Girraween High School

2026

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time: 10 minutes
- Working time: 2 Hours
- Write using a black pen
- NESA approved calculators may be used
- Laminated reference sheets are provided
- Answer multiple-choice questions by completely colouring in the appropriate circle on your multiple-choice answer sheet.
- Answer questions in the writing booklet provided and show all relevant mathematical reasoning and/or calculations.

Total Marks: 70

Section 1 (Pages 1 – 5) **10 Marks**

- Attempt Q1 - Q10
- Allow about 15 minutes for this section

Section 2 (Pages 6-10) **60 marks**

- Attempt Q11 - Q16
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1 – 10

Question 1

If $x^3 + 2x^2 + ax - 4$ has a remainder of 1 when divided by $x + 1$, then a equals

A) -8

B) -4

C) -2

D) -1

Question 2

A family of 10 is seated randomly around a circular table. What is the probability that the two youngest members of the family sit together.

A) $\frac{1}{5}$

B) $\frac{2}{5}$

C) $\frac{1}{9}$

D) $\frac{2}{9}$

Examination continues next page…

Question 3

What is the antiderivative of $\frac{3x^2+6x^5}{4+x^6}$?

A) $\frac{1}{2}\tan^{-1}\left(\frac{x^3}{2}\right)+\ln|4+x^6|+C$

B) $\frac{1}{3}\tan^{-1}\left(\frac{x^3}{2}\right)+\ln|4+x^6|+C$

C) $2\tan^{-1}\left(\frac{x^3}{2}\right)+\ln|4+x^6|+C$

D) $3\tan^{-1}\left(\frac{x^3}{2}\right)+\ln|4+x^6|+C$

Question 4

The members of a club voted for a new president. There were 12 candidates for the position of president and 3543 members voted. Each member voted for one candidate only. One candidate received more votes than anyone else and so became the new president. What is the smallest number of votes the new president could have received.

A) 298

B) 297

C) 296

D) 295

Examination continues next page…

Question 5

What is the derivative of $y = \cos^{-1}\sqrt{1-x}$?

A) $-\frac{2}{\sqrt{x(1-x)}}$

B) $\frac{2}{\sqrt{x(1-x)}}$

C) $-\frac{1}{2\sqrt{x(1-x)}}$

D) $\frac{1}{2\sqrt{x(1-x)}}$

Question 6

Consider the parametric equations $x = 5\cos\theta - 1$ and $y = 5\sin\theta + 3$. Which of these is the corresponding cartesian equation?

A) $x^2 + 2x + y^2 - 6y = 15$

B) $x^2 + 2x + y^2 - 6y = 25$

C) $x^2 - 2x + y^2 + 6y = 15$

D) $x^2 - 2x + y^2 + 6y = 25$

Examination continues next page…

Question 7

Given the function $f(x) = e^{-x} + ex, x \geq -1$. What is the gradient of the inverse function $y = f^{-1}(x)$ at $(1, 0)$?

A) $e - 1$

B) $\frac{1}{e-1}$

C) $\frac{e^2-1}{e}$

D) $\frac{e}{e^2-1}$

Question 8

What is the term independent of x in the expansion of $(1 + 4x^2)\left(2x + \frac{1}{x}\right)^{12}$?

A) $\binom{12}{6} \times 2^6$

B) $\binom{12}{6} \times 2^6 + \binom{12}{7} \times 2^6$

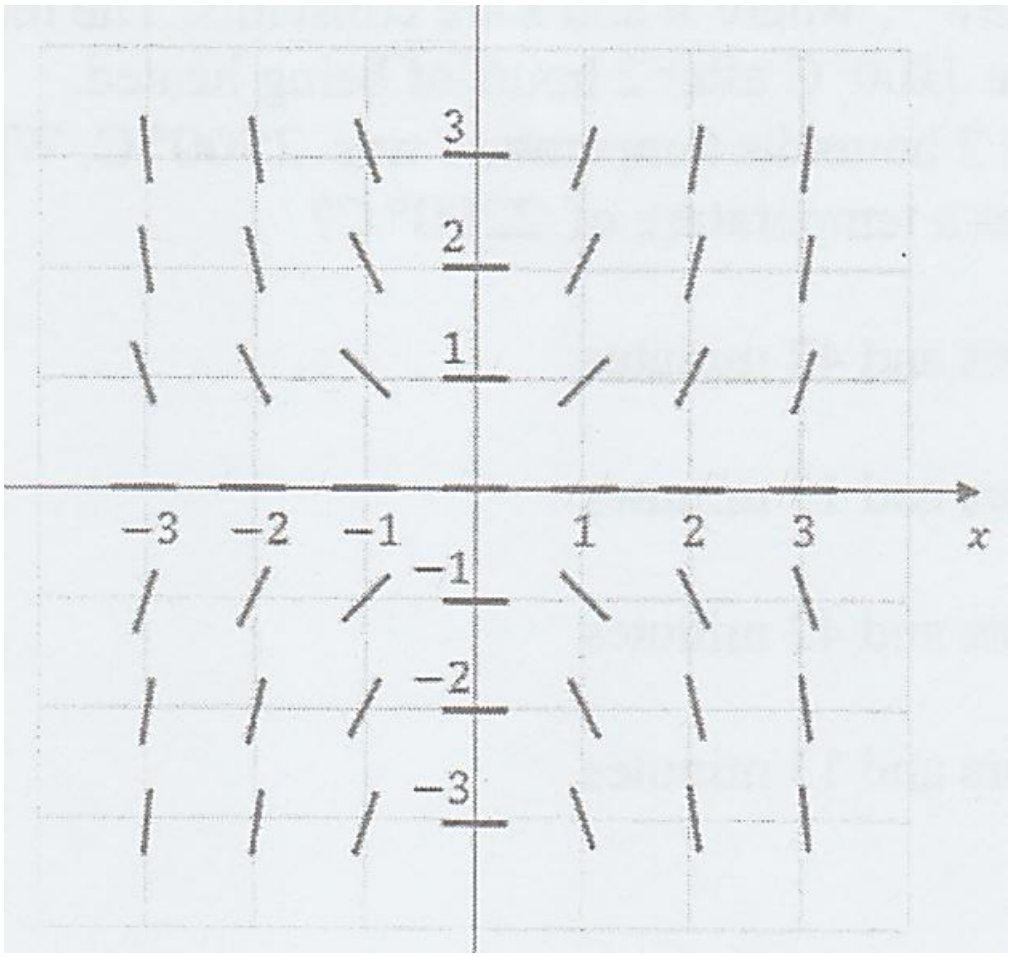
C) $\binom{12}{6} \times 2^6 + \binom{12}{7} \times 2^7$

