HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 1

Year 12 Higher School Certificate

Trial Examination Term 3 2026

STUDENT NUMBER: _____

TEACHER NAME: Ms Guan, Mrs Sztajer, Ms Murray, Ms Tourikis **(circle one)**

STUDENT NAME: _____

General Instructions:

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–14, show relevant mathematical reasoning and/ or calculations

Total Marks: 70

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6–11)

- Attempt Questions 11–14
- Start each question in a designated question writing booklet
- Write your student number on every writing booklet
- Allow about 1 hour and 45 minutes for this section

Question	*1-10*	*11*	*12*	*13*	*14*	*Total*
Total	*/10*	*/15*	*/15*	*/15*	*/15*	*/70*

Outcomes assessed: ME 12-1, 12-2, 12-3, 12-4, 12-7

Section I

10 marks
Attempt Questions 1 - 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1. Which of the following is an expression for $\int \cos^2 3x \, dx$?

   A. $\frac{x}{2} + \frac{\cos 6x}{6} + C$

   B. $\frac{x}{2} - \frac{\sin 6x}{12} + C$

   C. $\frac{x}{2} + \frac{\sin 6x}{12} + C$

   D. $\frac{x}{2} - \frac{\cos 6x}{6} + C$

2. Which vector is perpendicular to $\underset{\sim}{v} = -3\underset{\sim}{i} + 7\underset{\sim}{j}$?

   A. $\underset{\sim}{a} = 7\underset{\sim}{i} + 3\underset{\sim}{j}$

   B. $\underset{\sim}{b} = 7\underset{\sim}{i} - 3\underset{\sim}{j}$

   C. $\underset{\sim}{c} = 3\underset{\sim}{i} - 7\underset{\sim}{j}$

   D. $\underset{\sim}{d} = 3\underset{\sim}{i} + 7\underset{\sim}{j}$

3. The equation $y = e^{kx}$, where $k \in \mathbb{R}$, satisfies the differential equation

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0.$$

   The possible values of k are:

   A. $k = -2$ or $k = 3$

   B. $k = -1$ or $k = 6$

   C. $k = 2$ or $k = -3$

   D. $k = 6$ or $k = -1$

4. The vector $\underset{\sim}{v} = a\underset{\sim}{i} + b\underset{\sim}{j}$ has magnitude 12 and direction 240°T. The exact values of a and b are:

   A. $a = -6,\ b = 6\sqrt{2}$

   B. $a = -6,\ b = -6\sqrt{3}$

   C. $a = 6,\ b = 6\sqrt{3}$

   D. $a = -6\sqrt{3},\ b = -6$

5. A bookshelf contains 30 books of which 5 are cookbooks, 10 are craft books and the rest are language books. What is the minimum number of books that must be picked from the bookshelf blindfolded to be assured of picking at least one book from each category?

   A. 16

   B. 25

   C. 26

   D. 29

6. The displacement of the trajectory of a ball, $\underset{\sim}{r}(t)$, thrown from a height of 2 000 m at a speed of 240 m/s at angle θ, such that $\tan\theta = \frac{3}{4}$, is given below. The ball lands on the ground 3 600 m away from the origin, O, where $g = 10\ \text{ms}^{-2}$, after 20 seconds.

$$\underset{\sim}{r}(t) = \begin{bmatrix} 240t\cos\theta \\ 2000 + 240t\sin\theta - \frac{gt^2}{2} \end{bmatrix}$$

   What is the impact speed of the ball when it strikes the ground?

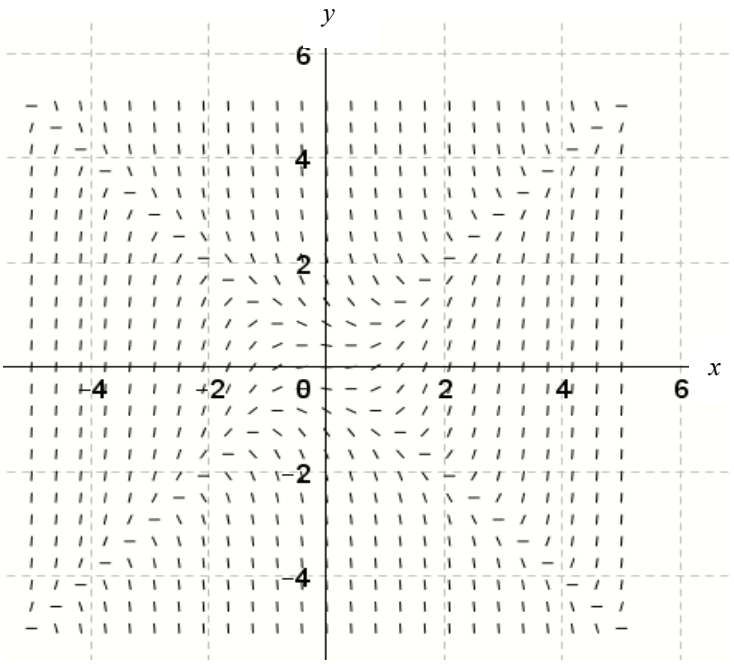
   A. 259 ms^{-2}

   B. 144 ms^{-2}

   C. 200 ms^{-2}

   D. 240 ms^{-2}

7. The direction (slope) field for a certain first order differential equation is shown below.



This differential equation could be:

A. $\frac{dy}{dx} = (x-y)^2$

B. $\frac{dy}{dx} = (x+y)^2$

C. $\frac{dy}{dx} = x(x+y)$

D. $\frac{dy}{dx} = (x-y)(x+y)$

8. What is the derivative of $f(x) = x\tan^{-1}\frac{1}{x} + \frac{1}{2}\log_e(x^2+1)$?

A. $\frac{x}{x^2+1}$

B. $\tan^{-1} x$

C. $\tan^{-1}\frac{1}{x}$

D. $\tan^{-1}\frac{1}{x} + \frac{2x}{x^2+1}$

9. Which of the following Cartesian equations represents the parametric equations

$x = 2\sin\theta - 5$ and $y = 3\cos\theta + 1$?

