Fort Street High School

NESA Number:

Name:

2026 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

Teacher	12MAX1	12MAX2	12MAX3	12MAX4	12MAX5	12MAXa
	☐	☐	☐	☐	☐	☐

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- Marks may be deducted for careless or badly arranged work.

Total marks: 70

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt Questions 11 – 15
- Allow about 1 hours and 45 minutes for this section

Write your student number on each answer booklet

Question	Mark
Multiple Choice	/10
Question 11	/11
Question 12	/9
Question 13	/10
Question 14	/11
Question 15	/10
Question 16	/9
Total	/70

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

Question 1.

Given that $\overrightarrow{OA} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$, what is $\overrightarrow{AB}$?

A. $\begin{pmatrix} -2 \\ 10 \end{pmatrix}$

B. $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$

C. $\begin{pmatrix} -6 \\ -4 \end{pmatrix}$

D. $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$

Question 2.

What is the range of the function $f(x) = 3\cos^{-1}\sqrt{x}$?

A. $\left[0, \frac{\pi}{2}\right]$

B. $[0, \pi]$

C. $\left[0, \frac{3\pi}{2}\right]$

D. $[0, 3\pi]$

Question 3.

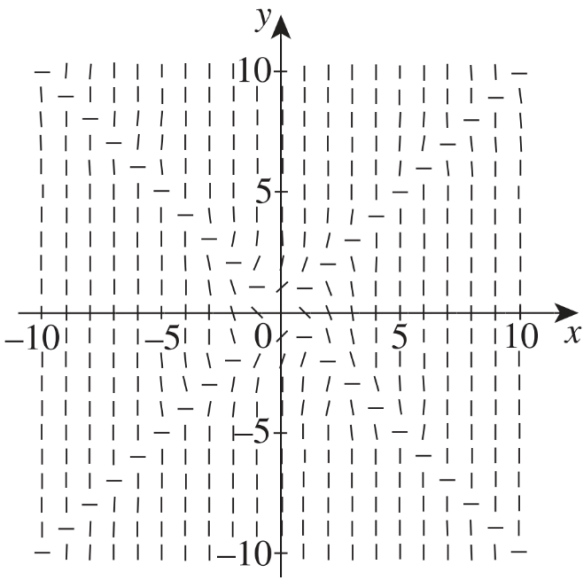
The direction (slope) field for a certain first order differential equation is shown. This differential equation could be:

A. $\frac{dy}{dx} = x^2 y$

B. $\frac{dy}{dx} = \frac{x^2}{y}$

C. $\frac{dy}{dx} = y^2 - x^2$

D. $\frac{dy}{dx} = -\frac{y^2}{x}$



Question 4.

How many years will it take for a bacteria population to reach 8000 if the population is modelled by:

$$P = \frac{10000}{1 + e^{-0.12(t-20)}}$$

A. $t = 30$

B. $t = 31.6$

C. $t = \ln 0.12$

D. $t = \ln 0.25$

Question 5.

Given $X \sim \text{Bin}(n, p)$, $\text{E}(X) = 24$ and $\text{Var}(X) = 18$, find the value of n.

A. 6

B. 24

C. 36

D. 96

Question 6.

Which of the following is the derivative of $\tan^{-1}(e^{2x})$?

A. $\frac{2e^{2x}}{1+e^{4x}}$

B. $\frac{2e^{2x}}{1+e^{2x}}$

C. $\frac{2e^{4x}}{1+e^{4x}}$

D. $\frac{e^{2x}}{1+e^{4x}}$

Question 7.

Fifteen cards are numbered 1 to 15 and placed on a table so that their numbers are hidden. The cards are flipped over one at a time. What is the minimum number of cards that must be flipped to ensure at least one pair with a difference of 8?

A. 8

B. 9

C. 10

D. 11

Question 8.

Which of the following is the solution set of the inequation $\frac{2}{|x-1|} \leq 3$?

A. $x < 1$ and $x \geq \frac{5}{3}$

B. $x \leq \frac{1}{3}$ and $x \geq \frac{5}{3}$

C. $1 < x \leq \frac{5}{3}$

D. $\frac{1}{3} \leq x \leq \frac{5}{3}$

Question 9.

The polynomial equation $x^3 - 2x^2 + 3x - 4 = 0$ has roots α, β and γ.

What is the exact value of $\alpha^3 + \beta^3 + \gamma^3$?

A. 2

B. 8

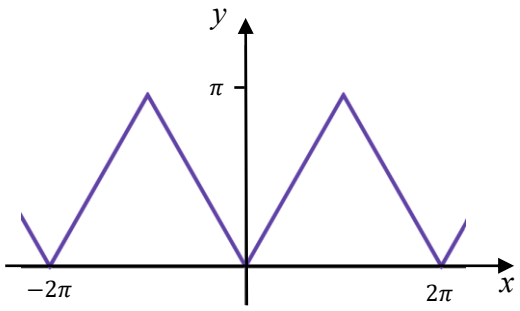
C. 10

D. 14

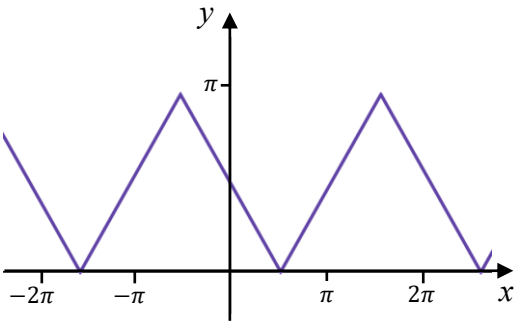
Question 10.

Which graph represents the function $y = \cos^{-1}(\cos x)$?

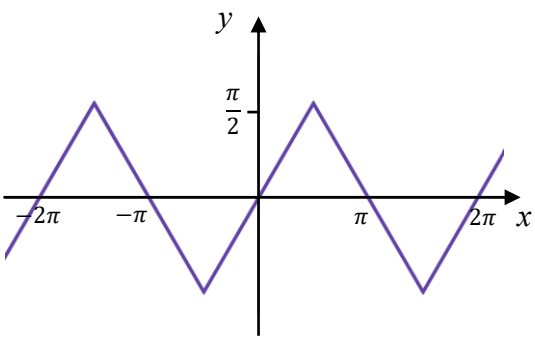
A.



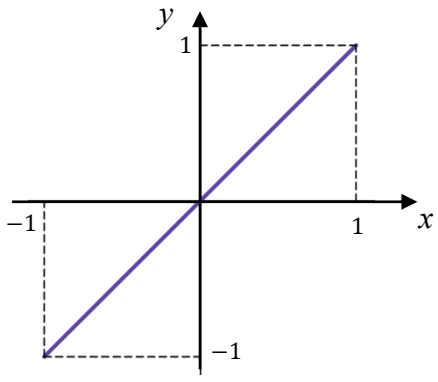
B.



C.



D.



Please turn to the next page for Question 11.

