STUDENT'S NAME: _____

TEACHER'S NAME: _____

2026

HURLSTONE AGRICULTURAL HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE ASSESMENT TASK 4

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using a blue or black pen
- NESA approved calculators may be used
- A reference sheet is provided at the end of this booklet
- For questions in Section II, use the relevant booklet for writing your solutions.
- Show all relevant mathematical reasoning and /or calculations.

Total marks: 70

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10. The multiple-choice answer sheet has been provided
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6 – 12)

- Attempt Questions 11 – 16, write your solutions in the spaces provided.
- You have been provided with 6 separate answer booklets, one for each question.
- Extra working pages are available if required.
- Allow about 1 hours and 45 minutes for this section.

This examination paper is not to be removed from the Examination Centre

Disclaimer: Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2026 HSC Mathematics Extension 1 Examination.

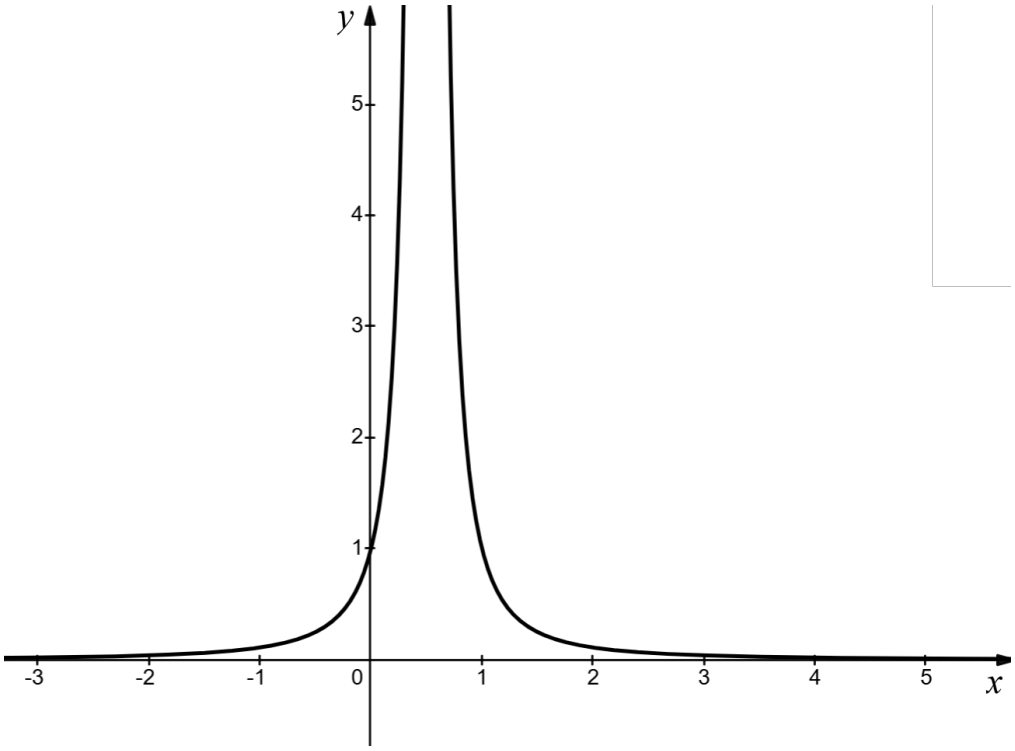
SECTION I (10 marks)

Attempt Questions 1 - 10

Allow about 15 minutes on this section.

Use the multiple-choice answer sheet provided for Questions 1 – 10.

1. The diagram shown below represents the graph of $y = \dfrac{1}{f(x)}$.



Which of the following could be $f(x)$?

(A) $(2x-1)^2$

(B) $2x^2-1$

(C) $(2x+1)^2$

(D) $2x^2+1$

2. If $f(x) = \sqrt{4-x^2}$ $-2 \le x \le 0$, then which of the following gives the inverse function?

(A) $x^2 + y^2 = 4$

(B) $y = \sqrt{4-x^2}$, $-2 \le x \le 2$

(C) $y = \sqrt{4-x^2}$, $0 \le x \le 2$

(D) $y = -\sqrt{4-x^2}$, $0 \le x \le 2$

3. Which expression is equal to $\sin x + \sqrt{3}\cos x$?

(A) $2\sin\left(x+\frac{\pi}{3}\right)$

(B) $2\sin\left(x-\frac{\pi}{3}\right)$

(C) $4\cos\left(x+\frac{\pi}{3}\right)$

(D) $4\cos\left(x-\frac{\pi}{3}\right)$

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4. Given $4\sin^2\left(\frac{\pi}{22}\right)+3=\sqrt{C}$, What is the value of $\cos\left(\frac{\pi}{11}\right)$ in terms of C?

(A) $\frac{\sqrt{C}-5}{2}$

(B) $\frac{5-\sqrt{C}}{2}$

(C) $\frac{3-\sqrt{C}}{4}$

(D) $\frac{\sqrt{C}-3}{4}$

---

5. Consider the vectors $\underset{\sim}{u} = 2\underset{\sim}{i}-5\underset{\sim}{j}$, $\underset{\sim}{v}= \underset{\sim}{i} + a\underset{\sim}{j}$ and $\underset{\sim}{w}= \underset{\sim}{i} + b\underset{\sim}{j}$.
If $\underset{\sim}{v} \parallel \underset{\sim}{u}$ and $\underset{\sim}{u} \perp \underset{\sim}{w}$, what are the possible values for a and b?