A. $\frac{(x-5)^2}{2} + \frac{(y+1)^2}{3} = 1$

B. $\frac{(x+5)^2}{2} + \frac{(y-1)^2}{3} = 1$

C. $\frac{(x-5)^2}{4} + \frac{(y+1)^2}{9} = 1$

D. $\frac{(x+5)^2}{4} + \frac{(y-1)^2}{9} = 1$

10. For the function $f(x)$, it is known that $g(1) = 3,\ g'(1) = 7$.

Let $f(x) = xg^{-1}(x)$, what is the value of $f'(3)$?

A. $\frac{10}{7}$

B. 4

C. $3\frac{1}{7}$

D. $\frac{8}{7}$

End of Section I

Section II

60 marks
Attempt Questions 11 to 14
Allow about 1 hour and 45 minutes for this section
Instructions

- Answer the questions in the appropriate writing booklet.
- In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a new Writing Booklet.

(a) Evaluate $\int_{-1}^{1} \frac{dx}{\sqrt{4-3x^2}}$. **2**

(b) Four Australians, three Germans and two Japanese are seated at random around a round table. With the seating organised at random, find the probability that:

(i) they are seated separately based on their nationality. **2**

(ii) only the Australians are seated together. **1**

(c) Consider the vectors $\underset{\sim}{a} = k\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + k\underset{\sim}{j}$, where $k \in \mathbb{R}$. **3**

Find the value(s) of k when the acute angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$ is $60°$.

(d) Given that $t = \tan\frac{\pi}{8}$,

(i) Show that $\frac{2t}{1-t^2} = 1$. **1**

(ii) Hence, find the exact value of $t = \tan\frac{\pi}{8}$. **2**

(e) (i) Show $\cos 3x - \cos 2x + \cos x = 4\cos^3 x - 2\cos^2 x - 2\cos x + 1$. **2**

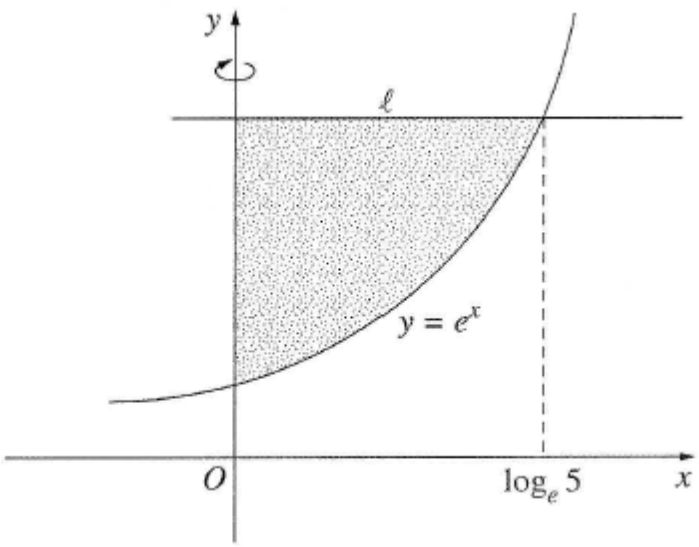
(ii) Hence, solve $\cos 3x - \cos 2x + \cos x = 1$ in the domain $[0,2\pi]$. **2**

Assessment task continues on the other page

Question 12 (15 marks) Use a new Writing Booklet.

(a) Find $\int \cos 7x \sin 3x \, dx$. **2**

(b) In the diagram below, the shaded region is bounded by the y-axis, the curve $y = e^x$ and a horizontal line ℓ that cuts the curve at a point whose x-coordinate is $\log_e 5$. **2**



Not to scale

A solid is formed by rotating the shaded region about the y-axis.

Write down a definite integral whose value is the volume of the solid. (Do NOT evaluate the integral)

(c) (i) Express $\sqrt{3}\sin\frac{x}{2} + \cos\frac{x}{2}$ as $A\cos\left(\frac{x}{2} - \alpha\right)$, where $A > 0$ and $0 \le \alpha \le \frac{\pi}{2}$. **2**

(ii) Hence, or otherwise, solve the equation $\sqrt{3}\sin\frac{x}{2} + \cos\frac{x}{2} = \frac{1}{2}$ for $0 \le x \le 2\pi$. **2**

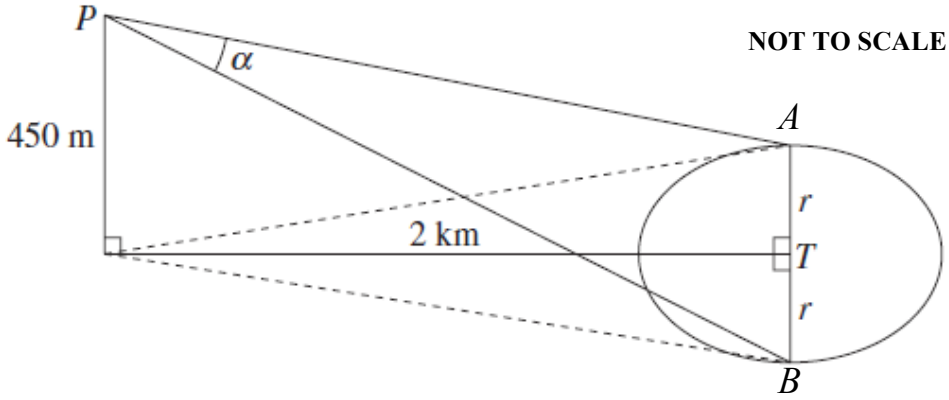
Write your answer(s) correct to three decimal places.

(d) When the polynomial $P(x)$ is divided by $(x+4)$ and $(x-1)$, the remainders are 5 and 9, respectively. Find the remainder when $P(x)$ is divided by $(x-1)(x+4)$. **3**

Question 12 continues on the next page

(e) An oil tanker at T is leaking oil which forms a circular oil slick around it. An observer

measures the oil slick from a position P, 450 metres above sea level and 2 kilometres horizontally from the centre of the oil slick. The triangle PAB is isosceles with $PA=PB$.



(i) At a certain time, the observer measures the angle, α, subtended by the diameter of the oil slick, to be 0.1 radians. Find an expression for the radius, r, at this time, in terms of α. **2**

(ii) At this time, $\frac{d\alpha}{dt} = 0.02$ radians per hour. Find the rate at which the radius of the oil slick is growing. **2**

Assessment task continues on the other page

Question 13 (15 marks) Use a new Writing Booklet.