D) $\binom{12}{6} \times 2^6 + \binom{12}{7} \times 2^9$

Examination continues next page…

Question 9

Which of the following differential equations is represented by the slope field below?



A) $y \times \frac{dy}{dx} = x$

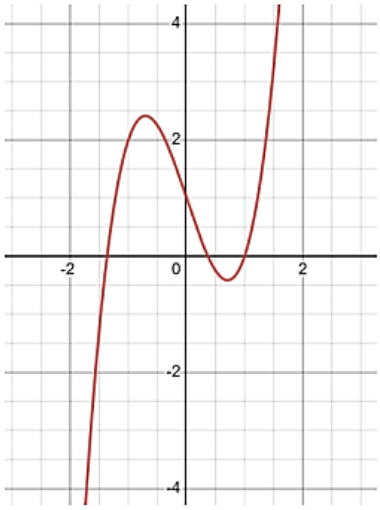
B) $x \times \frac{dy}{dx} = y$

C) $\frac{dy}{dx} + xy = 0$

D) $\frac{dy}{dx} - xy = 0$

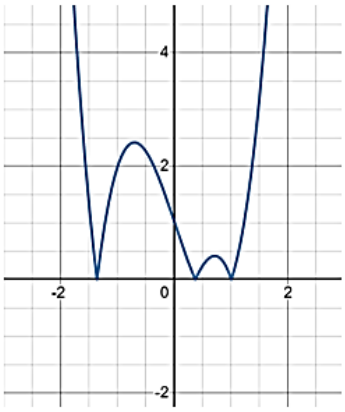
Question 10

The graph of the function $f(x)$ is drawn below.

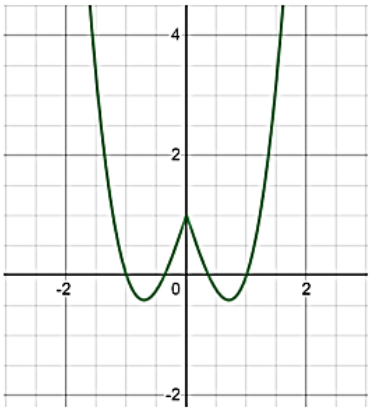


Which of the following best represents the graph of $y = |f(|x|)|$?

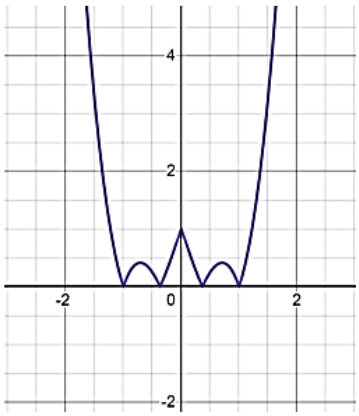
A)



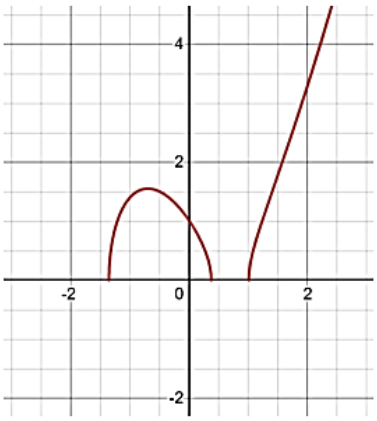
B)



C)



D)



Examination continues next page…

Section II

60 marks
Attempt Questions 11 - 16
Allow about 1 hour and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (10 marks) Start a new writing page

a) Solve the inequality $\frac{2x-1}{x+2} > 1$ 2

b) Evaluate $\lim_{x\to 0} \frac{\sin 4x \cos 4x}{16x}$ 2

c) Evaluate $\int_0^1 \sqrt{1-x^2}\, dx$ using the substitution $x = \sin\theta$ 3

d) Consider the polynomial $P(x) = 2x^4 + 5x^3 + 3x^2 - x - 1$. It is known that $P(x)$ has a triple root at $x = \alpha$.

i) By using differentiation, find the value of α. 2

ii) Hence solve $P(x) = 0$. 1

Examination continues next page…

Question 12 (10 marks) Start a new writing page

a) **(i)** Express $\sqrt{3}\cos\theta - \sin\theta$ in the form $A\cos(\theta + \alpha)$ where $A > 0$ and $0 < \alpha < \frac{\pi}{2}$. 2