Section II

60 marks

Attempt Questions 11–16

Allow about 1 hour and 45 minutes for this section

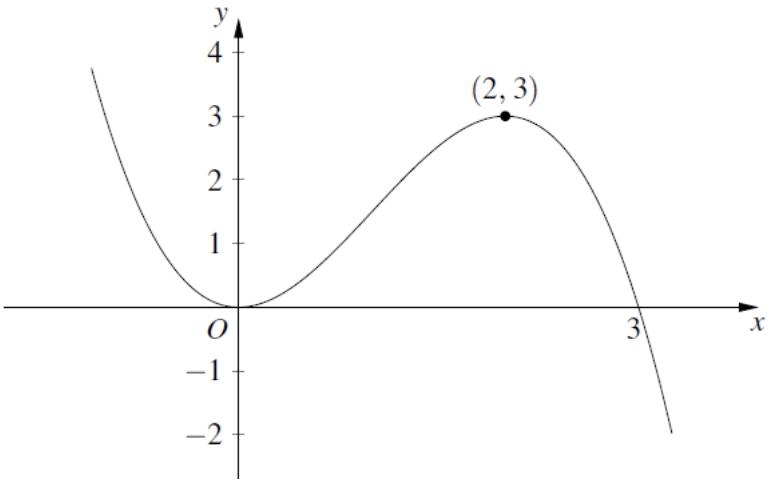
Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations

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Question 11 (11 marks) Use the Question 11 Writing Booklet

(a) The graph of $y = f(x)$ is shown.



On the diagrams provided in the writing booklet, sketch the following graphs:

i) $y = f(|x|)$ **2**

ii) $y = \sqrt{f(x)}$ **2**

(b) Solve $\frac{5}{x-4} \geq x$. **3**

(c) Given that $f(x) = (x-3)^2$ for $x \geq 3$,

i) Find the inverse function and state its domain and range. **2**

ii) Find the intersection point of $f(x)$ and $f^{-1}(x)$ for $x \geq 3$. **2**

Leave your answer in exact form.

End of Question 11

Please turn to the next page for Question 12.

Question 12 (9 marks) Use the Question 12 Writing Booklet

(a) A particle moves so that its velocity at time t is given by

$$\underset{\sim}{v}(t) = -4\sin(4t)\underset{\sim}{i} - 4\cos(4t)\underset{\sim}{j} \text{ for } 0 \leq t \leq \frac{\pi}{2}$$

(i) Given that $\underset{\sim}{r}(0) = -2\underset{\sim}{i} + 3\underset{\sim}{j}$, find the position vector $\underset{\sim}{r}(t)$ of the particle at any time t. **2**

(ii) Hence, find the Cartesian equation of the path followed by the particle. **2**

(b) Express $\sin 2x + 2\cos 2x - 2 = 0$ in the form $R\sin(2x + \alpha) = 2$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Do not solve the equation and leave α in exact form. **2**

(c) Find the particular solution to the differential equation $\sqrt{1 - x^2}\frac{dy}{dx} = 1 + x$ that passes through the point $(0, 1)$. **3**

End of Question 12

Please turn to the next page for Question 13.

Question 13 (10 marks) Use the Question 13 Writing Booklet

(a) Use the substitution $u = \ln x$ to find: **3**

$$\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$$

(b) *You may use the standard normal approximation table provided to answer this question.*

In Australia, 34% of households own cats as pets. A random sample of 500 Australian households is selected. Assuming the sample proportion is approximately normally distributed, find the probability that at least 190 of the sampled households own a pet cat. **3**

(c) A committee of 4 people is to be chosen at random from a group of 5 students, 4 teachers and 3 parents.

Find the probability that the committee will consist of:

i) No students. **2**

ii) At least 2 students, no more than 1 teacher and no more than 1 parent. **2**

End of Question 13

Please turn to the next page for Question 14.

Question 14 (11 marks) Use the Question 14 Writing Booklet

(a) Given the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} + 5\underset{\sim}{j}$:

i) Find the vector projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$. **2**

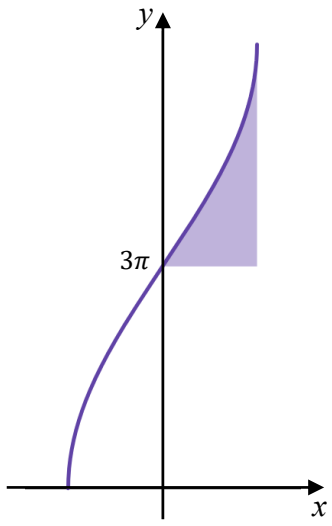
ii) Write the general form for the vector perpendicular to the vector projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$. **1**

(b) It is given that if k is a constant then **3**

$$\left(1 + k\sqrt{3}\right)^4 \equiv 892 - 336\sqrt{3}$$

Find the value of k.

(c) The volume of the solid of revolution is formed by rotating the area bounded by the curve $y = 6\cos^{-1}\left(-\frac{x}{4}\right)$, and the lines $x = 4$ and $y = 3\pi$ about the y axis as shown in the diagram below.



i) Show that $V = 48\pi^2 - 16\pi \int_{3\pi}^{6\pi} \cos^2\left(\frac{y}{6}\right) dy$. **2**

ii) Hence, find the volume of the solid. **3**

End of Question 14

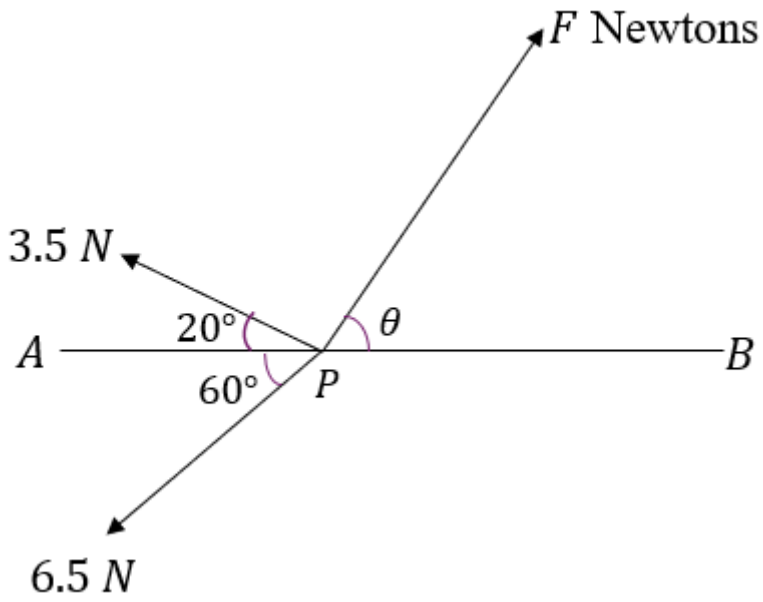
Please turn to the next page for Question 15.

Question 15 (10 marks) Use the Question 15 Writing Booklet

(a) Prove by mathematical induction that $3^n + 19^n$ is a multiple of 11, **3**

for any odd integer $n \geq 1$.

(b) Three light inextensible strings are tied to an object, P, and being pulled with forces $3.5\ N$, $6.5\ N$ and F Newtons on P. The third force, F, makes an angle θ with the horizontal line AB as shown in the diagram below.



Given that these three forces are in equilibrium, find the values of F and θ. **3**

Round your answers to 2 decimal places.

Question 15 continues on Page 11.