(A) $a=\frac{5}{2},\ b=\frac{2}{5}$

(B) $a=\frac{5}{2},\ b=-\frac{2}{5}$

(C) $a=-\frac{5}{2},\ b=\frac{2}{5}$

(D) $a=-\frac{5}{2},\ b=-\frac{2}{5}$

---

6. For what values of a and b does $y=e^{2x}+e^{-2x}$ satisfy the equation $\frac{d^2y}{dx^2}+a\frac{dy}{dx}+by=0$?

(A) $a=-1, b=-2$

(B) $a=-2, b=0$

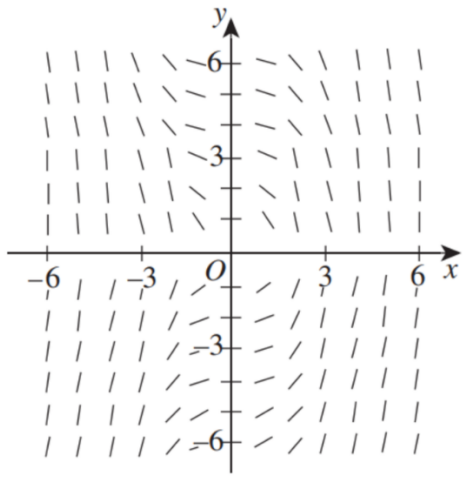
(C) $a=0, b=-4$

(D) $a=-2, b=-1$

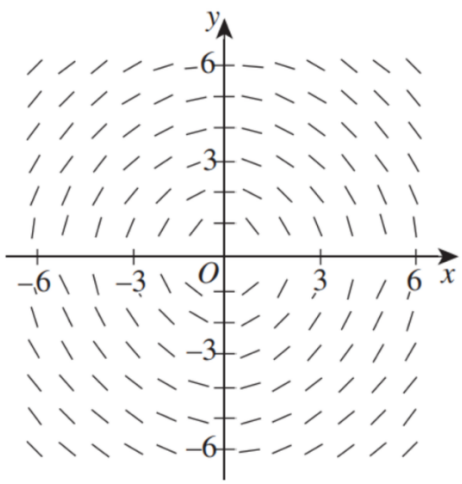
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7. Which of the following slope fields best represents the differential equation $\frac{dy}{dx} = \frac{3x^2}{2y}$?

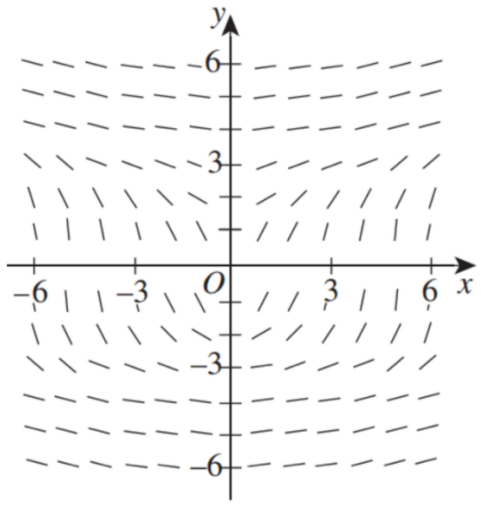
(A)



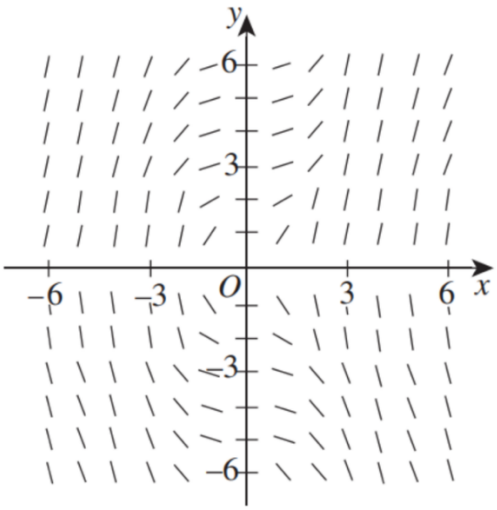
(B)



(C)



(D)



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8. How many ways can the letters in CABRAMATTA be arranged?

(A) $\frac{10!}{(2\times 4)!}$

(B) $\frac{10!}{(2+4)!}$

(C) $\frac{10!}{2!4!}$

(D) $10!$

---

9. What is the gradient of the tangent to the function $y = \sin^{-1} 2x$ at the point $\left(\frac{1}{4}, \frac{\pi}{6}\right)$?