(ii) Hence determine the maximum and minimum values of $f(\theta) = \sqrt{3}\cos\theta - \sin\theta$ 1

(iii) Find the first positive values of θ at which the maximum and minimum occurs. 2

b) Stoneware pottery is fired in a closed kiln to a temperature of $1280°C$. After reaching this temperature the kiln is turned off to avoid cracking, the cooling process is closely monitored in a surrounding environment whose temperature is maintained at $200°C$. The rate of cooling is approximately given by:

$$\frac{dT}{dt} = -k(T - 200)$$

where T is the temperature in $°C$, t is the time in hours and k is a positive constant.

i) Show that $T = 200 + Ae^{-kt}$, where A is a constant is a solution of this equation. 1

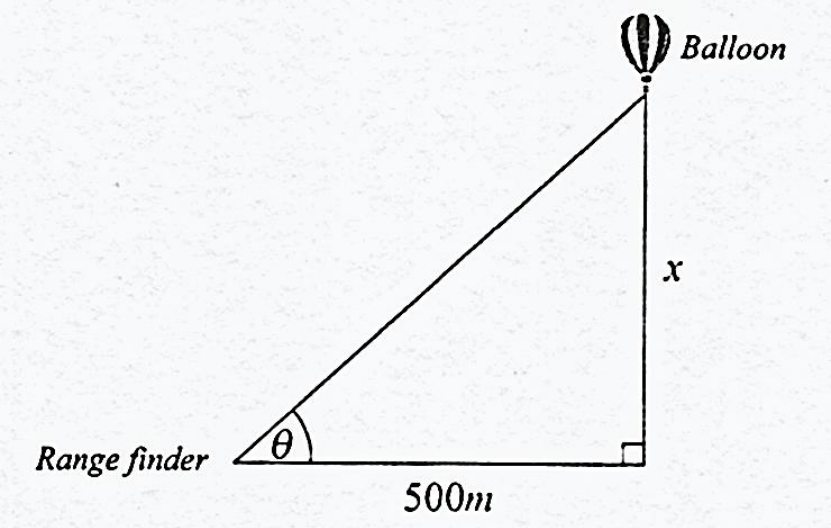
ii) If after 2 hours the pottery has cooled to $800°C$, find A and k (answer for k correct to 3 decimal places). 2

(iii) The door to the kiln is safe to open when the temperature of the pottery drops below $240°C$. How long after the kiln was turned off, to the nearest hour, can the door be opened? 2

Examination continues next page…

Question 13 (10 marks) Start a new writing page

a) A hot air balloon is rising straight up from a level field and is detected by a range finder 500 m from the lift-off point. At that point, the angle of elevation of the balloon is $\frac{\pi}{6}$, and the angle is increasing at the rate of 0.1 rad/min. How fast is the balloon rising at that moment? 3



b) The region in the first quadrant bounded by the curve $y = 2\tan^{-1}x$ and the $y-$ axis between $y = 0$ and $y = \frac{\pi}{2}$ is rotated through one complete revolution about the $y-$axis. Find the exact volume of the solid of revolution so formed. 3

c) **(i)** Use mathematical induction to prove that

$\frac{1}{1\times3} + \frac{1}{3\times5} + \frac{1}{5\times7} + \ldots + \frac{1}{(2n-1)\times(2n+1)} = \frac{n}{2n+1}$ for all positive integers $n \geq 1$. 2

(ii) Hence find in simplest fractional form the value of

$\frac{1}{11\times13} + \frac{1}{13\times15} + \frac{1}{15\times17} + \ldots + \frac{1}{119\times121}$ 2

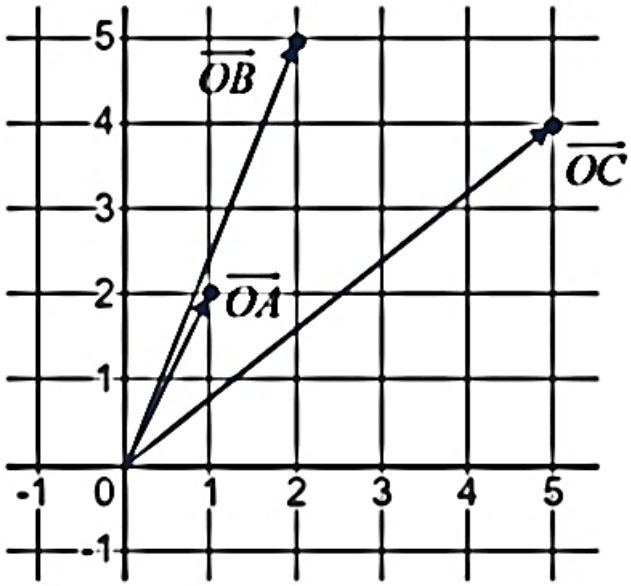
Examination continues next page…

Question 14 (10 marks) Start a new writing page

a) Find the exact value of $\int_0^{\frac{\pi}{3}} \cos 4x \cos 2x \, dx$ 2

b) Consider the vectors given by $\underset{\sim}{a} = p\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} + p\underset{\sim}{j}$, where p is a real number. If the acute angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is 30°, find the value(s) of p. 3

c) The coordinates of the position vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are (1,2), (2,5) and (5,4) respectively.