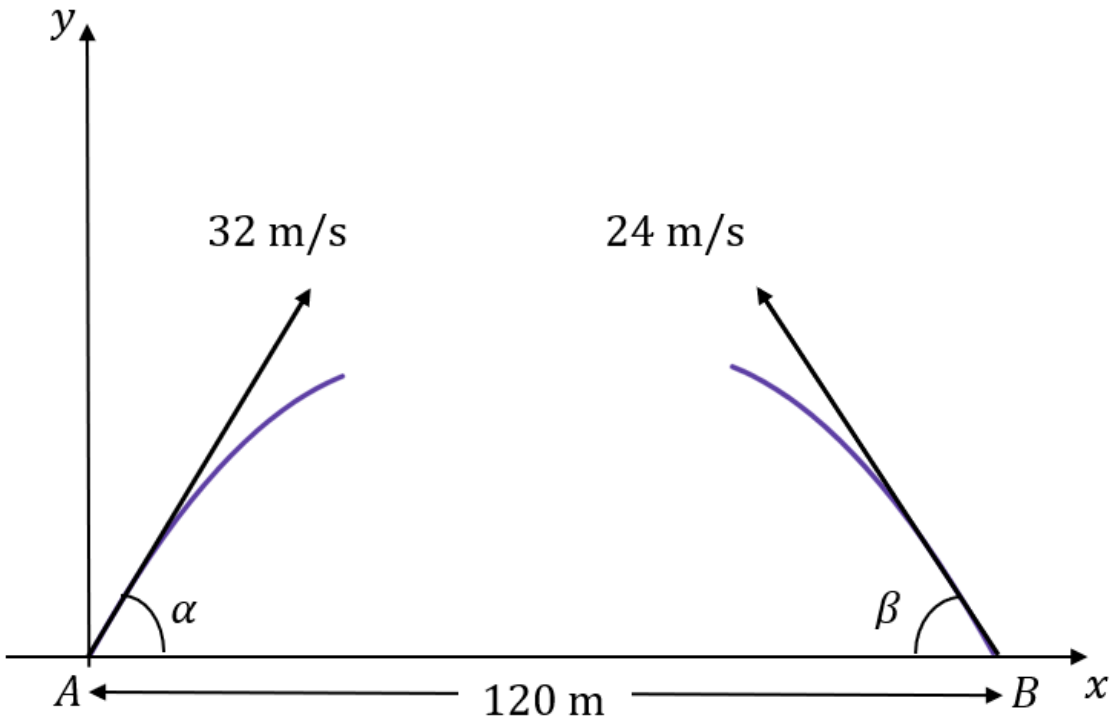
(c) Two particles, P and Q, are fired simultaneously from points A and B respectively which are located 120 metres apart on horizontal ground as shown in the diagram below. The particle P has an initial speed of 32 m/s and an angle of projection α, where $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$. The particle Q has an initial speed of 24 m/s with an angle of projection β where $\beta = \tan^{-1}\left(\frac{4}{3}\right)$.

Assume $g = 9.8\ \text{ms}^{-2}$ where g is the acceleration due to gravity.

The vector equation for the velocity of a projected particle M is given below:

$$\underset{\sim}{V_M} = U\cos\theta\ \underset{\sim}{i} + (U\sin\theta - gt)\underset{\sim}{j}$$

where U is the initial velocity and θ is the angle of projection. Do NOT prove this.



Find when and where particles P and Q collide, using mathematical calculations. **4**

End of Question 15

Please turn to the next page for Question 16.

Question 16 (9 marks) Use the Question 16 Writing Booklet

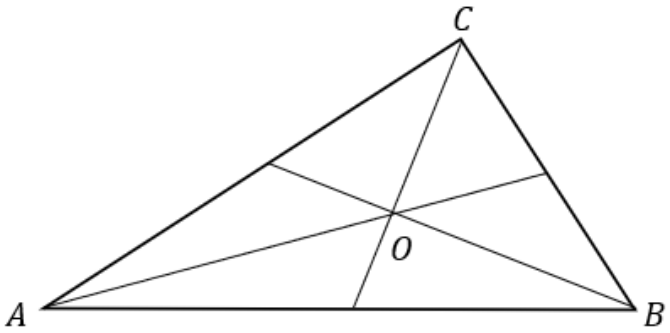
(a) Liquid is poured into a container at a constant rate of 3π cm^{3}s^{-1}.

The container initially contains some liquid where the height of the liquid in the container is 2 cm and the volume of the liquid, V cm^3, is given by:

$$V = \pi(h^2 + 4h - 2)$$

Find the exact rate at which the height of the liquid in the container is rising 1 minute after the liquid started pouring into the container. **4**

(b) O is the point inside ΔABC. Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{OC} = \underset{\sim}{c}$ and given that $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} = 0$, and $\left|\underset{\sim}{a}\right| = 6$, $\left|\underset{\sim}{b}\right| = 3$, $\left|\underset{\sim}{c}\right| = 4$:



(i) Show that $\left|\underset{\sim}{a}\right|^2 + \left|\underset{\sim}{b}\right|^2 + 2\underset{\sim}{a} \cdot \underset{\sim}{b} = \left|\underset{\sim}{c}\right|^2$ **2**

(ii) Hence, find the area of ΔAOB. **3**

End of paper

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Mathematics Extension 1

Solutions

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Given that $\overrightarrow{OA} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$, what is $\overrightarrow{AB}$?

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B. $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$

C. $\begin{pmatrix} -6 \\ -4 \end{pmatrix}$

D. $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$

Question 2.

What is the range of the function $f(x) = 3\cos^{-1}\sqrt{x}$?

A. $\left[0, \frac{\pi}{2}\right]$

B. $[0, \ \pi]$

C. $\left[0, \frac{3\pi}{2}\right]$

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Question 3.

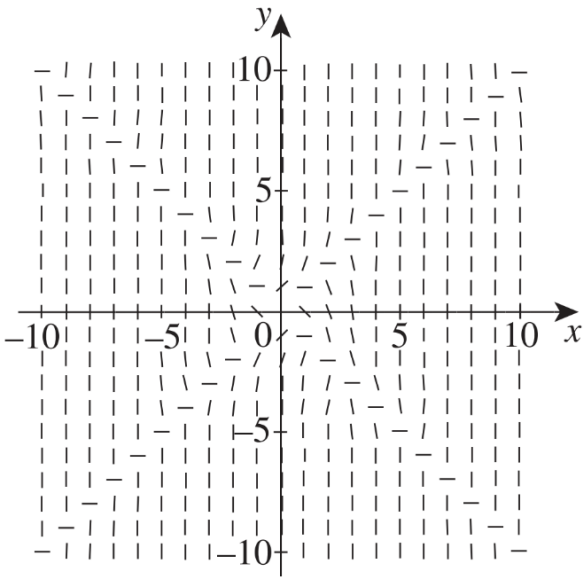
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C. $\frac{dy}{dx} = y^2 - x^2$

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Question 4.

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Question 5.

Given $X \sim \text{Bin}(n, p)$, $\text{E}(X) = 24$ and $\text{Var}(X) = 18$, find the value of n.

A. 6

B. 24

C. 36

D. 96

Question 6.

Which of the following is the derivative of $\tan^{-1}(e^{2x})$?