(A) $\frac{\pi}{6}$

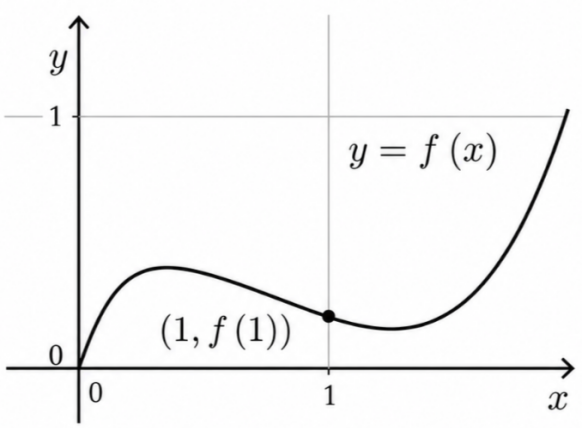
(B) $\frac{1}{\sqrt{3}}$

(C) 1

(D) $\frac{4}{\sqrt{3}}$

---

10. The diagram below shows $y = f(x)$.



Which of the following statements is true?

(A) $f'(1) < f(1) < f''(1) < 1$

(B) $1 < f(1) < f'(1) < f''(1)$

(C) $f''(1) < f(1) < 1 < f'(1)$

(D) $f'(1) < f''(1) < f(1) < 1$

~ End of Section 1 ~

SECTION II

60 marks

Attempt Questions 11 - 16

Allow about 1 hours and 45 minutes on this section.
For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.
Answer each question in a separate answer booklet.

Question 11 (10 marks) Use the Question 11 BOOKLET **Marks**

a) You have been given the sketches of the graphs of two functions, $y = g(x)$ and $y = h(x)$ as shown in the writing booklet.

On the number plane diagram provided in your answer booklet, draw the sketch of:

$$y = g(x) + h(x)$$

Include all relevant points in your sketch. **3**

b)

i. On the same set of axes, sketch the functions $y = |x-2|$ and $y = 2x$. **1**

ii. Hence solve $|x-2| < 2x$. **2**

c) If $f(x) = 3x + 5$, find the co-ordinates of the point where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect. **2**

d) Express $P(x)$ as a product of linear factors, given that $P(x) = 0$ has a multiple root.
$P(x) = 4x^3 + 12x^2 - 15x + 4$. **2**

~ End of Question 11 ~

Question 12 (10 marks) Use the Question 12 BOOKLET **Marks**

a) Solve the following equation for x:

$$\tan^{-1}\left(\frac{1}{x}\right)+\tan^{-1}\left(\frac{1}{x+2}\right)=\frac{\pi}{4}$$ **3**

b) Given $f(x)=2\arcsin\left(\frac{x}{3}\right)$:

i. State the domain and range of $f(x)$. **2**

ii. Sketch the graph of $y=|f(x)|$, clearly indicating the intercept/s and endpoint/s. **1**

c)

i. Show that $\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta$. **1**

ii. Hence, or otherwise, solve $\frac{2\tan\theta}{1+\tan^2\theta}=\sqrt{3}\cos\theta$ for $0\le\theta\le 2\pi$. **3**

~ End of Question 12 ~

Question 13 (10 marks) Use the Question 13 BOOKLET **Marks**

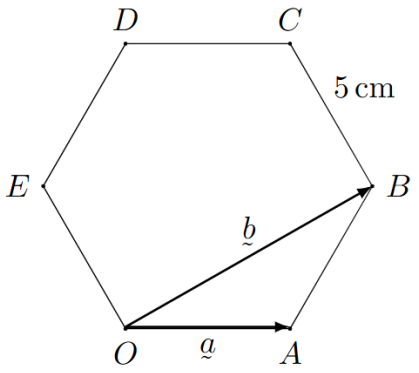
a) Using mathematical induction, prove the following is true for all positive integers n

$$1 + 6 + 11 + ... + (5n-4) = \frac{1}{2}n(5n-3)$$ **3**

b) Points A, B and C are such that $\overrightarrow{AB} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

Find the unit displacement vector in the direction $\overrightarrow{CB}$. **2**

c) A **regular hexagon** $OABCDE$ is drawn below.



The sides of the hexagon are 5 cm and $\angle OAB = 120^\circ$. Let $\overrightarrow{OA}$ be $\underset{\sim}{a}$ and $\overrightarrow{OB}$ be $\underset{\sim}{b}$.

i. Show that $\underset{\sim}{a} \cdot \underset{\sim}{b} = 37.5$ (geometric reasoning is not necessary). **2**

ii. Express $\overrightarrow{AD}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. **3**

~ End of Question 13 ~

Question 14 (10 marks) Use the Question 14 BOOKLET **Marks**

a) Solve the differential equation $\frac{dy}{dx} = xy$, given y > 0.

Express your answer in the form $y = e^{f(x)}$. **2**

b) Dr Kool wishes to find the temperature of a very hot substance using his thermometer, which only measures up to 100°C. Dr Kool takes a sample of the substance and places it in a room with a surrounding air temperature of 20°C and allows it to cool.