(i) Express $\overrightarrow{AB}$ in the form $x\underset{\sim}{i} + y\underset{\sim}{j}$. 1

(ii) Given that $ABCD$ is a parallelogram, use vector operations to show that the position vector $\overrightarrow{OD} = 4\underset{\sim}{i} + \underset{\sim}{j}$. 2

(iii) By using properties of vectors, show that $ABCD$ is a rectangle. 2

Examination continues next page…

Question 15 (10 marks) Start a new writing page

a) Show that $y = x\sin 3x$ is a solution of the differential equation $\frac{d^2y}{dx^2} + 9y - 6\cos 3x = 0$ 2

b) Solve the differential equation $(1 + e^{2x})\frac{dy}{dx} + (1 + y^2)e^x = 0$, given that when $x = 0$, $y = 1$. 3

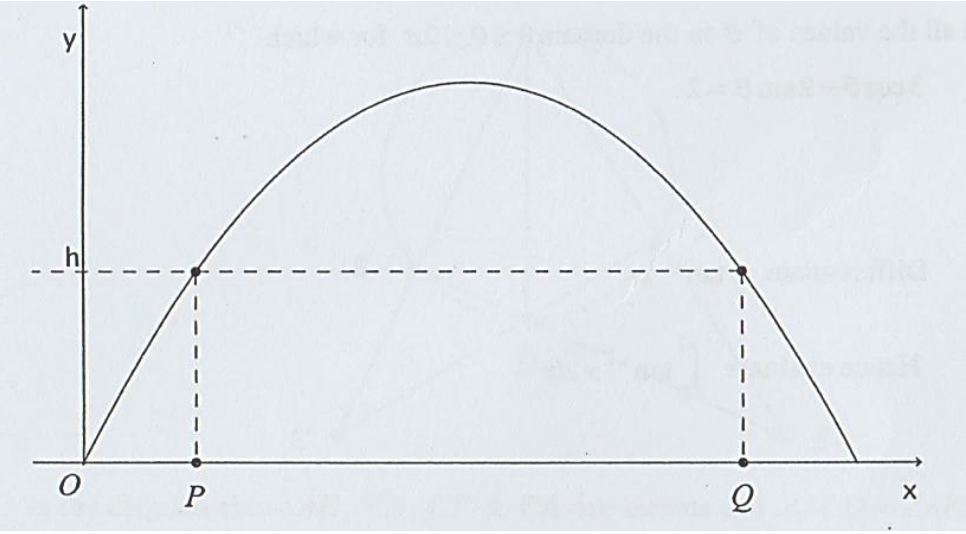
c) A certain biological culture is modelled by the differential equation $\frac{dP}{dt} = 0.002P(100 - P)$, where $P(t)$ is the percentage of the petri dish covered by the culture at time t minutes. When first observed, the culture occupied 5% of the petri dish.

(i) Given that $\frac{1}{0.002P(100-P)} = 5\left(\frac{1}{P} + \frac{1}{100-P}\right)$, find the equation of the size, P, of the culture in terms of t. 4

(ii) Find the percentage of the petri dish covered after 26 minutes. 1

Examination continues next page…

Question 16 (10 marks) Start a new writing page



A particle is projected with velocity V m/s from a point O at an angle of elevation α. The particle just clears two vertical trees P and Q, of height h metres at horizontal distance of p metres and q metres from O respectively. The acceleration due to gravity is taken as 10 m/s^2 and air resistance is ignored. The position of the particle at any time is given by $x = Vt\cos\alpha$ and $y = -5t^2 + Vt\sin\alpha$.

(i) Find the cartesian equation of the flight of the particle. 2

(ii) Show that $V^2 = \frac{5p^2(1+\tan^2\alpha)}{p\tan\alpha - h}$ 2

(iii) Show that $\tan\alpha = \frac{h(p+q)}{pq}$ 3

(iv) Hence show that $V^2 = \frac{5h(p+q)^2}{pq} + \frac{5pq}{h}$ 3

End of Examination.

12 Extension 1 Trial 2026 solutions

1. Remainder $= P(-1)$

$P(x) = x^3 + 2x^2 + ax - 4$

$P(-1) = (-1)^3 + 2(-1)^2 + a(-1) - 4$

$= -3 - a$

$-3 - a = 1$

$a = -4$ (B)

2. $\frac{8! \times 2!}{9!} = \frac{2}{9}$ (D)

3. $\int \frac{3x^2 + 6x^5}{4 + x^6}\, dx$

$= \int \frac{3x^2\, dx}{2^2 + (x^3)^2} + \int \frac{6x^5\, dx}{4 + x^6}$

$= I_1 + \ln|4 + x^6| + C$

Let $u = x^3$

$\frac{du}{dx} = 3x^2$

$3x^2\, dx = du$

$I_1 = \int \frac{du}{2^2 + u^2} = \frac{1}{2}\tan^{-1}\left(\frac{u}{2}\right)$ (A)

$I = \frac{1}{2}\tan^{-1}\left(\frac{x^3}{2}\right) + \ln|4 + x^6| + C$

4. 3543 votes are to be given to 12 candidates. If each candidate receives the same number of votes

$$\frac{3543}{12} = 295\frac{1}{4}$$

$295 \times 12 = 3540$

Three votes are remaining.

Case① three votes go to three different candidates.

In this case three candidates have 296 votes and 9 candidates have 295. So there is no clear winner.

Case② two votes go to the same candidate.

In this case there is a candidate with 297 votes, another one with 296 votes and 10 candidates with 295 votes.

∴ the smallest number of votes for a majority is 297.