A. $\frac{2e^{2x}}{1+e^{4x}}$

B. $\frac{2e^{2x}}{1+e^{2x}}$

C. $\frac{2e^{4x}}{1+e^{4x}}$

D. $\frac{e^{2x}}{1+e^{4x}}$

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Fifteen cards are numbered 1 to 15 and placed on a table so that their numbers are hidden. The cards are flipped over one at a time. What is the minimum number of cards that must be flipped to ensure at least one pair with a difference of 8?

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B. 9

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Question 9.

The polynomial equation $x^3 - 2x^2 + 3x - 4 = 0$ has roots α, β and γ.

What is the exact value of $\alpha^3 + \beta^3 + \gamma^3$?

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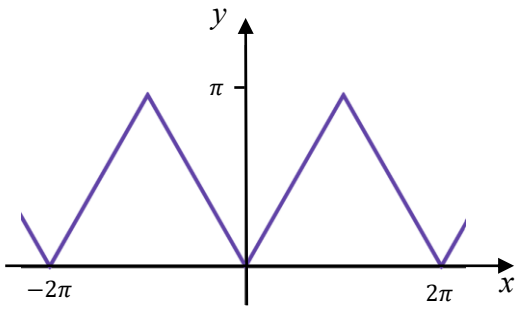
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D. 14

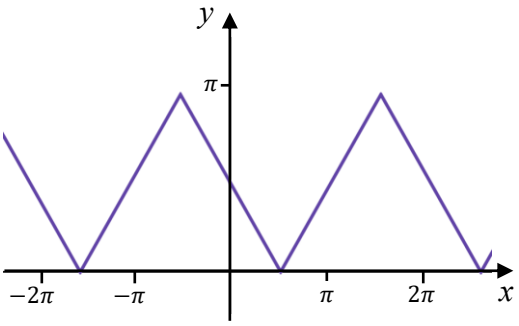
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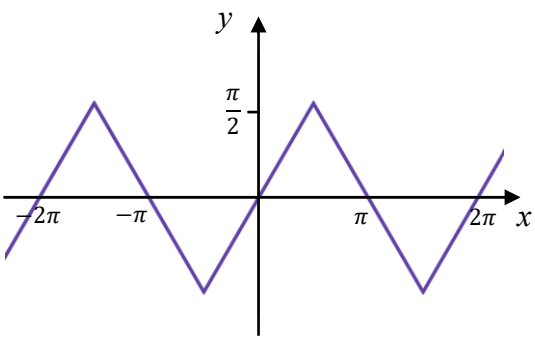
A.



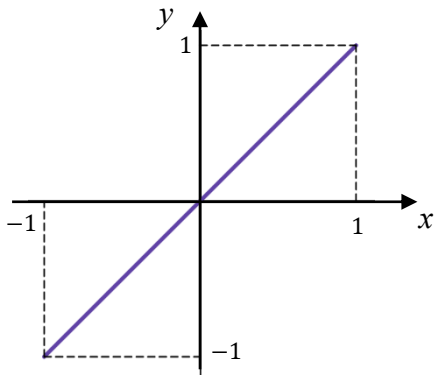
B.



C.



D.



Please turn to the next page for Question 11.

Section II

60 marks

Attempt Questions 11–16

Allow about 1 hour and 45 minutes for this section

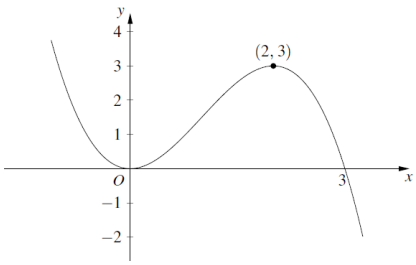
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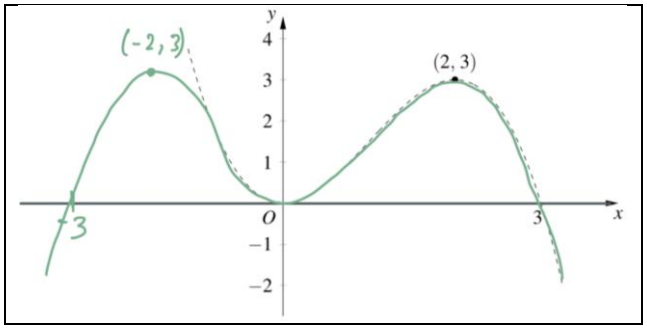
Question 11 (11 marks) Use the Question 11 Writing Booklet

(a) The graph of $y = f(x)$ is shown.



On the diagrams provided in the writing booklet, sketch the following graphs:

i) $y = f(|x|)$ **2**

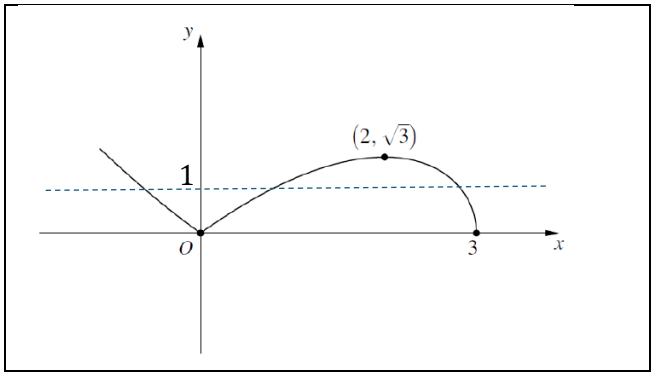


Marking guidelines:

1 mark – COMPLETE general shape

1 mark – Essential features **including** intercepts and points

ii) $y = \sqrt{f(x)}$ **2**



Marking guidelines:

1 mark – COMPLETE general shape

1 mark – Essential features **including** intercepts and points

(b) Solve $\frac{5}{x-4} \geq x$. **3**

$\frac{5}{x-4} \geq x$

$\frac{5(x-4)^2}{(x-4)} \geq x(x-4)^2$

$5(x-4) \geq x(x-4)^2$

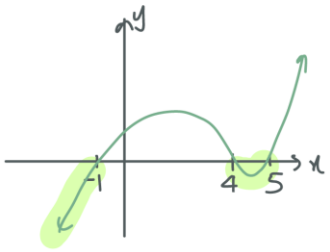
$x(x-4)^2 - 5(x-4) \leq 0$

$(x-4)[x(x-4)-5] \leq 0$

$(x-4)(x^2-4x-5) \leq 0$

$(x-4)(x-5)(x+1) \leq 0$

$\therefore x \leq -1, \quad 4 < x \leq 5$



Marking guidelines:

1 mark – Multiplying the square of the denominator on both sides.

1 mark – Simplifying the inequality correctly.

1 mark – Correct answer.

(c) Given that $f(x) = (x-3)^2$ for $x \geq 3$,

i) Find the inverse function and state its domain and range. **2**

ii) Find the intersection point of $f(x)$ and $f^{-1}(x)$ for $x \geq 3$. **2**

Leave your answer in exact form.

ci) Let $y = (x-3)^2$

$x = (y-3)^2$

$y - 3 = \pm\sqrt{x}$

$y = 3 + \sqrt{x}$ for $x \geq 0$

$\therefore$ D: $x \in [0, \infty)$

R: $y \in [3, \infty)$

Marking guidelines:

1 mark – Correctly states the inverse function.

