{(x+k)!}{k!} = \frac{(x+k+1)!}{(x+1)k!}$ ✓

LHS: $\frac{(x+0)!}{0!} + \frac{(x+1)!}{1!} + \frac{(x+2)!}{2!} + \ldots + \frac{(x+k-1)!}{(k-1)!} + \frac{(x+k)!}{k!}$

$= \frac{(x+k)!}{(x+1)(k-1)!} + \frac{(x+k)!}{k!}$, from assumption. ✓

$= (x+k)! \left[\frac{1}{(x+1)(k-1)!} + \frac{1}{k(k-1)!} \right]$

Do NOT write in this area.

Q14(b) cont.

$= (x+k)! \left[\frac{k+(x+1)}{(x+1)k!}\right]$

$= \frac{(x+k+1)(x+k)!}{(x+1)k!}$

$= \frac{(x+k+1)!}{(x+1)k!}$ ✓

$=$ RHS

Since statement is true for $n=1$ and true for $n=k$, then it is also true for $n=k+1$.
∴ by MI, the statement is true for $n \geq 1$.

(c) $(1-x^2)^n = \binom{n}{0} + \binom{n}{1}(-x^2) + \binom{n}{2}(-x^2)^2 + \binom{n}{\frac{n}{2}}(-x^2)^{\frac{n}{2}} + \ldots + \binom{n}{n}(-x^2)^n$ ✓

$(1-x^2)^n = (1-x)^n (1+x)^n$

RHS: $(1-x)^n \times (1+x)^n$

$= \left(\binom{n}{0} + \binom{n}{1}(-x) + \binom{n}{2}(-x^2) + \ldots + \binom{n}{n}(-x)^n\right) \times$

$\left(\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{n}x^n\right)$ ✓

x^n terms:

$\binom{n}{0}\times\binom{n}{n}x^n + \binom{n}{1}(-x) \times \binom{n}{n-1}x^{n-1} + \binom{n}{2}(-x)^2 \times \binom{n}{n-2}x^{n-2} + \ldots$

$+ \binom{n}{n}(-x)^n \times \binom{n}{0}$

Do NOT write in this area.

Q14(c) cont.

$= x^n \left(\binom{n}{0}\binom{n}{n} - \binom{n}{1}\binom{n}{n-1} + \binom{n}{2}\binom{n}{n-2} + \ldots + \binom{n}{n}\binom{n}{0} \right)$ ✓

since $\binom{n}{k} = \binom{n}{n-k}$

$\therefore x^n \left(\binom{n}{0}^2 - \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots + \binom{n}{n}^2 \right)$

x^n term in LHS:

$\binom{n}{\frac{n}{2}} (-1)^{\frac{n}{2}} x^n$

Equating x^n coefficients on both sides:

$\binom{n}{0}^2 - \binom{n}{1}^2 + \binom{n}{2}^2 + \ldots + \binom{n}{n}^2 = \binom{n}{\frac{n}{2}} (-1)^{\frac{n}{2}}$ ✓

(d) Using the 30°-60° exact triangle at impacts, $x = \sqrt{3}y$.

$\therefore 40\sqrt{2}t = \sqrt{3}(-5t^2 + 40\sqrt{2}t)$ ✓

$40\sqrt{2}t = -5\sqrt{3}t^2 + 40\sqrt{6}t$

$5\sqrt{3}t^2 + 40\sqrt{2}t - 40\sqrt{6}t = 0$ ✓

$5t(\sqrt{3}t + 8\sqrt{2} - 8\sqrt{6}) = 0$

$t = 0$ or $\sqrt{3}t + 8(\sqrt{2} - \sqrt{6}) = 0$

$\sqrt{3}t = -8(\sqrt{2} - \sqrt{6})$

$t = \dfrac{-8(\sqrt{2} - \sqrt{6})}{\sqrt{3}}$

$\doteq 4.78$ sec. ✓

Do NOT write in this area.

Q14(d) cont.

$\dot{x} = \frac{d}{dt}(40\sqrt{2}t)$

$= 40\sqrt{2}$

$\dot{y} = \frac{d}{dt}(-5t^2 + 40\sqrt{2}t)$

$= -10t + 40\sqrt{2}$

At impact, $\dot{x} = 40\sqrt{2}$, $\dot{y} = -10\left(\frac{8\sqrt{6} - \sqrt{2}}{\sqrt{3}}\right) + 40\sqrt{2}$

$\doteq 8.75 \text{ m/sec}$ (1dp)

Since $\dot{y} > 0$, the golf ball is still rising at impact.

Let the angle to the horizontal at impact be α.

$\tan\alpha = \frac{\dot{y}}{\dot{x}}$

$= \frac{8.75}{40\sqrt{2}}$

$= 8.793976\ldots^\circ$

$\therefore \theta = 30^\circ - 8.79^\circ$

$= 21.2060\ldots^\circ$

$\doteq 21^\circ$ (nearest degree) ✓

$\therefore$ the golfball impacts the hill at an angle of 21°.

Do NOT write in this area.