(B)

Check

$297 + 296 + (10 \times 295) = 3543$

5. $y = \cos^{-1}\sqrt{1-x}$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-(1-x)}} \times \frac{1}{2\sqrt{1-x}} \times -1$$

$$= \frac{1}{2\sqrt{x(1-x)}}$$ (D)

6. $\cos\theta = \frac{x+1}{5}$, $\sin\theta = \frac{y-3}{5}$

$\sin^2\theta + \cos^2\theta = 1$

$$\frac{(x+1)^2}{25} + \frac{(y-3)^2}{25} = 1$$

$$(x+1)^2 + (y-3)^2 = 25$$

$$x^2 + 2x + 1 + y^2 - 6y + 9 = 25$$ (A)

$$x^2 + 2x + y^2 - 6y = 15$$

7. $y = e^{-x} + ex$

$$x = e^{-y} + ey$$

$$= \frac{1}{e^y} + ey$$

$$\frac{dx}{dy} = e^{-y} \times -1 + e$$

$$= -e^{-y} + e$$

when $x = 1, y = 0$

$$\frac{dx}{dy} = -1 + e = e - 1$$

$$\frac{dy}{dx} = \frac{1}{e-1}$$ (B)

8. Terms independent of x

$= 1 \times {}^{12}C_6 (2x)^6 \left(\frac{1}{x^6}\right) + 4x^2 \times {}^{12}C_7 (2x)^5 \left(\frac{1}{x}\right)^7$ ©

9. D

10. C

Question 11 (10 marks)

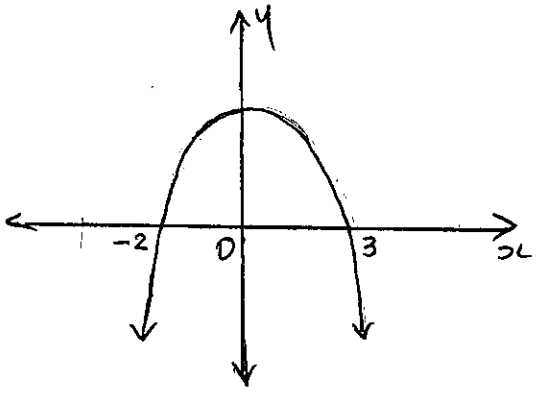
a) $\frac{2x-1}{x+2} > 1$

$(2x-1)(x+2) > (x+2)^2, \quad x \neq -2$

$(x+2)^2 - (2x-1)(x+2) < 0$

$(x+2)(-x+3) < 0$ 1

$x = -2, 3$



From the graph

$x < -2$ or $x > 3$ 1

b) $\lim_{x \to 0} \frac{\sin 4x \cos 4x}{16x}$

$= \lim_{x \to 0} \frac{\sin 8x}{2} \times \frac{1}{16x}$ 1

$= \lim_{x \to 0} \frac{\sin 8x}{8x} \times \frac{1}{4}$

$= \frac{1}{4} \quad \left(\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right)$] 1

c) $I = \int_0^1 \sqrt{1-x^2}\, dx$

Let $x = \sin\theta$

$\frac{dx}{d\theta} = \cos\theta$

$dx = \cos\theta\, d\theta$

when $x=0$, $\theta = 0$

when $x=1$, $\theta = \frac{\pi}{2}$

$I = \int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2\theta}\, \cos\theta\, d\theta$

$= \int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$ |

$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (1+\cos 2\theta)\, d\theta$

$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$ |

$= \frac{1}{2}\left\{\left(\frac{\pi}{2} + \frac{\sin\pi}{2}\right) - (0+0)\right\}$

$= \frac{1}{2}\left(\frac{\pi}{2}\right) = \underline{\underline{\frac{\pi}{4}}}$ |

d (i) $P(x) = 2x^4 + 5x^3 + 3x^2 - x - 1$

$P'(x) = 8x^3 + 15x^2 + 6x - 1$

$P''(x) = 24x^2 + 30x + 6$

For a triple root $x = \alpha$

$P(\alpha) = P'(\alpha) = P''(\alpha) = 0$

$P''(\alpha) = 0$

$24\alpha^2 + 30\alpha + 6 = 0$

$6(4\alpha^2 + 5\alpha + 1) = 0$

$4\alpha^2 + 5\alpha + 1 = 0$

$(\alpha + 1)(4\alpha + 1) = 0$

$\alpha = -1$ or $\alpha = \frac{-1}{4}$

$P(-1) = 2 \times 1 - 5 + 3 + 1 - 1$

$= 0$

$\therefore \alpha = -1$ is the triple root

(ii) Let $-1, -1, -1$ and β be the roots of $P(x)$.