1 mark – Correctly states the domain **and** range.

cii) $(x-3)^2 = x$

$x^2 - 6x + 9 = x$

$x^2 - 7x + 9 = 0$

$x = \frac{7 \pm \sqrt{(-7)^2 - 4(9)(1)}}{2(1)}$

$= \frac{7 \pm \sqrt{49-36}}{2}$

$= \frac{7 \pm \sqrt{13}}{2}$

$\because x \geq 3$

$\therefore x = \frac{7+\sqrt{13}}{2}$

$\therefore$ Point of intersection is $\left(\frac{7+\sqrt{13}}{2}, \frac{7+\sqrt{13}}{2}\right)$

Marking guidelines:

1 mark – Equates $f(x)$ with x **AND** solves using the quadratic formula.

1 mark – Correctly states the point of intersection given the restricted domain.

Question 12 (9 marks) Use the Question 12 Writing Booklet

(a) A particle moves so that its velocity at time t is given by

$$\underset{\sim}{v}(t) = -4\sin(4t)\underset{\sim}{i} - 4\cos(4t)\underset{\sim}{j} \text{ for } 0 \leq t \leq \frac{\pi}{2}$$

(i) Given that $\underset{\sim}{r}(0) = -2\underset{\sim}{i} + 3\underset{\sim}{j}$, find the position vector $\underset{\sim}{r}(t)$ of the particle at any time t. **2**

ai) $\underset{\sim}{v}(t) = -4\sin(4t)\underset{\sim}{i} - 4\cos(4t)\underset{\sim}{j}$ for $0 \leq t \leq \frac{\pi}{2}$

$\underset{\sim}{r}(t) = \cos(4t)\underset{\sim}{i} - \sin(4t)\underset{\sim}{j} + \underset{\sim}{c}$

Given $\underset{\sim}{r}(0) = -2\underset{\sim}{i} + 3\underset{\sim}{j}$

$-2\underset{\sim}{i} + 3\underset{\sim}{j} = \cos(4\times0)\underset{\sim}{i} - \sin(4\times0)\underset{\sim}{j} + \underset{\sim}{c}$

$-2\underset{\sim}{i} + 3\underset{\sim}{j} = \underset{\sim}{i} - 0 + \underset{\sim}{c}$

$\underset{\sim}{c} = -3\underset{\sim}{i} + 3\underset{\sim}{j}$

Sub $\underset{\sim}{c}$ into $\underset{\sim}{r}(t)$

$\therefore \underset{\sim}{r}(t) = (-3 + \cos(4t))\underset{\sim}{i} + (3 - \sin(4t))\underset{\sim}{j}$

Marking guidelines:

1 mark – Correctly integrates to find $\underset{\sim}{r}(t)$.

1 mark – Correctly finds $\underset{\sim}{c}$ using initial conditions and substituting into $\underset{\sim}{r}(t)$.

(ii) Hence, find the Cartesian equation of the path followed by the particle. **2**

aii) $x = -3 + \cos(4t)$ $\quad y = 3 - \sin(4t)$

$\cos(4t) = x + 3$ (1) $\quad \sin(4t) = 3 - y$ (2)

(1)^2 + (2)^2

$\cos^2(4t) + \sin^2(4t) = (x+3)^2 + (3-y)^2$

$(x+3)^2 + (3-y)^2 = 1$ (Pythagorean Trig. Identity)

OR $(x+3)^2 + (y-3)^2 = 1$

Marking guidelines:

1 mark – Correctly equates x and y components and uses Pythagorean Trig Identity.

1 mark – Correct equation

(b) Express $\sin 2x + 2\cos 2x - 2 = 0$ in the form $R\sin(2x + \alpha) = 2$, **2**

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. Do not solve the equation and leave α in exact form.

b) $\sin 2x + 2\cos 2x = 2$

$R\sin(2x+\alpha) = R(\sin 2x\cos\alpha + \sin\alpha\cos 2x)$

$= R\sin 2x\cos\alpha + R\sin\alpha\cos 2x$

Equating:

$R\cos\alpha = 1$ $\qquad$ $R\sin\alpha = 2$

$R^2\cos^2\alpha = 1^2$ (1) $\qquad$ $R^2\sin^2\alpha = 2^2$ (2)

$(1)^2 + (2)^2$

$R^2\cos^2\alpha + R^2\sin^2\alpha = 1 + 4$

$R^2(\cos^2\alpha + \sin^2\alpha) = 5$

$R^2 = 5$

$R = \sqrt{5}$ since $R > 0$

Now (2) ÷ (1)

$\frac{R\sin\alpha}{R\cos\alpha} = 2$

$\tan\alpha = 2$

$\alpha = \tan^{-1}2$

$\therefore \sin 2x + 2\cos 2x = \sqrt{5}\sin[2x + (\tan^{-1}2)]$

$\therefore \sqrt{5}\sin(2x + \tan^{-1}2) = 2$

Marking guidelines:

1 mark – Significant progress towards finding R and α.

1 mark – Correctly finds R and α and writes in the form $R\sin(2x + \alpha) = 2$

(c) Find the particular solution to the differential equation $\sqrt{1-x^2}\frac{dy}{dx} = 1 + x$ that passes through the point $(0, 1)$. **3**

c) $\sqrt{1-x^2}\frac{dy}{dx} = 1+x \qquad x \neq \pm 1$

$\frac{dy}{dx} = \frac{1+x}{\sqrt{1-x^2}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}$

$= \frac{1}{\sqrt{1-x^2}} + x(1-x^2)^{-\frac{1}{2}}$

$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{2}(-2x)(1-x^2)^{-\frac{1}{2}}$

$y = \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2}\int(-2x)(1-x^2)^{-\frac{1}{2}}dx$

$= \sin^{-1}x - \frac{1}{2}\times 2(1-x^2)^{\frac{1}{2}} + C$

At $(0, 1)$,

$1 = \sin^{-1}0 - (1-0^2)^{\frac{1}{2}} + C$

$C = 2$

$\therefore y = \sin^{-1}x - \sqrt{1-x^2} + 2$

Marking guidelines:

1 mark – Correctly rearranges the differential equation to make $\frac{dy}{dx}$ the subject AND simplifies.

1 mark – Significant progress made in integrating for y.

1 mark – Substitutes the point $(0, 1)$ to find the complete equation for y.

Question 13 (10 marks) Use the Question 13 Writing Booklet

(a) Use the substitution $u = \ln x$ to find: **3**

$$\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$$

a) $\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$ Given $u = \ln x$

$du = \frac{1}{x}\,dx$

$= \int \frac{1}{\sqrt{1-u^2}}\,du$

$= \sin^{-1} u + C$

$= \sin^{-1}(\ln x) + C$ where $0 \leq \ln x \leq 1$

$1 \leq x \leq e$

Marking guidelines:

1 mark – Correctly finds $\frac{du}{dx}$ AND rewrite the integral in terms of u.

1 mark – Integrates correctly.