(a) Use the substitution $u = \log_e x$ to evaluate $\int_e^{e^2} \frac{2}{x(\log_e x)^2}\, dx$. **3**

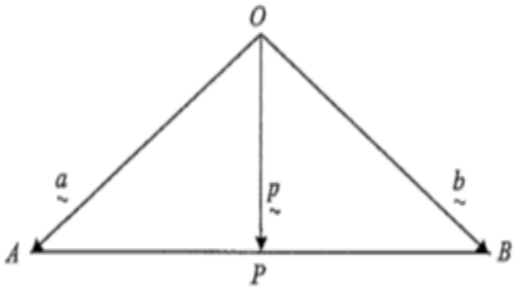
(b) Use mathematical induction to prove that: **3**

$$1+4+16+...+4^n = \frac{1}{3}(4^{n+1}-1) \text{ for } n \geq 0.$$

(c) A small fishing boat can travel at 10 km/h in still water. It is being steered due east but there is a current running south at 5 km/h and a breeze blowing the boat on a bearing of N30E at 4km/h. Find the resultant velocity of the boat, giving the speed correct to 2 decimal places and the direction correct to the nearest degree. **3**

(d) Solve $\frac{dy}{dx} = 6x^2 y$ given that the point $(0, e)$ lies on the function and $y \neq 0$. **3**

(e) In the diagram below, OAB is a triangle with $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OB} = \underset{\sim}{b}$ and $OA = OB$. **3**
P is a point on AB such that $\overrightarrow{OP} = \underset{\sim}{p}$ and OP bisects $\angle AOB$.



Use vector methods to show that $OP \perp AB$.

Assessment task continues on the other page

Question 14 (15 marks) Use a new Writing Booklet.

(a) Consider the function $f(x) = \cos^{-1}\left(\frac{x}{2}\right)$.

(i) State the domain and range of $f(x)$. **2**

(ii) By sketching $f(x)$, determine whether $f(x)$ is odd, even or neither. Justify your answer. **2**

(iii) Hence, or otherwise, sketch the curve $y = \frac{1}{f(x)}$, without using calculus. **2**

Clearly label any asymptotes, intercepts with the coordinate axes and any endpoints.

(b) In the town of Hornsville, the population, P, in the year 2000 was estimated to be 7 500. The population growth is given by the logistic equation:

$$\frac{dP}{dt} = 0.03P\left(\frac{C-P}{C}\right),$$

where t is the number of years after 2000 and C is the carrying capacity.
In the year 2025, the population was estimated to be 10 000.

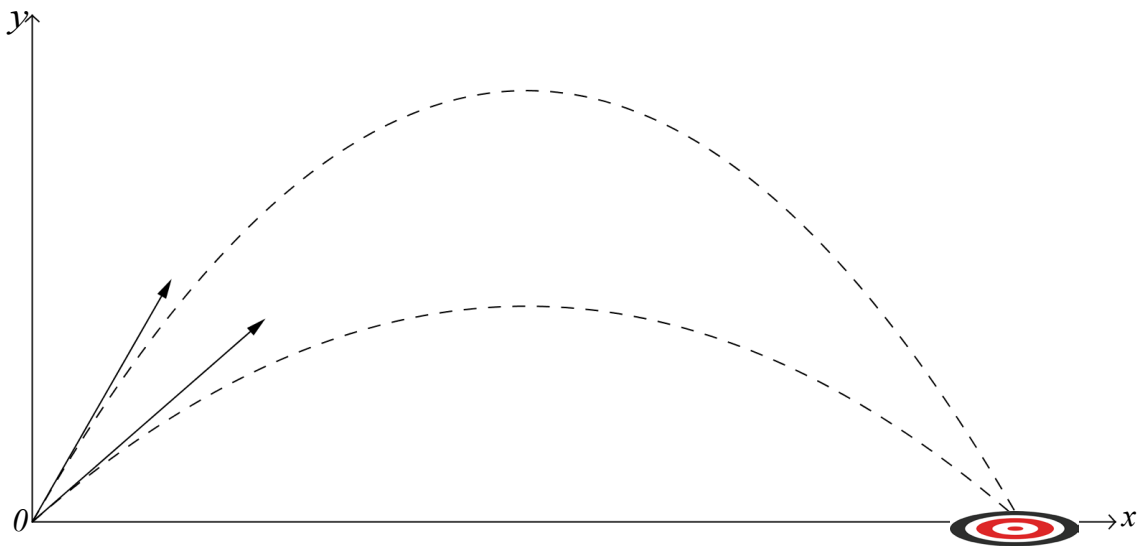
(i) Show that $\frac{C}{P(C-P)} = \frac{1}{P} + \frac{1}{C-P}$. **1**

(ii) Use part (i), or otherwise, to find the carrying capacity, C, correct to the nearest thousand. **3**

Question 14 continues on the next page.

(c) Two archers, A and B, stand at the same spot, 0. Each archer shoots one arrow at the same time at a target which lies on the same horizontal plane as the point of projection of the arrows.

The archers shoot with the same initial velocity, 25m/s, but with different angles of projection, with archer A's angle of projection being 60°. Their arrows hit the target at different times.



You may assume that the position vector, $\underset{\sim}{r}(t)$, of the path of each arrow after projection from 0, after t seconds is given by:

$$\underset{\sim}{r}(t) = Vt\cos\theta\,\underset{\sim}{i} + \left(Vt\sin\theta - 5t^2\right)\underset{\sim}{j} \qquad \textbf{DO NOT PROVE THIS}$$

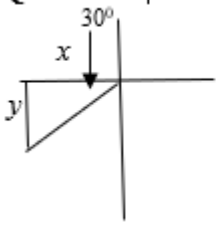
(i) Show that the distance of the target from the archers is $\dfrac{125\sqrt{3}}{4}$ metres. **2**

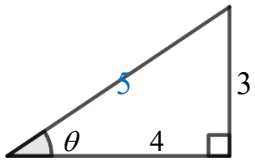
(ii) Showing full working, prove that the angle of elevation of archer B's arrow is equal to the complement of the angle of elevation of archer A's arrow. **2**

(iii) Find the exact time which elapses between when the two arrows hit the target. **1**