After 6 minutes the temperature of the substance is 80°C, and after a further 2 minutes it is 50°C. If $T(t)$ is the temperature of the substance after t minutes, then Newton's law of cooling states that T satisfies the equation:

$$\frac{dT}{dt} = k\left(T - A\right)$$

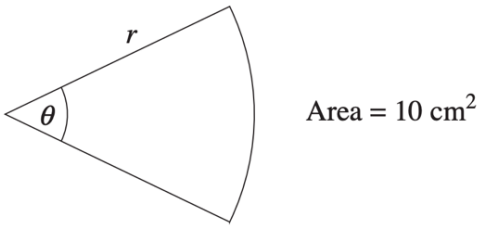
where k is a constant and A is the surrounding air temperature.

i. Verify that $T = A + Be^{kt}$ satisfies the above equation. **1**

ii. Show that $k = -\frac{\ln 2}{2}$, and find the value of B. **3**

iii. Hence find the initial temperature of the substance. **1**

c) The radius, r cm, and angle, θ radians, of a sector vary in such a way that its area remains a constant 10 cm^2.



The angle θ is increasing at a constant rate of 2 radians per second.
Find the rate at which the radius is changing when the radius is 4 cm. **3**

~ End of Question 14~

Question 15 (10 marks) Use the Question 15 BOOKLET **Marks**

a) Using the Pigeonhole Principle, prove that if 5 points are placed anywhere inside a square with a side length of 2 units that there must be a pair of points that are at most $\sqrt{2}$ units apart. **2**

b) A biased coin is flipped 100 times with the probability of landing a head 0.3. Determine the expected number of heads and the standard deviation. **2**

c) A machine has a probability of 0.1 of producing a defective item.
A batch of 1000 items was sampled.

It is assumed the sample proportion, $\hat{p}$, is approximately normally distributed with $\mu = p$ and $\sigma^2 = \dfrac{p(1-p)}{n}$.

What is the probability that the sample proportion lies between 0.09 and 0.11? **3**

You may use the z-score probability table that is on the next page.

d)

i. Show that

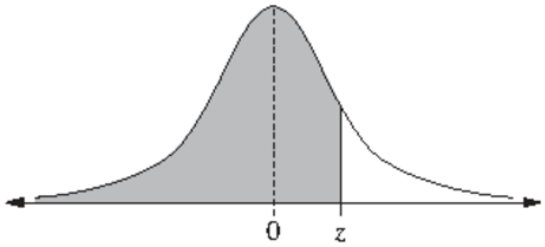
$$(1+x)^{2n} + (1-x)^{2n} = 2\left[\binom{2n}{0} + \binom{2n}{2}x^2 + \binom{2n}{4}x^4 + \ldots + \binom{2n}{2n}x^{2n}\right]$$

where n is a positive integer. **2**

ii. Hence evaluate $1 + {}^{20}C_2 + {}^{20}C_4 + \cdots + {}^{20}C_{20}$. **1**

~ Question 15 Continues onto the next page ~

Table of values $P(Z \leq z)$ for the normal distribution $N(0,1)$



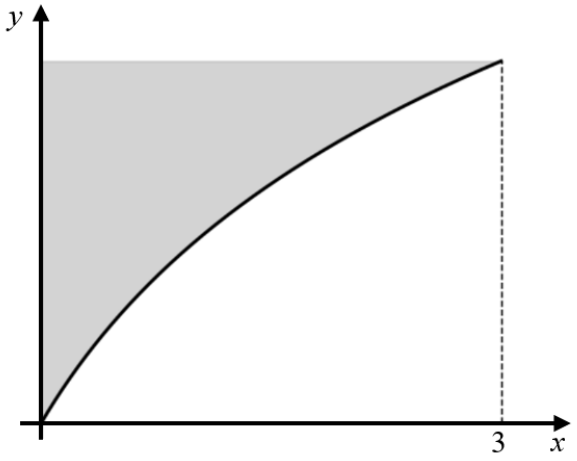
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995

~ End of Question 15~

Question 16 (10 marks) Use the Question 16 BOOKLET **Marks**

a) For the curve $y = f(x)$, it is given that $f'(x) = \sin^2 x$ and the curve passes through the point $\left(\frac{\pi}{4}, \frac{\pi}{8}\right)$. Find the equation of the curve. **3**

b) The region bounded by the curve $y = \ln(x+1)$ and the y-axis for $0 \le x \le 3$, as shown below, is rotated about the y-axis. Find the exact volume of the solid formed. **3**



c)

i. Differentiate $\left(\tan^{-1}\left(\frac{x}{3}\right)\right)^2$ with respect to x. **2**

ii. Hence, or otherwise, find the exact value of $\frac{1}{\pi}\int_0^{\sqrt{3}} \frac{\tan^{-1}\left(\frac{x}{3}\right)}{x^2+9}\,dx$. **2**

~ End of Question 16~

~ End of Trial Examination ~

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

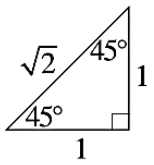
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

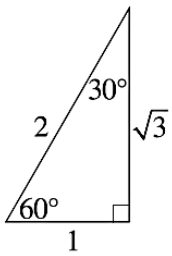
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

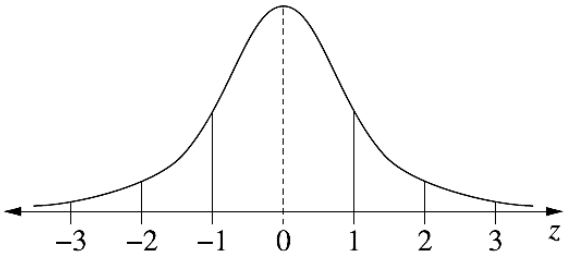
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow \ P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c \quad \text{where } n \neq -1$$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^n e^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$