1 mark – Writes answer in terms of x

(b) *You may use the standard normal approximation table provided to answer this question.*

In Australia, 34% of households own cats as pets. A random sample of 500 Australian households is selected. Assuming the sample proportion is approximately normally distributed, find the probability that at least 190 of the sampled households own a pet cat. **3**

b) $E(\hat{p}) = p = 0.34$ $n = 500$ $\hat{p} = \frac{190}{500} = 0.38$

$\sigma^2 = \frac{p(1-p)}{n}$

$= \frac{0.34 \times 0.66}{500}$

$= 0.0004488$

$P(\hat{p} \geq 0.38) = P\left(z \geq \frac{0.38-0.34}{\sqrt{0.0004488}}\right)$

$= P(z \geq 1.89)$

$= 1 - P(z < 1.89)$

$= 1 - 0.97062$ (from table)

$= 0.02938$

$\therefore 2.938\%$

Marking guidelines:

1 mark – Correctly finds $E(\hat{p})$ and $\sigma_{\hat{p}}$.

1 mark – Significant progress when calculating the z score.

1 mark – Uses table to find the final answer.

(c) A committee of 4 people is to be chosen at random from a group of 5 students, 4 teachers and 3 parents.

Find the probability that the committee will consist of:

i) No students. **2**

ci) Total $= {}^{12}C_4$
$= 495$

Committee without students $= {}^{7}C_4$
$= 35$

$\therefore P(\text{no students}) = \frac{35}{495}$
$= \frac{7}{99}$

Marking guidelines:

1 mark – Significant progress when calculating the probability.

1 mark – Correct simplified answer.

ii) At least 2 students, no more than 1 teacher and no more than 1 parent. **2**

cii)

Students	Teachers	Parent	
4	0	0	${}^5C_4 \times {}^4C_0 \times {}^3C_0 = 5$
3	1	0	${}^5C_3 \times {}^4C_1 \times {}^3C_0 = 40$
3	0	1	${}^5C_3 \times {}^4C_0 \times {}^3C_1 = 30$
2	1	1	${}^5C_2 \times {}^4C_1 \times {}^3C_1 = 120$
			Total $= 195$

$\therefore P(\text{at least 2 students, no more than 1 teacher, parent}) = \frac{195}{495}$
$= \frac{13}{33}$

Marking guidelines:

1 mark – Significant progress when calculating the probability.

1 mark – Correct simplified answer.

Question 14 (11 marks) Use the Question 14 Writing Booklet

(a) Given the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j}$ and $\underset{\sim}{b} = -2\underset{\sim}{i} + 5\underset{\sim}{j}$:

i) Find the vector projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$. **2**

ai) $\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b} = \dfrac{\underset{\sim}{b}\cdot\underset{\sim}{a}}{|\underset{\sim}{a}|^2}\cdot\underset{\sim}{a}$

$= \left[(-2\underset{\sim}{i}+5\underset{\sim}{j})\cdot\dfrac{(2\underset{\sim}{i}-3\underset{\sim}{j})}{\sqrt{2^2+(-3)^2}}\right]\cdot\dfrac{(2\underset{\sim}{i}-3\underset{\sim}{j})}{\sqrt{2^2+(-3)^2}}$

$= \left[\dfrac{1}{\sqrt{13}}(-2\underset{\sim}{i}+5\underset{\sim}{j})\cdot(2\underset{\sim}{i}-3\underset{\sim}{j})\right]\cdot\left[\dfrac{1}{\sqrt{13}}(2\underset{\sim}{i}-3\underset{\sim}{j})\right]$

$= \dfrac{1}{13}\left[(-2)(2)+(5)(-3)\right]\cdot(2\underset{\sim}{i}-3\underset{\sim}{j})$

$= -\dfrac{19}{13}(2\underset{\sim}{i}-3\underset{\sim}{j})$

Marking guidelines:

1 mark – Attempt to use the vector projection formula by correctly using the dot product or equivalent merit.

2 mark – Correct simplified answer.

ii) Write the general form for the vector perpendicular to the vector projection of $\underset{\sim}{b}$ onto $\underset{\sim}{a}$. **1**

aii) $\therefore$ Perpendicular vector $= \lambda(3\underset{\sim}{i}+2\underset{\sim}{j})$ where $\lambda \in \mathbb{R}$

Marking guidelines:

1 mark – Correct vector perpendicular to the vector a.

(b) It is given that if k is a constant then **3**

$$\left(1 + k\sqrt{3}\right)^4 \equiv 892 - 336\sqrt{3}$$

Find the value of k.

b) $(1+k\sqrt{3})^4 \equiv 892 - 336\sqrt{3}$

$1 + 4k\sqrt{3} + 6\times 3k^2 + 4\times 3\sqrt{3}k^3 + 9k^4 = 892 - 336\sqrt{3}$

$1 + 18k^2 + 9k^4 = 892$ and $4k + 12k^3 = -336$ (1)

$9k^4 + 18k^2 - 891 = 0$

$k^4 + 2k^2 - 99 = 0$

$(k^2+11)(k^2-9) = 0$

$k = \pm 3 \quad (k^2 \neq -11)$

Now sub $k=3$ into (1)

$4\times 3 + 12\times 3^3 = 12 + 12\times 27$

$= 336$

$\neq -336$

Sub $k=-3$

$4\times(-3) + 12\times(-3)^3 = -12 - 324$

$= -336$

$\therefore k = -3$

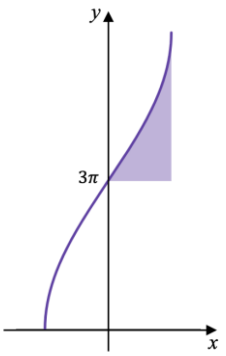
Marking guidelines:

1 mark – Correct expansion of the binomial OR correctly stated one of the two polynomial equations in k.

2 marks – correctly stated both polynomial equations in k, correctly factorised the quartic polynomials reducible to quadratic or correctly solved for the value of k^2.

3 marks – Correct solutions.

(c) The volume of the solid of revolution is formed by rotating the area bounded by the curve $y = 6\cos^{-1}\left(-\frac{x}{4}\right)$, and the lines $x = 4$ and $y = 3\pi$ about the y axis as shown in the diagram below.



i) Show that $V = 48\pi^2 - 16\pi \int_{3\pi}^{6\pi} \cos^2\left(\frac{y}{6}\right) dy$. **2**

ci) $V = \pi r^2 h - \pi \int_a^b y^2 \, dy$

$x=0, \; y = 3\pi$
$x=4, \; y = 6\pi$

Given $y = 6\cos^{-1}\left(-\frac{x}{4}\right)$

$\frac{y}{6} = \cos^{-1}\left(-\frac{x}{4}\right)$

$x = -4\cos\left(\frac{y}{6}\right)$

$V = \pi \times 4^2 \times 3\pi - \pi \int_{3\pi}^{6\pi} \left[-4\cos\left(\frac{y}{6}\right)\right]^2 dy$

$= 48\pi^2 - \pi \int_{3\pi}^{6\pi} 16\cos^2\left(\frac{y}{6}\right) dy$

$\therefore V = 48\pi^2 - 16\pi \int_{3\pi}^{6\pi} \cos^2\left(\frac{y}{6}\right) dy$

Marking guidelines:

1 mark – correctly expressed x in terms of y OR use the correct formula to calculate the volume of the cylinder and attempted to use the volume of the solid of revolution rotated about the y-axis.