End of assessment task

Hornsby Girls High School
Year 12 Mathematics Extension 1 HSC Trial 2026 Solutions

Solutions	Marker's Comments
Question 1 C $\int \cos^2 3x\ dx = \int\left(\frac{\cos 6x+1}{2}\right)dx$ $= \frac{1}{2}\left(\frac{\sin 6x}{6}+x\right)$ $= \frac{\sin 6x}{12}+\frac{x}{2}+C$	Mostly well done (90%)
Question 2 A Switch the values of x and y + switch the sign so that $x_1x_2+y_1y_2=0$	Mostly well done (95%)
Question 3 A $y = e^{kx},\ k \in R,$ $\frac{dy}{dx} = ke^{kx}$ $\frac{d^2y}{dx^2} = k^2e^{kx}$ For $\frac{d^2y}{dx^2}-\frac{dy}{dx}-6y=0$ $k^2e^{kx}-ke^{kx}-6e^{kx}=0$ $e^{kx}\left(k^2-k-6\right)=0$ Since $e^{kx}>0$ for all real x, $\therefore k^2-k-6=0$ $(k-3)(k+2)=0$ $\therefore k=3$ or $k=-2$	Mostly well done (88.75%)
Question 4 D  x=-12cos30° $y=-12\sin 30°$ $x=-6\sqrt{3}$ $y=-6$	Poorly done (48.75%) 50% of cohort chose B Area of concern: 1) students had $x=12\sin 30°$ and $y=-12\cos 30°$
Question 5 C Assume 15 Language + 10 Craft books are chosen. Then there must be 25+1=26 to be sure that at least one of each is chosen.	Mostly well done (88.75%)

Solutions	Marker's Comments
Question 6 **C**  $\tan\theta = \frac{3}{4}$ $\underset{\sim}{r}(t) = \begin{bmatrix} 240t\cos\theta \\ 2000 + 240t\sin\theta - \frac{gt^2}{2} \end{bmatrix}$ $\underset{\sim}{v}(t) = \begin{bmatrix} 240\cos\theta \\ 240\sin\theta - gt \end{bmatrix}$ $\therefore \underset{\sim}{v}(20) = \begin{bmatrix} 240\left(\frac{4}{5}\right) \\ 240\left(\frac{3}{5}\right) - 10(20) \end{bmatrix}$ $= \begin{bmatrix} 192 \\ -56 \end{bmatrix}$ $\therefore \left\lvert \underset{\sim}{v}(20) \right\rvert = \sqrt{(192)^2 + (-56)^2}$ $= 200\ \text{ms}^{-1}$	Poorly done (58.75%) Area of concern: 1) students had not used a right-angle triangle to work out and had $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$ and $y = -12\cos 30°$ 2) Student did not find the magnitude of velocity vector for speed.
Question 7 **D** Testing points (2,2),(-2,2),(2,-2),(-2,-2) in each equation	Mostly well done (85%)
Question 8 **C** $f(x) = x\tan^{-1}\frac{1}{x} + \frac{1}{2}\log_e(x^2+1)$ $f'(x) = \tan^{-1}\frac{1}{x} \times 1 + x \times \frac{-x^{-2}}{\left(\frac{1}{x}\right)^2 + 1} + \frac{1}{2}\cdot\frac{2x}{x^2+1}$ $f'(x) = \tan^{-1}\frac{1}{x} + x \times \frac{1}{x^2} \times \frac{-1}{\frac{1}{x^2}+1} + \frac{x}{x^2+1}$ $f'(x) = \tan^{-1}\frac{1}{x} + x \times \frac{1}{x^2} \times \frac{-1}{\frac{1+x^2}{x^2}} + \frac{x}{x^2+1}$ $f'(x) = \tan^{-1}\frac{1}{x} + x \times \frac{1}{x^2} \times \frac{-x^2}{1+x^2} + \frac{x}{x^2+1}$ $f'(x) = \tan^{-1}\frac{1}{x} + \frac{-x}{1+x^2} + \frac{x}{x^2+1}$ $f'(x) = \tan^{-1}\frac{1}{x}$	Fairly well done (70%) Majority chose D incorrectly (25%) Area of concern: 1) students did not notice $x\tan^{-1}\frac{1}{x}$ is a product of two functions and chose D.

Solutions	Marker's Comments
Question 9 **D** $x = 2\sin\theta - 5$ $2\sin\theta = x + 5$ $\sin\theta = \frac{x+5}{2}$ $\sin^2\theta = \frac{(x+5)^2}{4}$ $y = 3\cos\theta + 1$ $3\cos\theta = y - 1$ and $\cos\theta = \frac{y-1}{3}$ $\cos^2\theta = \frac{(y-1)^2}{9}$ since $\sin^2\theta + \cos^2\theta = 1$ then $\frac{(x+5)^2}{4} + \frac{(y-1)^2}{9} = 1$	Mostly well done (95%) 5% chose B incorrectly, forgetting to square the denominator values.
Question 10 **A** $g^{-1}(3) = 1,\ \frac{d}{dx}g^{-1}(3) = \frac{1}{g'(1)} = \frac{1}{7}$ $f'(x) = 1g^{-1}(x) + x\frac{d}{dx}g^{-1}(x)$ $f'(3) = 1g^{-1}(3) + x\frac{d}{dx}g^{-1}(3)$ $= 1 + 3\cdot\frac{1}{7}$ $= \frac{10}{7}$	Fairly well done (61.25%) Majority chose C incorrectly (23.75%) Students need to make sure they reverse differentiation of inverse functions.