HURLSTONE AGRICULTURAL HIGH SCHOOL

STUDENT'S NAME: _____

TEACHER'S NAME: _____

2026

HSC ASSESSMENT TASK 4
TRIAL EXAMINATION

Mathematics Extension1

Section I - MULTIPLE-CHOICE ANSWER SHEET

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ✕ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ✕ B ✕ (correct) C ○ D ○

1.	**A**	○	**B**	○	**C**	○	**D** ○
2.	**A**	○	**B**	○	**C**	○	**D** ○
3.	**A**	○	**B**	○	**C**	○	**D** ○
4.	**A**	○	**B**	○	**C**	○	**D** ○
5.	**A**	○	**B**	○	**C**	○	**D** ○
6.	**A**	○	**B**	○	**C**	○	**D** ○
7.	**A**	○	**B**	○	**C**	○	**D** ○
8.	**A**	○	**B**	○	**C**	○	**D** ○
9.	**A**	○	**B**	○	**C**	○	**D** ○
10.	**A**	○	**B**	○	**C**	○	**D** ○

Multiple Choice Solutions

Question 1:

The reciprocal function is always positive, eliminating B and D.

The reciprocal function approaches $+\infty$ for a positive x value, eliminating C

Answer: A

Question 2:

The restricted domain of $f(x)$ means that it only describes the quarter-circle in the 2nd quadrant. Reflecting in the line $y=x$ gives the inverse function in the 4th quadrant.

Answer: D

Question 3:

Let $\sin x + \sqrt{3}\cos x = R\sin(x+\alpha)$

$$R = \sqrt{1^2 + \left(\sqrt{3}\right)^2} = 2$$

$$\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

$$\therefore \sin x + \sqrt{3}\cos x = 2\sin\left(x + \frac{\pi}{3}\right)$$

Answer: A

Question 4:

$$4\sin^2\left(\frac{\pi}{22}\right)+3=\sqrt{C}$$

$$\sin^2\left(\frac{\pi}{22}\right)=\frac{\sqrt{C}-3}{4}$$

We know $\cos(2\theta)=1-2\sin^2\theta$

Also, $\frac{\pi}{11}=2\times\frac{\pi}{22}$

$$\therefore \cos\left(\frac{\pi}{11}\right)=\cos\left(2\times\frac{\pi}{22}\right)=1-2\sin^2\left(\frac{\pi}{22}\right)$$

$$=1-2\times\left(\frac{\sqrt{C}-3}{4}\right)$$

$$=1-\frac{\sqrt{C}-3}{2}$$

$$=\frac{5-\sqrt{C}}{2}$$

Answer: B

Question 5:

$\underset{\sim}{v}$ is a scalar multiple of $\underset{\sim}{u}$. So $a=-\frac{5}{2}$.

$\underset{\sim}{u}\times\underset{\sim}{w}=0$. So, $b=\frac{2}{5}$.

Answer C

Question 6:

$$y = e^{2x} + e^{-2x}$$

$$\frac{dy}{dx} = 2e^{2x} - 2e^{-2x}$$

$$\frac{d^2y}{dx^2} = 4e^{2x} + 4e^{-2x}$$

now sub into equation:

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$$

$$4e^{2x} + 4e^{-2x} + a\left(2e^{2x} - 2e^{-2x}\right) + b\left(e^{2x} + e^{-2x}\right) = 0$$

$$e^{2x}\left(4 + 2a + b\right) + e^{-2x}\left(4 - 2a + b\right) = 0 = e^{2x}\left(0\right) + e^{-2x}\left(0\right)$$

ie $4 + 2a + b = 0$ (1)

$4 - 2a + b = 0$ (2)

(1) + (2) $8 + 2b = 0$

$b = -4$

$a = 0$

Answer C

Question 7:

$$\frac{dy}{dx} = \frac{3x^2}{2y}$$

- numerator always positive, and
- denominator negative when y is negative
- denominator positive when y is positive
  i.e. gradient is negative below x-axis, and positive above x-axis

Answer is D

Question 8:

There are 10 letter in total, with 2 T's and 4 S's. So $\frac{10!}{2!4!}$.

Answer C

Question 9:

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-(2x)^2}} = \frac{2}{\sqrt{1-4x^2}}$$

At $x = \frac{1}{4}$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4\left(\frac{1}{4}\right)^2}} = \frac{2}{\frac{\sqrt{3}}{2}} = \frac{4}{\sqrt{3}}$$

Answer D

Question 10:

Looking at the graph at $x = 1$,

The point $(1, f(1))$ is above the x-axis but below $y = 1$, so

$0 < f(1) < 1$

Next, the graph is decreasing at $x = 1$. Hence the gradient is negative.

$f'(1) < 0$

Around $x = 1$, the graph is changing concavity which indicates an inflection point.