Sum of roots:

$-3 + \beta = \frac{-5}{2}$

$\beta = \frac{-5}{2} + 3 = \frac{1}{2}$

Solutions are $\underline{\underline{x = -1, -1, -1, \frac{1}{2}}}$

Question 12 (10 marks)

(a)(i) $\sqrt{3}\cos\theta - \sin\theta = A\cos(\theta+\alpha)$

$= A\cos\theta\cos\alpha - A\sin\theta\sin\alpha$

$\sqrt{3} = A\cos\alpha$

$1 = A\sin\alpha$

$A^2\cos^2\alpha = 3$

$A^2\sin^2\alpha = 1$

$A^2(\sin^2\alpha+\cos^2\alpha) = 4$

$A = 2$ 1

$\sin\alpha = \frac{1}{2}$

$\cos\alpha = \frac{\sqrt{3}}{2}$

$\alpha = \frac{\pi}{6}$ 1

$\sqrt{3}\cos\theta - \sin\theta = 2\cos(\theta+\frac{\pi}{6})$

(ii) Maximum value $= 2$
Minimum value $= -2$] 1

(iii) For maximum $\cos(\theta+\frac{\pi}{6}) = 1$

$\theta + \frac{\pi}{6} = 0$

$\theta = \frac{-\pi}{6} + 2\pi = \underline{\underline{\frac{11\pi}{6}}}$ 1

For minimum, $\cos(\theta+\frac{\pi}{6}) = -1$

$\theta + \frac{\pi}{6} = \pi$

$\theta = \pi - \frac{\pi}{6} = \underline{\underline{\frac{5\pi}{6}}}$ 1

(b)(i) LHS $= \frac{dT}{dt}$

$= Ae^{-kt}\times -k$

$= -kAe^{-kt}$

RHS $= -k(T-200)$

$= -k \times Ae^{-kt}$

$= -kAe^{-kt}$ 1

LHS = RHS $\therefore T = 200 + Ae^{-kt}$ is a solution.

(ii) when $t=0$, $1280 = 200 + A$

$$A = 1080$$

when $t=2$, $T = 800^\circ C$

$$800 = 200 + 1080e^{-kt}$$

$$600 = 1080e^{-kt}$$

$$\frac{5}{9} = e^{-2k}$$

$$-2k = \log_e \frac{5}{9}$$

$$k = \frac{-1}{2}\log_e \frac{5}{9}$$

$$= \underline{0.294}$$

(iii) $240 = 200 + 1080e^{-kt}$

$$\frac{40}{1080} = e^{-kt}$$

$$-kt = \log_e\left(\frac{4}{108}\right)$$

$$t = \frac{-1}{k}\log_e\left(\frac{4}{108}\right)$$

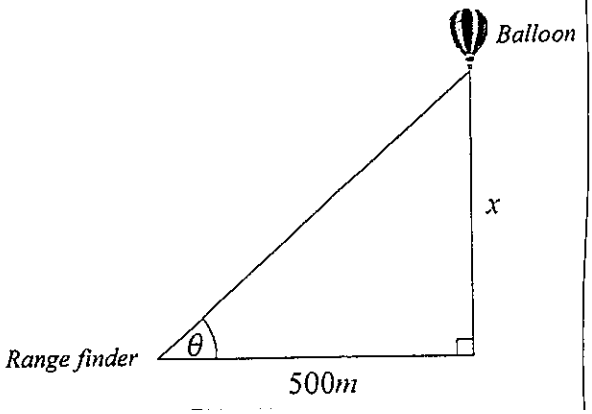
$$= 11.21$$

$\therefore$ the door is safe to open

after 12 hrs

Question 13 (10 marks)

(a)



$\tan\theta = \frac{x}{500}$

$x = 500\tan\theta$

$\frac{dx}{dt} = \frac{dx}{d\theta} \times \frac{d\theta}{dt}$

$\frac{dx}{d\theta} = 500\sec^2\theta$ 1

$\frac{d\theta}{dt} = 0.1$ rad/min when $\theta = \frac{\pi}{6}$

$\frac{dx}{dt} = 500\sec^2\theta \times 0.1$ 1

$= 50(1+\tan^2\theta)$

$= 50\left(1+\left(\tan\frac{\pi}{6}\right)^2\right)$

$= 50\left(1+\frac{1}{3}\right)$

$= \frac{200}{3}$ metres/min 1

(b) $y = 2\tan^{-1} x$

$\tan^{-1} x = \frac{y}{2}$

$x = \tan \frac{y}{2}$

$V = \pi \int_0^{\frac{\pi}{2}} \tan^2 \frac{y}{2}\, dy$ 1

$= \pi \int_0^{\frac{\pi}{2}} \left(\sec^2 \frac{y}{2} - 1\right) dy$

$= \pi \left[2\tan \frac{y}{2} - y\right]_0^{\frac{\pi}{2}}$ 1

$= \pi \left\{\left(2\tan \frac{\pi}{4} - \frac{\pi}{2}\right) - (2\tan 0 - 0)\right\}$

$= \pi \left(2 - \frac{\pi}{2}\right)$ U3

OR

$= 2\pi - \frac{\pi^2}{2}$ U3

OR

$= \frac{4\pi - \pi^2}{2}$ U3] 1

(c)(i) step1 $n=1$

$LHS = \frac{1}{1\times 3} = \frac{1}{3}$; $RHS = \frac{1}{2\times 1+1} = \frac{1}{3}$

$LHS = RHS$

$\therefore$ the result is true for $n=1$

step 2 Assume the result is true for $n=k$

$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \cdots + \frac{1}{(2k-1)(2k+1)} = \frac{k}{2k+1}$ — (1)

To prove for $n = k+1$

$$\frac{1}{1\times3} + \frac{1}{3\times5} + \cdots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2(k+1)-1)(2(k+1)+1)} = \frac{k+1}{2(k+1)+1}$$

$$\frac{1}{1\times3} + \frac{1}{3\times5} + \cdots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2k+1)(2k+3)} = \frac{k+1}{2k+3} \quad \text{— (2)}$$

Step 3

LHS of (2)

$$= \frac{1}{1\times3} + \frac{1}{3\times5} + \cdots + \frac{1}{(2k-1)(2k+1)} + \frac{1}{(2k+1)(2k+3)}$$

$$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)} \quad \text{(by assumption (1))}$$

$$= \frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2+3k+1}{(2k+1)(2k+3)}$$

$$= \frac{(k+1)(2k+1)}{(2k+1)(2k+3)} = \frac{k+1}{2k+3} = \text{RHS of (2)}$$

If the result is true for $n=k$, then it is true for $n = k+1$.