SECTION II

Solutions	Marker's Comments
Question 11 (a) $\int_{-1}^{1}\frac{dx}{\sqrt{4-3x^2}}=\frac{1}{\sqrt{3}}\left[\sin^{-1}\left(\frac{\sqrt{3}x}{2}\right)\right]_{-1}^{1}$ $=\frac{1}{\sqrt{3}}\left[\sin^{-1}\left(\frac{\sqrt{3}(1)}{2}\right)-\sin^{-1}\left(\frac{\sqrt{3}(-1)}{2}\right)\right]$ $=\frac{1}{\sqrt{3}}\left[\frac{\pi}{3}-\left(-\frac{\pi}{3}\right)\right]$ $=\frac{1}{\sqrt{3}}\frac{2\pi}{3}$ $=\frac{2\pi}{3\sqrt{3}}$	Mostly well done.
(b) (i) $\frac{4!\times3!\times2!\times2}{8!}$	Need to revise the topic of Permutations and Combinations. Some students forgot circular arrangements are (n-1)!. Some mistakes in permutation of the groups.
(b) (ii) $\frac{4!\times5!}{8!}=\frac{1}{14}$	Not very well done. Need to revise circular arrangements with grouping.
(c) $\underset{\sim}{a}\cdot\underset{\sim}{b}=\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert\cos60°$ $k\times2+2\times k=\sqrt{k^2+2^2}\sqrt{2^2+k^2}\cos60°$ $4k=(k^2+4)\times\frac{1}{2}$ $8k=k^2+4$ $k^2-8k+4=0$ $(k-4)^2-16+4=0$ $(k-4)^2=12$ $k-4=\pm\sqrt{12}$ $k=4\pm2\sqrt{3}$	Very well done. Some mistakes solving the quadratic equation.
(d) (i) $\tan\left(2\times\frac{\pi}{8}\right)=\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$ $\tan\frac{\pi}{4}=\frac{2t}{1-t^2}$ $\therefore\ 1=\frac{2t}{1-t^2}$ as required	Mostly well done. Many students understood the t-formula.

(d) (ii) $1 = \dfrac{2t}{1-t^2}$ $1-t^2 = 2t$ $t^2 + 2t - 1 = 0$ $t = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2}$ $= \dfrac{-2 \pm \sqrt{8}}{2}$ $= -1 \pm \sqrt{2}$ since $t = \tan\dfrac{\pi}{8} > 0$, $\therefore t = -1 + \sqrt{2}$	Mostly well done. The negative solution should be rejected because $\tan\dfrac{\pi}{8}$ is positive.
(e) (i) $LHS = \cos 3x + \cos x - \cos 2x$ $= 2\cos\left(\dfrac{3x+x}{2}\right)\cos\left(\dfrac{3x-x}{2}\right) - \cos 2x$ $= 2\cos 2x \cos x - \cos 2x$ $= \cos 2x(2\cos x - 1)$ $= (2\cos^2 x - 1)(2\cos x - 1)$ $= 4\cos^3 x - 2\cos^2 x - 2\cos x + 1$ $= RHS$	Mostly well done. Sufficient steps need to be shown to prove the equation.
(e) (ii) $4\cos^3 x - 2\cos^2 x - 2\cos x + 1 = 1$ $4\cos^3 x - 2\cos^2 x - 2\cos x = 0$ $\cos x(2\cos^2 x - \cos x - 1) = 0$ $\cos x(2\cos x + 1)(\cos x - 1) = 0$ $\cos x = 0,\ \cos x = \dfrac{-1}{2},\ \cos x = 1, \quad 0 \le x \le 2\pi$ $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, 0, 2\pi$	Mostly well done. Some careless mistakes.

Question 12

(a)

$$\int \cos 7x \sin 3x \, dx = \int \sin(7x+3x) - \sin(7x-3x)\, dx$$

$$= \int \sin 10x - \sin 4x \, dx \quad ①$$

$$= -\frac{\cos 10x}{10} - \left(-\frac{\cos 4x}{4}\right) + C$$

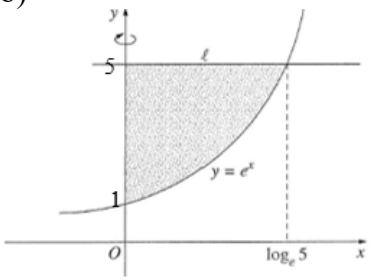
$$= -\frac{\cos 10x}{10} + \frac{\cos 4x}{4} + C \quad ①$$

Mostly well done

Area of concern:

1) Some students did not recognise the integrand as product of trigonometry
2) Some students did not know how to convert to sum of trigonometry.

(b)



When $x = \log_e 5$, $y = e^{\log_e 5}$

$\therefore y = 5$ ①

When $x = 0$, $\therefore y = e^0 = 1$

$$V = \pi \int_1^5 x^2 \, dy \qquad \text{where } y = e^x$$

$$\therefore x = \log_e y$$

$$V = \pi \int_1^5 (\log_e y)^2 \, dy \quad ①$$

Mostly well done.

Area of concern:

1) Biggest error involved is with using the volume of a cylinder subtracting the unshaded volume where students had the unshaded area rotated about the x-axis.
2) Some students did not recognise the upper limit is $y = 5$ for the integration.

(c)(i) $\sqrt{3}\sin\frac{x}{2} + \cos\frac{x}{2} = A\cos\left(\frac{x}{2} - \alpha\right)$ where $A > 0$

and $0 \le \alpha \le \frac{\pi}{2}$.

$$\sqrt{3}\sin\frac{x}{2} + \cos\frac{x}{2} = A\cos\frac{x}{2}\cos\alpha + A\sin\frac{x}{2}\sin\alpha$$

Equating like terms, $A\cos\alpha = 1$ — ①

$A\sin\alpha = \sqrt{3}$ — ②

①² + ②² $A^2\cos^2\alpha + A^2\sin^2\alpha = (1)^2 + \left(\sqrt{3}\right)^2$

$A^2(\cos^2\alpha + \sin^2\alpha) = 1 + 3$

$A^2(1) = 4$

$A = 2, \ A > 0$ ①

② ÷ ① $\frac{A\sin\alpha}{A\cos\alpha} = \frac{\sqrt{3}}{1}$

Mostly done well.