$f''(1) < 0$

$\therefore f'(1) < f''(1) < f(1) < 1$

Answer D

Question 11

Outcomes Addressed in this Question

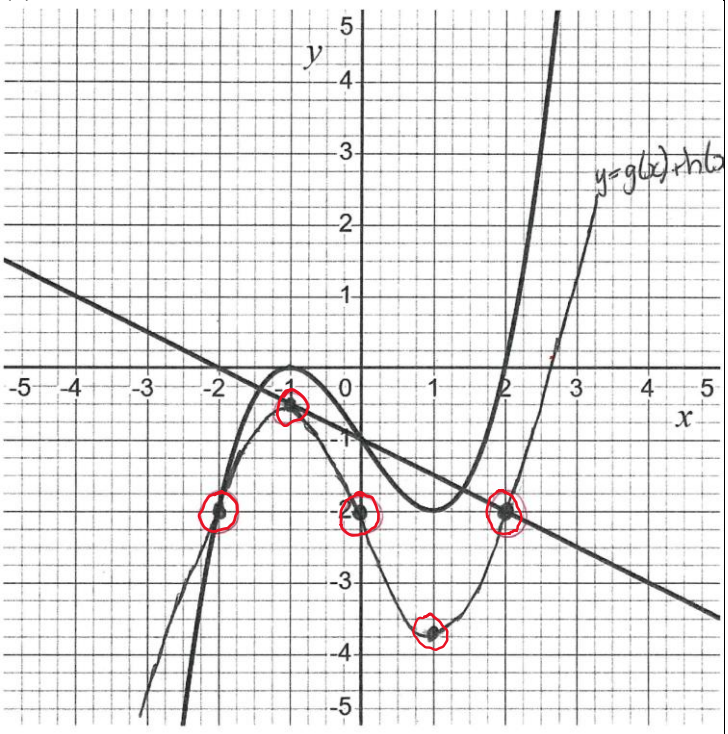
ME11-1 uses algebraic and graphical concepts in the modelling and solving of problems involving functions and their inverses

ME11-2 manipulates algebraic expressions and graphical functions to solve problems

Question 5

(a) 11-2

(a)



Note: Many good responses "missed" the correct point when x=-2. It's important that the sketch of the sum intersect one curve exactly when the other function is equal to zero.

Marking Guidelines

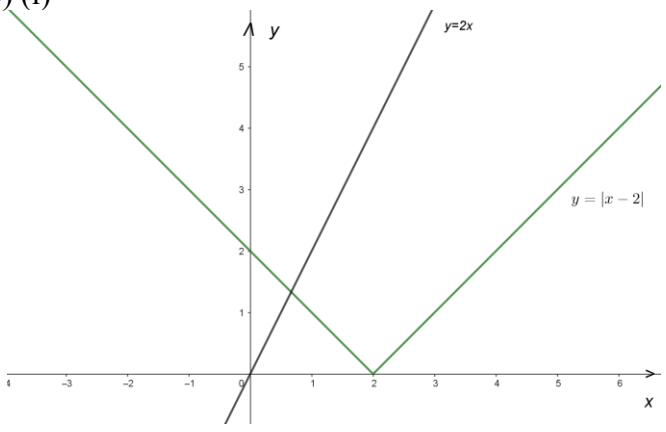
(a) 3 marks: Correct sketch that includes:
A cubic shape that crosses to go above the x-axis as $x \to \infty$.
Correct behaviour of the curve at the circled points on the given solution.

2 marks: A correct shape with correct behaviour at more than 2 of the circled points.
Or a curve including all 5 points indicated, but not crossing the x-axis as $x \to \infty$.

1 mark: A correctly oriented cubic curve not passing through at least 2 of the required points.

(b)(i) 11-2

(b) (i)



(b) (i)1 mark: Correct sketch.
Note: Scale does not need to be perfect, but the line $y=2x$ must be steeper than the right branch of the "V".
Imperfect scale for most sketches also means that the x-value for the point of intersection should be determined algebraically rather than by sight.

(ii)

(ii) The sketch shows that the only point of intersection occurs with the left branch of the absolute value function. Thus, to solve the inequality, we are concerned with the x-value only.

$2x = 2 - x \qquad \to x = \frac{2}{3}$

Hence solution: $|x-2| < 2x$ when $x > \frac{2}{3}$

(ii) **2 marks:** Correct solution.
1 mark: Correct x-value at boundary, or correct progress from incorrect sketch/algebra.