2 marks – correct solution including justifying how to calculate the upper limit of 6π of the definite integral.

ii) Hence, find the volume of the solid. **3**

cii) $V = 48\pi^2 - 16\pi \int_{3\pi}^{6\pi} \frac{1}{2}\left(1+\cos\left(\frac{y}{3}\right)\right) dy$

$= 48\pi^2 - 8\pi \left[y + 3\sin\left(\frac{y}{3}\right)\right]_{3\pi}^{6\pi}$

$= 48\pi^2 - 8\pi \left[\left(6\pi + 3\sin\left(\frac{6\pi}{3}\right)\right) - \left(3\pi + 3\sin\left(\frac{3\pi}{3}\right)\right)\right]$

$= 48\pi^2 - 8\pi \left[6\pi + 3\times 0 - 3\pi - 3\times 0\right]$

$= 48\pi^2 - 8\pi \times 3\pi$

$= 24\pi^2$ units^3

Marking guidelines:

1 mark – Correctly used the double angle formula to simplify the integrand OR found the correct antiderivative after attempting to use the double angle formula

2 marks – Correctly used the double angle formula AND found the correct antiderivative OR correctly found $24\pi^2$ by either correct use of the double angle formula or correct antiderivative.

3 marks – Correct solution

Question 15 (10 marks) Use the Question 15 Writing Booklet

(a) Prove by mathematical induction that $3^n + 19^n$ is a multiple of 11, for any odd integer $n \geq 1$. **3**

a) Step 1: Prove for $n=1$

$3^n + 19^n = 3 + 19$
$= 22$
$= 2 \times 11$ $\therefore$ Divisible by 11

Step 2: Assume true for $n=k$, k is an odd integer

i.e. $3^k + 19^k = 11M$ where M is an integer
$3^k = 11M - 19^k$

Step 3: Prove true for $n=k+2$

i.e. $3^{k+2} + 19^{k+2} = 11N$ where N is an integer

$LHS = 3^{k+2} + 19^{k+2}$
$= 3^2 \cdot 3^k + 19^2 \cdot 19^k$
$= 9(11M - 19^k) + 19^2 \cdot 19^k$
$= 99M - 9 \cdot 19^k + 361 \cdot 19^k$
$= 99M + 352 \cdot 19^k$
$= 11(9M + 32 \cdot 19^k)$
$= 11N$ where $N = 9M + 32 \cdot 19^k$ since M & k are integers

$\therefore$ By mathematical induction, since $3^k + 19^k$ is true for $n=1$, then true for all odd integers $n \geq 1$.

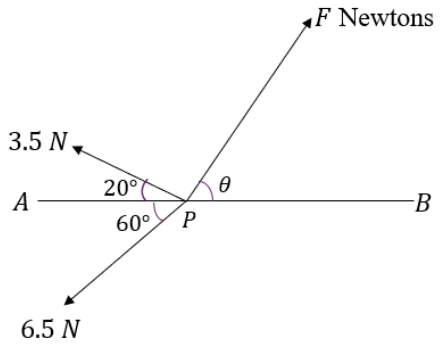
Marking guidelines:

1 mark – Proof of base case.

1 mark – Assumption step and setup of inductive step.

1 mark – Complete proof with concluding statement.

(b) Three light inextensible strings are tied to an object, P, and being pulled with forces $3.5\ N$, $6.5\ N$ and F Newtons on P. The third force, F, makes an angle θ with the horizontal line AB as shown in the diagram below.



Given that these three forces are in equilibrium, find the values of F and θ. **3**

Round your answers to 2 decimal places.

b) Horizontal

$3.5\cos 20 + 6.5\cos 60 = F\cos\theta$

$F\cos\theta = 3.25 + 3.5\cos 20$ (1)

Vertical

$3.5\sin 20 + F\sin\theta = 6.5\sin 60$

$F\sin\theta = 6.5\sin 60 - 3.5\sin 20$

$F\sin\theta = \frac{13\sqrt{3}}{4} - 3.5\sin 20$ (2)

(2)^2 + (1)^2

$F^2\cos^2\theta + F^2\sin^2\theta = (3.25 + 3.5\cos 20)^2 + \left(\frac{13\sqrt{3}}{4} - 3.5\sin 20\right)^2$

$F^2 = 62.4009\ldots$

$F = 7.8994\ldots$

≈ 7.90

Sub F into (1)

$\theta = \cos^{-1}\left(\frac{3.25 + 3.5\cos 20}{F}\right)$

$= 34.1294\ldots$

$\approx 34.13^\circ$

$\therefore F \approx 7.90$ and $\theta = 34.13^\circ$

Marking guidelines:

1 mark – Correctly finds the horizontal and vertical component.

1 mark – Significant progress towards solving F and θ.

1 mark – Correctly finds F and θ.

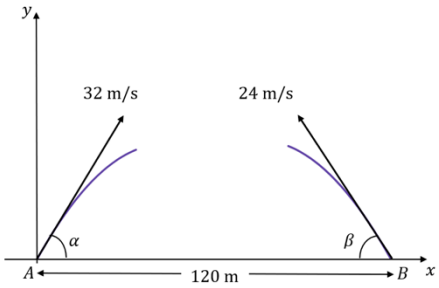
(c) Two particles, P and Q, are fired simultaneously from points A and B respectively which are located 120 metres apart on horizontal ground as shown in the diagram below. The particle P has an initial speed of 32 m/s and an angle of projection α, where $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$. The particle Q has an initial speed of 24 m/s with an angle of projection β where $\beta = \tan^{-1}\left(\frac{4}{3}\right)$.

Assume $g = 9.8\ \text{ms}^{-2}$ where g is the acceleration due to gravity.

The vector equation for the velocity of a projected particle M is given below:

$$\underset{\sim}{V_M} = U\cos\theta\ \underset{\sim}{i} + (U\sin\theta - gt)\underset{\sim}{j}$$

where U is the initial velocity and θ is the angle of projection. Do NOT prove this.



Find when and where particles P and Q collide, using mathematical calculations. **4**

c)

Particle P

$\underset{\sim}{V_P} = 32\cos\alpha\ \underset{\sim}{i} + (32\sin\alpha - 9.8t)\underset{\sim}{j}$

$= \left(32\times\frac{4}{5}\right)\underset{\sim}{i} + \left(32\times\frac{3}{5} - 9.8t\right)\underset{\sim}{j}$

$= \frac{128}{5}\underset{\sim}{i} + \left(\frac{96}{5} - 9.8t\right)\underset{\sim}{j}$

$\underset{\sim}{r_P} = \frac{128t}{5}\underset{\sim}{i} + \left(\frac{96t}{5} - \frac{9.8t^2}{2}\right)\underset{\sim}{j} + \underset{\sim}{c}$

$\therefore \underset{\sim}{r_P} = \frac{128t}{5}\underset{\sim}{i} + \left(\frac{96t}{5} - 4.9t^2\right)\underset{\sim}{j}$

Particle Q

$\underset{\sim}{V_Q} = 24\cos\beta + (24\sin\beta - 9.8t)\underset{\sim}{j}$

$= 24\left(-\frac{3}{5}\right)\underset{\sim}{i} + \left[24\times\frac{4}{5} - 9.8t\right]\underset{\sim}{j}$

$= -\frac{72}{5}\underset{\sim}{i} + \left(\frac{96}{5} - 9.8t\right)\underset{\sim}{j}$

$\underset{\sim}{r_Q} = -\frac{72t}{5}\underset{\sim}{i} + \left(\frac{96t}{5} - \frac{9.8t^2}{2}\right)\underset{\sim}{j} + \underset{\sim}{c}$

At $t = 0$

$\underset{\sim}{r_Q} = \left(-\frac{72t}{5}\underset{\sim}{i} + 120\right)\underset{\sim}{i} + \left(\frac{96t}{5} - 4.9t^2\right)\underset{\sim}{j}$

Equating components

$\frac{128t}{5} = -\frac{72t}{5} + 120$

$\frac{200t}{5} = 120$

$t = 3$

At $t = 3$

$x = \frac{128t}{5}$

$= 76.8$

$y = \frac{96t}{5} - 4.9t^2$

$= \frac{96\times3}{5} - 4.9\times(3)^2$

$= 13.5$

∴ P and Q will collide when $t = 3$ seconds. The collision will be 76.8m away from P at a height of 13.5m.