Area of concern:

1) some students again showed they did not know exact values for trigonometry:
$\alpha = \tan^{-1}\sqrt{3} = \frac{\pi}{6}$
2) Some student did not recognise that the identity is $\sqrt{3}\sin\frac{x}{2} + \cos\frac{x}{2} = A\cos\left(\frac{x}{2} - \alpha\right)$

and not $\sqrt{3}\sin\frac{x}{2} + \cos\frac{x}{2} = A\sin\left(\frac{x}{2} + \alpha\right)$

(c)(i)cont. $\tan\alpha = \sqrt{3}$

$$\alpha = \frac{\pi}{3} \quad ①$$

$$\therefore \sqrt{3}\sin\frac{x}{2} + \cos\frac{x}{2} = 2\cos\left(\frac{x}{2} - \frac{\pi}{3}\right)$$

(c)(ii) $\sqrt{3}\sin\frac{x}{2} + \cos\frac{x}{2} = \frac{1}{2}$

$$2\cos\left(\frac{x}{2} - \frac{\pi}{3}\right) = \frac{1}{2}, \quad 0 \le x \le 2\pi$$

$$0 \le \frac{x}{2} \le \pi$$

$$-\frac{\pi}{3} \le \frac{x}{2} - \frac{\pi}{3} \le \pi - \frac{\pi}{3}$$

$$-\frac{\pi}{3} \le \frac{x}{2} - \frac{\pi}{3} \le \frac{2\pi}{3}$$

$$\cos\left(\frac{x}{2} - \frac{\pi}{3}\right) = \frac{1}{4} \quad ①$$

$$\frac{x}{2} - \frac{\pi}{3} = -\cos^{-1}\frac{1}{4}, \ \cos^{-1}\frac{1}{4}$$

$$\frac{x}{2} = -\cos^{-1}\frac{1}{4} + \frac{\pi}{3}, \ \cos^{-1}\frac{1}{4} + \frac{\pi}{3}$$

$$\frac{x}{2} = 2.36531, \quad 0 \le \frac{x}{2} \le \pi$$

$$x = 4.73062...$$

$\therefore x = 4.731$ (to 3 d.p.)①

Mostly done well.
Area of concern:
1) some students had more than one answer as they did not consider the restricted domain.

(d) $P(x) = (x-1)(x+4)Q(x) + ax + b$,

where $\{a, b\} \in R$ ①

Given: $P(-4) = 5$ and $P(1) = 9$

$-4a + b = 5$ — ① $\quad a + b = 9$ — ②

② - ① $\quad 5a = 4$

$$\therefore a = \frac{4}{5} \text{ — ③ ①}$$

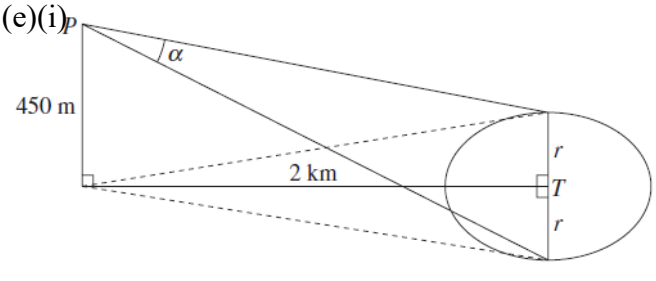
Sub③ into ② $b = 9 - \frac{4}{5}$

$$\therefore b = \frac{41}{5}$$

Hence the remainder is $\frac{4}{5}x + \frac{41}{5}$ ①

Very poorly done.
Area of concern:
1) many students forgotten the remainder theorem; the remainder function with a quadratic divisor being is a linear function.

(e)(i)



Very poorly done.
Area of concern:
1) many students did not pay attention to the fact that α is in ΔPAB and 2km is in ΔOAB. So many errors is assuming $PT = 2$ km.
2) Common mistakes made by students were the lack attention to detail that 2 km is not

$PT = \sqrt{(0.45)^2 + (2)^2}$

$PT = 2.05$ km ①

$\tan\frac{\alpha}{2} = \frac{r}{PT}$

$r = PT\tan\frac{\alpha}{2}$

$\therefore r = 2.05\tan\frac{\alpha}{2}$ ①

(e)(ii) Given: $\frac{d\alpha}{dt} = 0.02$ radian per hour.

$\frac{dr}{dt} = ?$ when $r = 102.6$ m, $\alpha = 0.1$ radians

$\frac{dr}{dt} = \frac{dr}{d\alpha} \times \frac{d\alpha}{dt}$ where $r = 2.05\tan\frac{\alpha}{2}$

$\frac{dr}{d\alpha} = \frac{2.05}{2}\sec^2\frac{\alpha}{2}$

$\frac{dr}{d\alpha} = 1.025\sec^2\frac{\alpha}{2}$ ①

$\frac{dr}{dt} = 1.025\sec^2\frac{(0.1)}{2} \times 0.02$

$= 0.02055\ldots$ km per hour

$\therefore \frac{dr}{dt} = 0.0206$ km per hour ①

2 m, many found $PT = \sqrt{(450)^2 + (2)^2}$

3) a few students used cosine rule and did not know how to continue, not realising that $\cos\alpha = \cos 2\alpha$ and can obtain the answer.

4) A few students made the mistakes of substituting $\alpha = 0.1$ rad and later in part (ii) realised they did not have a variable α to differentiate, and became lost.

Very poorly done.
Area of concern:

1) many students did not substitute $\alpha = 0.1$ rad.

Question 13

(a)

$u = \ln x$

$\therefore du = \frac{1}{x}dx = \frac{1}{e^u}dx$

$$\int_e^{e^2} \frac{2}{x(\ln x)^2}dx = \int_1^2 \frac{2}{e^u u^2}\cdot e^u du$$

$$= \int_1^2 \frac{2}{u^2}du$$

$$= 2\left[-\frac{1}{u}\right]_1^2$$

$$= 1$$

(b) Step 1: To prove the statement true for $n = 0$

$\text{LHS} = 4^0 = 1$

$\text{RHS} = \frac{1}{3}[4^{0+1} - 1] = 1$

Result true for $n = 0$

Step 2: Assume the result true for $n = k$

$$1 + 4 + 16 + \ldots + 4^k = \frac{1}{3}[4^{k+1} - 1]$$

Step 3: To prove the result true for $n = k + 1$

a) Very well done by students

b) Many students proved true for *n=1* instead of *n=0,* which is incorrect for this question.