Hence by the principle of mathematical induction, the result is true for all positive integers $n \geq 1$.

(ii) $\frac{1}{1\times3}+\frac{1}{3\times5}+\cdots+\frac{1}{(2\times60-1)(2\times60+1)}=\frac{60}{2\times60+1}=\frac{60}{121}$ |

$\frac{1}{1\times3}+\frac{1}{3\times5}+\cdots+\frac{1}{(2\times5-1)(2\times5+1)}=\frac{5}{2\times5+1}=\frac{5}{11}$ |

$\frac{11}{11\times13}+\frac{1}{13\times15}+\cdots+\frac{1}{119\times121}=\frac{60}{121}-\frac{5}{11}$

$=\frac{5}{121}$ |

Question 14 (10 marks)

(a) $\cos A\cos B=\frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right]$

$\cos 4x\cos 2x=\frac{1}{2}\left[\cos(4x-2x)+\cos(4x+2x)\right]$

$=\frac{1}{2}\left[\cos 2x+\cos 6x\right]$

$\int_0^{\frac{\pi}{3}}(\cos 4x\cos 2x)\,dx$

$=\frac{1}{2}\int_0^{\frac{\pi}{3}}(\cos 2x+\cos 6x)\,dx$

$=\frac{1}{2}\left[\frac{1}{2}\sin 2x+\frac{1}{6}\sin 6x\right]_0^{\frac{\pi}{3}}$

$=\frac{1}{2}\left\{\left(\frac{1}{2}\sin\frac{2\pi}{3}+\frac{1}{6}\sin 2\pi\right)-0\right\}$

$=\frac{1}{2}\left(\frac{1}{2}\times\frac{\sqrt{3}}{2}+\frac{1}{6}\times 0-0\right)$

$=\frac{\sqrt{3}}{8}$

(b) $|\underset{\sim}{a}| = \sqrt{p^2+1}$

$|\underset{\sim}{b}| = \sqrt{p^2+1}$

$\underset{\sim}{a} \cdot \underset{\sim}{b} = (p \times 1) + (1 \times p)$

$= 2p$

$2p = \sqrt{p^2+1} \times \sqrt{p^2+1} \times \frac{\sqrt{3}}{2}$ 1

$2p = (p^2+1)\frac{\sqrt{3}}{2}$

$4p = (p^2+1)\sqrt{3}$

$\sqrt{3}p^2 - 4p + \sqrt{3} = 0$ 1

$-3, -1$ $\quad mn = 3$

$m+n = -4$

$\sqrt{3}p^2 - 3p - p + \sqrt{3} = 0$

$\sqrt{3}p(p-\sqrt{3}) - (p-\sqrt{3}) = 0$

$(p-\sqrt{3})(\sqrt{3}p - 1) = 0$

$p = \sqrt{3}$ or $p = \frac{1}{\sqrt{3}}$ 1

(c)(i) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$= \underset{\sim}{i} + 3\underset{\sim}{j}$

(ii) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$= \overrightarrow{OA} + \overrightarrow{BC}$

Now $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$

$= \begin{pmatrix} 5 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\overrightarrow{OD} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ 1 \end{pmatrix} = 4\underset{\sim}{i} + \underset{\sim}{j}$

(iii) $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA}$

$= \begin{pmatrix} 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$\overrightarrow{AB} \cdot \overrightarrow{AD} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

$= (1 \times 3) + (3 \times -1)$

$= 0$

$\therefore \overrightarrow{AB} \perp \overrightarrow{AD}$

ABCD is a rectangle.

Question 15 (10 marks)

$y = x \sin 3x$

$y' = x \times (\cos 3x) 3 + \sin 3x$

$= 3x \cos 3x + \sin 3x$

$y'' = 3x(-\sin 3x) \times 3 + (\cos 3x) \times 3 + (\cos 3x) \times 3$

$= -9x \sin 3x + 6 \cos 3x$

$LHS = y'' + 9y - 6\cos 3x$

$= -9x \sin 3x + 6\cos 3x + 9x \sin 3x - 6\cos 3x$

$= 0$

$\therefore$ $y = x \sin 3x$ is a solution.

(b) $(1 + e^{2x})\, dy = -(1+y^2) e^x\, dx$

$$\frac{dy}{1+y^2} = -\frac{e^x\, dx}{1+e^{2x}}$$

$$\int \frac{dy}{1+y^2} = -\int \frac{e^x\, dx}{1+e^{2x}}$$

$$\int \frac{dy}{1+y^2} = -\int \frac{du}{1+u^2} \quad \text{where } u = e^x$$

$$\tan^{-1}(y) = -\tan^{-1}(e^x) + C$$

when $x = 0$, $y = 1$

$$\tan^{-1}(1) = -\tan^{-1}(e^0) + C$$

$$\frac{\pi}{4} = -\frac{\pi}{4} + C \qquad \therefore C = \frac{\pi}{2}$$

Solution is $\tan^{-1}(y) + \tan^{-1}(e^x) = \frac{\pi}{2}$

(c) (i)