Marking guidelines:

1 mark – Correctly expresses velocity for both P and Q

1 mark – Significant progress towards finding displacement for P and Q.

1 mark – Correctly equates components AND makes significant progress towards finding x and y.

1 mark – Correctly finds when and where P and Q will collide.

Question 16 (9 marks) Use the Question 16 Writing Booklet

(a) Liquid is poured into a container at a constant rate of 3π cm^{3}s^{-1}.

The container initially contains some liquid where the height of the liquid in the container is 2 cm and the volume of the liquid, V cm^3, is given by:

$$V = \pi(h^2 + 4h - 2)$$

Find the exact rate at which the height of the liquid in the container is rising 1 minute after the liquid started pouring into the container. **4**

a) METHOD 1

$V = \pi(h^2+4h-2)$ $\quad \frac{dV}{dt} = 3\pi$

$\frac{dV}{dh} = \pi(2h+4)$

Initially, $h=2$

$V = \pi(4+8-2)$

$= 10\pi$

A 10π cm^3 volume of water already in the tank

Add the volume when liquid is poured until 1 min.

$3\pi \times 60 = 180\pi$

Total after 1 minute $= 10\pi + 180\pi$

$= 190\pi$

$\frac{dV}{dh} = \frac{dV}{dt} \times \frac{dt}{dh}$

$\pi(2h+4) = 3\pi \times \frac{dt}{dh}$

$\frac{dt}{dh} = \frac{\pi(2h+4)}{3\pi}$

$= \frac{2h+4}{3}$

$\frac{dh}{dt} = \frac{3}{2h+4}$

$190\pi = \pi(h^2+4h-2)$

$190 = h^2+4h-2$

$h^2+4h-192 = 0$

$(h+16)(h-12) = 0$

$h = -16$ or $h = 12$ (but $h > 0$)

$\therefore h = 12$

At $h = 12$

$\therefore \frac{dh}{dt} = \frac{3}{2\times 12+4} = \frac{3}{28}$ cm/s

Marking guidelines:

1 mark – Correctly applies the chain rule.

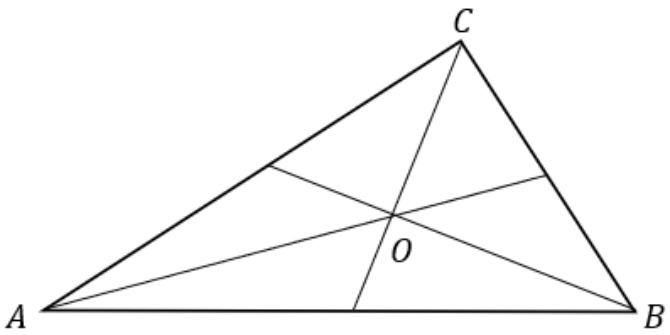
1 mark – Some progress.

1 mark – Successfully solves for h.

1 mark – Correctly finds the exact rate.

(Alternative solutions available)

(b) O is the point inside ΔABC. Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $\overrightarrow{OC} = \underset{\sim}{c}$ and given that $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} = 0$, and $\left|\underset{\sim}{a}\right| = 6$, $\left|\underset{\sim}{b}\right| = 3$, $\left|\underset{\sim}{c}\right| = 4$:



(i) Show that $\left|\underset{\sim}{a}\right|^2 + \left|\underset{\sim}{b}\right|^2 + 2\underset{\sim}{a} \cdot \underset{\sim}{b} = \left|\underset{\sim}{c}\right|^2$ **2**

$\overrightarrow{OA} = \underset{\sim}{a}$ $\overrightarrow{OB} = \underset{\sim}{b}$ $\overrightarrow{OC} = \underset{\sim}{c}$

$\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c} = 0$

$\underset{\sim}{a} + \underset{\sim}{b} = -\underset{\sim}{c}$

$|\underset{\sim}{a} + \underset{\sim}{b}| = |-\underset{\sim}{c}|$

$|\underset{\sim}{a} + \underset{\sim}{b}|^2 = |\underset{\sim}{c}|^2$

$(\underset{\sim}{a} + \underset{\sim}{b}) \cdot (\underset{\sim}{a} + \underset{\sim}{b}) = |\underset{\sim}{c}|^2$

$(\underset{\sim}{a} \cdot \underset{\sim}{a} + \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{b} \cdot \underset{\sim}{a} + \underset{\sim}{b} \cdot \underset{\sim}{b}) = |\underset{\sim}{c}|^2$

$|\underset{\sim}{a}|^2 + \underset{\sim}{a} \cdot \underset{\sim}{b} + \underset{\sim}{a} \cdot \underset{\sim}{b} + |\underset{\sim}{b}|^2 = |\underset{\sim}{c}|^2$

$|\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + 2\underset{\sim}{a} \cdot \underset{\sim}{b} = |\underset{\sim}{c}|^2$

$\therefore$ Hence proved.

Marking guidelines:

1 mark – Some progress made.

1 mark – Correctly shows all steps in proof.

(ii) Hence, find the area of ΔAOB. **3**

ii) From i)

$|\underset{\sim}{c}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{b}|^2 + 2\underset{\sim}{a}\cdot\underset{\sim}{b}$

$4^2 = 6^2 + 3^2 + 2\underset{\sim}{a}\cdot\underset{\sim}{b}$

$2\underset{\sim}{a}\cdot\underset{\sim}{b} = -29$

$\underset{\sim}{a}\cdot\underset{\sim}{b} = -\frac{29}{2}$

Let $\theta = \angle AOB$

Using the dot product

$\underset{\sim}{a}\cdot\underset{\sim}{b} = |\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta$

$-\frac{29}{2} = 6\times 3\cos\theta$

$\cos\theta = -\frac{29}{36}$

Since $0<\theta<\pi$ we have $\sin\theta > 0$

$\sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-\left(-\frac{29}{36}\right)^2}$

$= \frac{\sqrt{455}}{36}$

$\therefore$ Area of ΔAOB

$= \frac{1}{2}\times|\underset{\sim}{a}|\times|\underset{\sim}{b}|\sin\theta$

$= \frac{1}{2}\times 6\times 3\times\frac{\sqrt{455}}{36}$

$= \frac{\sqrt{455}}{4}$

Marking guidelines:

1 mark – Uses the identity from part i) to find $\underset{\sim}{a}\cdot\underset{\sim}{b}$.

1 mark – Correctly applies the dot product to find $\cos\theta$.

1 mark – Correctly finds the area of ΔAOB.

End of paper