$1 + 4 + 16 + \ldots + 4^k + 4^{k+1} = \frac{1}{3}\left[4^{(k+1)+1} - 1\right]$

$$\begin{aligned}\text{LHS} &= 1 + 4 + 16 + \cdots + 4^k + 4^{k+1} \\ &= \frac{1}{3}[4^{k+1} - 1] + 4^{k+1} \\ &= \frac{1}{3}[4^{k+1} - 1] + \frac{3}{3}4^{k+1} \\ &= \frac{1}{3}[4^{k+1} - 1 + 3 \times 4^{k+1}] \\ &= \frac{1}{3}[4 \times 4^{k+1} - 1] \\ &= \frac{1}{3}\left[4^{(k+1)+1} - 1\right] \\ &= \text{RHS}\end{aligned}$$

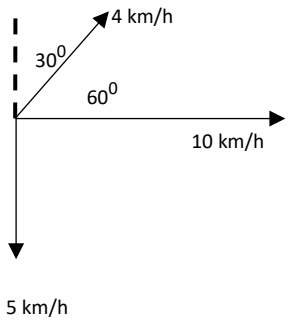
Result is true for $n = k + 1$

Step 4: If true for $n=k$, proven true for $n=k+1$.

Since true for $n=0$, true for $n=0+1=1$, $n=1+1=2$ and so on for all integers $n \geq 0$

Some students need to improve their conclusion

(c)



$10\underset{\sim}{i} + 0\underset{\sim}{j}$

$0\underset{\sim}{i} - 5\underset{\sim}{j}$

$4\cos 60^\circ \underset{\sim}{i} + 4\sin 60^\circ \underset{\sim}{j}$

$= 4 \times \frac{1}{2}\underset{\sim}{i} + 4 \times \frac{\sqrt{3}}{2}\underset{\sim}{j}$

$= 2\underset{\sim}{i} + 2\sqrt{3}\underset{\sim}{j}$

$\left(10 + 2\right)\underset{\sim}{i} + \left(-5 + 2\sqrt{3}\right)\underset{\sim}{j}$

$= 12\underset{\sim}{i} + \left(2\sqrt{3} - 5\right)\underset{\sim}{j}$

$|v| = \sqrt{12^2 + (2\sqrt{3} - 5)^2}$

$= \sqrt{146.3589838}$

$= 12.09789171$

$= 12.10\,km/h$

Many students made an error with the breeze and used

$4\cos 30^\circ \underset{\sim}{i} + 4\sin 30^\circ \underset{\sim}{j}$

$= 4 \times \frac{\sqrt{3}}{2}\underset{\sim}{i} + 4 \times \frac{1}{2}\underset{\sim}{j}$

$= 2\sqrt{3}\underset{\sim}{i} + 2\underset{\sim}{j}$

Which gave 13.79 km/h and

$\tan\theta = -0.2228147177$

$\theta = -12.56115361^\circ$

Bearing is $90^\circ + 12.56...^\circ = 103^\circ$ to nearest degree

Some used $0\underset{\sim}{i} + 5\underset{\sim}{j}$ instead of $0\underset{\sim}{i} - 5\underset{\sim}{j}$

$\tan\theta = \dfrac{2\sqrt{3}-5}{12}$ $\tan\theta = -0.1279915321$ $\theta = -7.293718952^\circ$ Bearing is $90^\circ + 7.29...^\circ = 97^\circ$ to nearest degree	Several found the angle but did not find the bearing.
(d) $\dfrac{dy}{dx} = 6x^2 y$ $\dfrac{1}{y}dy = 6x^2 dx$ $\int \dfrac{1}{y}dy = \int 6x^2 dx$ $\ln\lvert y\rvert = \dfrac{6x^3}{3} + c$ $\ln\lvert y\rvert = 2x^3 + c$ $y = e$ when $x = 0$ $\ln e = 0 + c$ $1 = 0 + c$ $c = 1$ $\ln\lvert y\rvert = 2x^3 + 1$ $\lvert y\rvert = e^{2x^3+1}$ $y = \pm e^{2x^3+1}$ But $(0, e)$ is on the function so $y>0$ $y = e^{2x^3+1}$	Many students failed to discuss $\pm$ and why the negative case was discarded.
(e) Given OP bisects $\angle AOB$, and $OA=OB$ $\angle AOP = \angle BOP$ $\therefore \dfrac{\underset{\sim}{a}\cdot\underset{\sim}{p}}{\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{p}\rvert} = \dfrac{\underset{\sim}{b}\cdot\underset{\sim}{p}}{\lvert\underset{\sim}{b}\rvert\lvert\underset{\sim}{p}\rvert}$ and $\lvert\underset{\sim}{a}\rvert = \lvert\underset{\sim}{b}\rvert$ so $\dfrac{\underset{\sim}{a}\cdot\underset{\sim}{p}}{\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{p}\rvert} = \dfrac{\underset{\sim}{b}\cdot\underset{\sim}{p}}{\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{p}\rvert}$ $\therefore \underset{\sim}{a}\cdot\underset{\sim}{p} = \underset{\sim}{b}\cdot\underset{\sim}{p}$ $0 = \underset{\sim}{b}\cdot\underset{\sim}{p} - \underset{\sim}{a}\cdot\underset{\sim}{p}$ $0 = \underset{\sim}{p}\cdot(\underset{\sim}{b} - \underset{\sim}{a})$ But $\underset{\sim}{b} - \underset{\sim}{a} = \overrightarrow{AB}$ $\therefore OP \perp AB$	$\underset{\sim}{a}\cdot\underset{\sim}{p} = \underset{\sim}{b}\cdot\underset{\sim}{p}$ **does not mean** $\underset{\sim}{a} = \underset{\sim}{b}$

OR

$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$

$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$

In ΔOAP and ΔOBP

$\angle AOP = \angle BOP$ given OP bisects $\angle AOB$

$OA=OB$ given

OP is common

$\Delta OAP \equiv \Delta OBP \ (SAS)$

$\therefore \overrightarrow{AP} = \overrightarrow{PB}$

and $\overrightarrow{AP} + \overrightarrow{PB} = \overrightarrow{AB}$

$\overrightarrow{AP} + \overrightarrow{AP} = \overrightarrow{AB}$

so $\overrightarrow{AP} = \frac{1}{2}\overrightarrow{AB}$

$\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$= \underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{b} - \underset{\sim}{a})$

$= \underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$

$= \frac{1}{2}\underset{\sim}{a} + \frac{1}{2}\underset{\sim}{b}$

$\overrightarrow{OP} = \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$

$\overrightarrow{AB} \cdot \overrightarrow{OP} = (\underset{\sim}{b} - \underset{\sim}{a}) \cdot \frac{1}{2}(\underset{\sim}{a} + \underset{\sim}{b})$