$$\frac{dP}{dt} = 0.002P(100-P)$$

$$\frac{dt}{dP} = \frac{1}{0.002P(100-P)}$$

$$\int dt = \int \frac{dP}{0.002P(100-P)}$$

$$t = \int 5\left(\frac{1}{P} + \frac{1}{100-P}\right)dP \quad 1$$

$$= 5\int\left(\frac{1}{P} + \frac{1}{100-P}\right)dP$$

$$= 5\left(\ln P - \ln(100-P)\right) + C \quad \text{since } P \text{ is positive}$$

$$= 5\ln\frac{P}{100-P} + C \quad 1$$

when $t=0$, $P=5$

$$0 = 5\ln\frac{5}{95} + C$$

$$C = -5\ln\frac{1}{19} = -5\ln 19^{-1}$$

$$= -5 \times -1 \ln 19$$

$$= 5\ln 19$$

$$t = 5 \ln \frac{P}{100-P} + 5 \ln 19 \quad 1$$

$$= 5 \ln \frac{19P}{100-P}$$

$$\frac{t}{5} = \ln \frac{19P}{100-P}$$

$$e^{\frac{t}{5}} = \frac{19P}{100-P}$$

$$(100-P)e^{\frac{t}{5}} = 19P$$

$$100e^{\frac{t}{5}} - Pe^{\frac{t}{5}} = 19P$$

$$100e^{\frac{t}{5}} = 19P + Pe^{\frac{t}{5}}$$

$$= P(19 + e^{\frac{t}{5}})$$

$$P = \frac{100e^{\frac{t}{5}}}{19 + e^{\frac{t}{5}}} \quad 1$$

(ii) when $t = 26$

$$P = \frac{100e^{\frac{26}{5}}}{19 + e^{\frac{26}{5}}}$$

$$= 90.51$$

$$= \underline{91\%} \quad 1$$

Question 16 (10 marks)

(i) $x = Vt\cos\alpha$ — ① $\quad y = -5t^2 + Vt\sin\alpha$ — ②

From ① $t = \dfrac{x}{V\cos\alpha}$

substitute in ②

$$y = -\frac{5x^2}{V^2\cos^2\alpha} + \frac{Vx\sin\alpha}{V\cos\alpha}$$

$$= \frac{-5x^2}{V^2}\sec^2\alpha + x\tan\alpha$$

$$= \frac{-5x^2(1+\tan^2\alpha)}{V^2} + x\tan\alpha \quad \text{— ③}$$

(ii) substitute (p, h) in ③

$$h = \frac{-5p^2(1+\tan^2\alpha)}{V^2} + p\tan\alpha$$

$$h - p\tan\alpha = -\frac{5p^2}{V^2}(1+\tan^2\alpha)$$

$$-(p\tan\alpha - h) = -\frac{5p^2}{V^2}(1+\tan^2\alpha)$$

$$p\tan\alpha - h = \frac{5p^2}{V^2}(1+\tan^2\alpha)$$

$$V^2(p\tan\alpha - h) = 5p^2(1+\tan^2\alpha)$$

$$V^2 = \frac{5p^2(1+\tan^2\alpha)}{p\tan\alpha - h}$$

(iii) We have $V^2 = \frac{5p^2(1+\tan^2\alpha)}{p\tan\alpha - h}$ ——— ④

Similarly by substituting (q, h) in ③

we get $V^2 = \frac{5q^2(1+\tan^2\alpha)}{q\tan\alpha - h}$ ——— ⑤

From ④ and ⑤ we get

$$\frac{5p^2(1+\tan^2\alpha)}{p\tan\alpha - h} = \frac{5q^2(1+\tan^2\alpha)}{q\tan\alpha - h} \quad 1$$

$$\frac{p^2}{p\tan\alpha - h} = \frac{q^2}{q\tan\alpha - h}$$

$$p^2(q\tan\alpha - h) = q^2(p\tan\alpha - h)$$

$$p^2q\tan\alpha - p^2h = q^2p\tan\alpha - q^2h$$

$$p^2q\tan\alpha - q^2p\tan\alpha = p^2h - q^2h$$

$$\tan\alpha(p^2q - q^2p) = h(p^2 - q^2) \quad 1$$

$$\tan\alpha = \frac{h(p+q)(p-q)}{pq(p-q)}$$

$$= \frac{h(p+q)}{pq} \quad 1$$

(iv) From (ii) and (iii) we have

$$V^2 = \frac{5p^2(1+\tan^2\alpha)}{p\tan\alpha - h} \quad \text{and} \quad \tan\alpha = \frac{h(p+q)}{pq}$$

$$V^2 = \frac{5p^2\left(1+\dfrac{h^2(p+q)^2}{p^2q^2}\right)}{p\,h\dfrac{(p+q)}{pq} - h} \quad 1$$

$$= \frac{5p^2\left(\dfrac{p^2q^2+h^2(p+q)^2}{p^2q^2}\right)}{\dfrac{ph(p+q)-pqh}{pq}}$$

$$= \frac{5p^2\left(p^2q^2+h^2(p+q)^2\right)}{p^2q^2} \times \frac{pq}{ph(p+q)-pqh} \quad 1$$

$$= \frac{5p^2\left(p^2q^2+h^2(p+q)^2\right)}{pq \times p^2h} \quad 1$$

$$= \frac{5p^2q^2}{pqh} + \frac{5h^2(p+q)^2}{pqh} = \frac{5pq}{h} + \frac{5h(p+q)^2}{pq}$$