$= \frac{1}{2}\left[\underset{\sim}{a} \cdot \underset{\sim}{b} + |\underset{\sim}{b}|^2 - |\underset{\sim}{a}|^2 - \underset{\sim}{a} \cdot \underset{\sim}{b}\right]$

$= \frac{1}{2}\left[|\underset{\sim}{b}|^2 - |\underset{\sim}{a}|^2\right]$

But $OA=OB$ so $|\underset{\sim}{a}| = |\underset{\sim}{b}|$

$\overrightarrow{AB} \cdot \overrightarrow{OP} = \frac{1}{2}\left[|\underset{\sim}{b}|^2 - |\underset{\sim}{a}|^2\right]$

$= \frac{1}{2}\left[|\underset{\sim}{a}|^2 - |\underset{\sim}{a}|^2\right]$

$= \frac{1}{2} \times 0$

$\overrightarrow{AB} \cdot \overrightarrow{OP} = 0$

$\therefore \overrightarrow{AB} \perp \overrightarrow{OP}$

Many students did not explain why

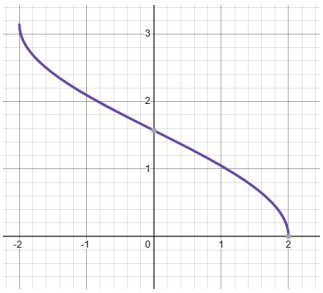
$\overrightarrow{AP} = \frac{1}{2}\overrightarrow{AB}$

Some stated that OP bisects $\angle AOB$ but that is insufficient. *OA=OB* must be used as well.

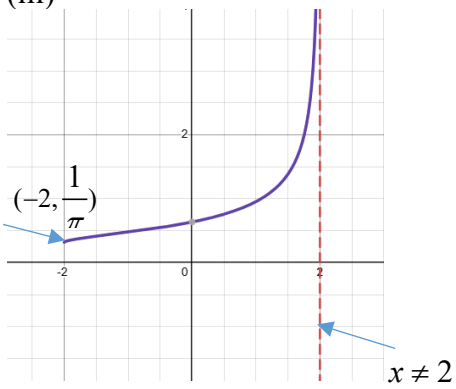
Question 14

(a) i) D:[-2,2] ; R:[0,π]

(ii) The function is neither odd nor even.



(iii)



(a) (i) Done well. However, some students divided by 2 instead of multiplying by 2.

(ii) The graph was drawn well however quite a few students stated that the function was odd because it had rotational symmetry.

(iii) Generally well done.

(b) i) RHS $= \frac{1}{P} + \frac{1}{C-P}$

$= \frac{C-P+P}{P(C-P)}$

$= \frac{C}{P(C-P)}$

=LHS

(b) (i) Very well done.

(ii) Poorly done. Many students didn't substitute t = 25 when P=10000.

(b) ii) $\int \frac{C}{P(C-P)}\,dP = \int 0.03\,dt$ $\int\left(\frac{1}{P}+\frac{1}{C-P}\right)dP = 0.03t + k$ $\ln\lvert P\rvert - \ln\lvert C-P\rvert = 0.03t + k$ $\ln\left\lvert\frac{P}{C-P}\right\rvert = 0.03t + k$ $\frac{P}{C-P} = \pm e^{0.03t+k}$ $= Ae^{0.03t}\ (where\ A = \pm e^{k})$ $when\ t = 0, P = 7500$ $when\ t = 25, P = 10000$ $\frac{7500}{C-7500} = Ae^{0}$.....1 $\frac{10000}{C-10000} = Ae^{0.75}$.....2 *equating* 1 *and* 2 $\frac{7500e^{0.75}}{C-7500} = \frac{10000}{C-10000}$ $10000(C-7500) = 7500e^{0.75}(C-10000)$ $10000C - 75000000 = 7500e^{0.75}C - 158775001.2$ $83775001.25 = 5877.5500125C$ $C = 14253.50906$ $C = 14000\ (correct\ to\ the\ nearest\ thousand)$	
(c) i) $Solve\ y = 0$ $0 = 25t\sin 60° - 5t^2$ $0 = 5t\left(\frac{5\sqrt{3}}{2} - t\right)$ $t = 0, t = \frac{5\sqrt{3}}{2}$ $sub.t = \frac{5\sqrt{3}}{2}\text{in}\ x$ $x = 25\times\frac{5\sqrt{3}}{2}\times\cos 60°$ $x = \frac{125\sqrt{3}}{4}m\ (as\ required)$	(c) (i) Very well done.

(c) ii) *Method* 1 *sub.* $\theta = 30°$ in $y = 0$ $25t\sin 30° - 5t^2 = 0$ $5t\left(\frac{5}{2} - t\right) = 0$ $t \neq 0, t = \frac{5}{2}$ *sub.* $t = \frac{5}{2}$ *and* $\theta = 30°$ *in* x $x = 25 \times \frac{5}{2} \times \cos 30°$ $x = \frac{125\sqrt{3}}{4} m$ same distance as part i) *Method* 2 $25t\sin\theta - 5t^2 = 0$ $5t(5\sin\theta - t) = 0$ $t \neq 0, t = 5\sin\theta$ *sub.* $t = 5\sin\theta$ *in* x $x = 25 \times 5\sin\theta\cos\theta$ *sub.* $x = \frac{125\sqrt{3}}{4}$ *in* x $\frac{125\sqrt{3}}{4} = \frac{125}{2}\sin 2\theta$ $\sin 2\theta = \frac{\sqrt{3}}{2}$ $(Q1 + Q2)$ *acute* $2\theta = \frac{\pi}{3}, \frac{2\pi}{3}$ $\theta = \frac{\pi}{6}, \frac{\pi}{3}$ $\theta = 30°$, *which is the complement of* $60°$.	(ii) Generally well done.
(c) iii) $t = \frac{5\sqrt{3}}{2}$ *for archer A* $t = \frac{5}{2}$ *for archer B* $\therefore$ *time lapse between the two is* : $t = \frac{5}{2}(\sqrt{3} - 1)s$	(iii) Very well done.