(c) 11-1	**(c)** The linear function $y=3x+5$ will only intersect its inverse on the line $y=x$. Thus, you only need to solve: $x=3x+5\rightarrow x=-\frac{5}{2}$ Hence the co-ordinates of the intersection are $\left(-\frac{5}{2},-\frac{5}{2}\right)$. Most responses did not take this simple path but instead solved an equation after finding $f^{-1}(x)$, and then performing a substitution. This time consumption could affect performance in other parts of the exam.	**(c) 2 marks:** Correct solution. **1 mark:** A correct equation involving only x terms. No error carried forward from incorrect x where $y\neq x$.
(d) 11-2	(d) $P(x)=4x^3+12x^2-15x+4$ $P'(x)=12x^2+24x-15$ $=3(4x^2+8x-5)$ $=3(2x+5)(2x-1)$ *** So possible double-roots occur when $P'(x)=0$, i.e. when $x=-\frac{5}{2}$ or $\frac{1}{2}$. The more likely value, when we compare this to the original $P(x)$, is $x=\frac{1}{2}$. $P\left(\frac{1}{2}\right)=4\left(\frac{1}{8}\right)+12\left(\frac{1}{4}\right)-15\left(\frac{1}{2}\right)+4$ $=\frac{1}{2}+3-7\frac{1}{2}+4=0$ Hence $x=\frac{1}{2}$ **is the repeated root**, and since $P(x)$ has degree 3, there is no other multiple root possible. Hence ***P*(*x*) has a factor equal to** $(2x-1)^2$ *** $P(x)=(2x-1)^2(x+4)$ by inspection of the leading term and the constant term +4 in $P(x)$. Note: • The 2 lines that I have labelled *** are related to each other; Many responses use $\left(x-\frac{1}{2}\right)$ as a factor and hence did not acknowledge that the polynomial is not monic. Inserting the leading coefficient of 4 creates factors equal to $(2x-1)$ that are easier to work with. • The **root of an equation** is the **value** that makes it true. Many responses incorrectly stated that the double root was either $\left(x-\frac{1}{2}\right),(2x-1)$ or $P\left(\frac{1}{2}\right)$.	**(d) 2 marks:** Complete factorisation using any relevant method. **1 mark:** Finding at least one correct factor. Many different methods were used. The one shown here initially uses the information from the question, uses very little trial and error, avoids long division, and finds factors of a non-monic polynomial in a straightforward way.

Year 12	Mathematics Extension 1	Assessment Task 4 2026
Question 12	Solutions and Marking Guidelines	

Outcomes Addressed in this Question:

ME11-3 applies concepts and techniques of inverse trigonometric functions and simplifying expressions involving compound angles in the solution of problem

ME12-3 applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations

Solutions	Marking Guidelines

Q12 a)

$$\tan^{-1}\left(\frac{1}{x}\right)+\tan^{-1}\left(\frac{1}{x+2}\right)=\frac{\pi}{4}$$

Let $A=\tan^{-1}\left(\frac{1}{x}\right)$ and $B=\tan^{-1}\left(\frac{1}{x+2}\right)$

$$\therefore \tan A=\frac{1}{x} \text{ and } \tan B=\frac{1}{x+2}$$

$$\text{and } A+B=\frac{\pi}{4}$$

$$\tan\left(A+B\right)=\tan\left(\frac{\pi}{4}\right)$$

$$\frac{\tan A+\tan B}{1-\tan A\times\tan B}=1$$

$$\frac{\frac{1}{x}+\frac{1}{x+2}}{1-\left(\frac{1}{x}\right)\left(\frac{1}{x+2}\right)}=1$$

$$\frac{1}{x}+\frac{1}{x+2}=1-\left(\frac{1}{x}\right)\left(\frac{1}{x+2}\right)$$

$$\frac{x+x+2}{x\left(x+2\right)}=\frac{x\left(x+2\right)-1}{x\left(x+2\right)}$$

$$2x+2=x^2+2x-1$$

$$x^2=3$$

$$x=\pm\sqrt{3}$$

Marker's Notes:

- Students should recognise that compound angle identity for tangent, is an efficient method for solving the equation.

3 marks: Correctly uses the compound angle formula for tangent, solves the resulting equation in x, and obtains both solutions for x.

2 marks: Correctly uses the compound angle formula for tangent, and obtains the correct equation in x, but makes a minor algebraic error or obtains only one of the two solutions.

1 mark: Demonstrates some understanding of the compound angle formula for tangent, by correctly substituting the inverse tangent expressions or making relevant progress towards an equation in x.

Q12 b) (i)

For $f(x) = 2\arcsin\left(\frac{x}{3}\right)$

Domain: $[-3,3]$ and Range: $[-\pi,\pi]$

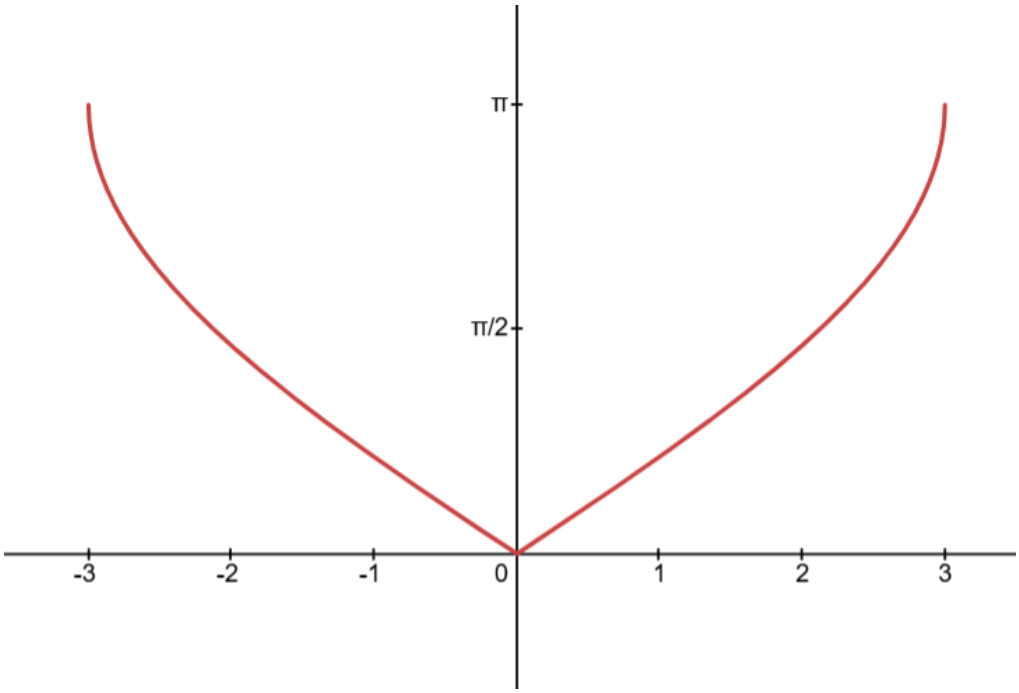
Marker's Notes:

- Accepted equivalent inequality notation
- Excluding the boundary values for both domain and range or mixing up domain and range were common mistakes

2 marks: Correctly states both the domain and the range.

1 mark: Correctly states either the domain or the range.

Q12 b) (ii)



Marker's Notes:

Common errors included

- The cusp at the origin was not clearly shown.
- The vertical tangent behaviour at the endpoints was not shown.

1 mark: Sketches the graph with the correct overall shape, showing the intercept and endpoints.

Q12 c) (i)

$$\begin{aligned}\text{LHS} &= \frac{2\tan\theta}{1+\tan^2\theta} \\ &= \frac{2\dfrac{\sin\theta}{\cos\theta}}{\sec^2\theta} \\ &= \frac{2\sin\theta}{\cos\theta} \div \frac{1}{\cos^2\theta} \\ &= 2\sin\theta\cos\theta \\ &= \sin 2\theta = RHS\end{aligned}$$

Marker's Notes:

- Simply stating the double-angle identity without showing the required working.
- Some students let $t = \tan\theta$ and used results involving t results, but did not explicitly show how these results were applied to transform the LHS into the RHS.
- Where $t = \tan\theta$ is introduced, sufficient working should be shown to clearly connect the t-results back to the trigonometric expression.

1 mark: Correctly manipulates the LHS using relevant trigonometric identity/ies to obtain the RHS.

Q12 c) (ii)

$$\frac{2\tan\theta}{1+\tan^2\theta} = \sqrt{3}\cos\theta$$

$$\sin 2\theta = \sqrt{3}\cos\theta$$

$$2\sin\theta\cos\theta - \sqrt{3}\cos\theta = 0$$

$$\cos\theta\left(2\sin\theta - \sqrt{3}\right) = 0$$

$$\cos\theta = 0 \quad \text{or} \quad \sin\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or} \quad \theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

but $\theta \neq \dfrac{\pi}{2}$ and $\theta \neq \dfrac{3\pi}{2}$ as $\tan\theta$ is undefined for these values.

$\therefore \theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ are the only solutions.

Marker's Notes:

- Some students included $\theta = \dfrac{\pi}{2}$ and $\theta = \dfrac{3\pi}{2}$, overlooking that $\tan\theta$ is undefined at these values. k
- Students should check proposed solutions against the domain of the original equation.
- Dividing by $\cos\theta$ is valid here only after recognising that $\cos\theta = 0$ corresponds to values outside the domain of the original equation.

3 marks: Correctly uses the result from part (i), solves the resulting trigonometric equation, and obtains both valid solutions, excluding values for which the original expression is undefined.

2 marks: Correctly forms and solves the resulting trigonometric equation, but either includes extraneous solutions or excludes them without valid justification or makes a minor error in obtaining the valid solutions.

1 mark: Correctly uses the result from part (i) to obtain at least one valid solution.

	Solutions	Marking Guidelines
(a)	Prove statement true for $n=1$ $LHS = 5(1)-4$ $=1$ $RHS = \frac{1}{2}(1)-\left(5(1)-3\right)$ $=1$ $LHS = RHS$ $\therefore$ statement true for $n=1$ Assume true for $n=k$, where k is a positive integer $1+6+11+...+\left(5k-4\right)=\frac{1}{2}k\left(5k-3\right)$ Prove true for $n=k+1$, i.e. $1+6+11+...+\left(5k-4\right)+\left(5\left(k+1\right)-4\right)=\frac{1}{2}\left(k+1\right)\left(5\left(k+1\right)-3\right)$ $1+6+11+...+\left(5k-4\right)+\left(5k+1\right)=\frac{1}{2}\left(k+1\right)\left(5k+2\right)$ $LHS = 1+6+11+...+\left(5k-4\right)+\left(5k+1\right)$ $= \frac{1}{2}k\left(5k-3\right)+\left(5k+1\right)$ ... from step 2 $= \frac{1}{2}\left(5k^2-3k+10k+2\right)$ $= \frac{1}{2}\left(5k^2+7k+2\right)$ $= \frac{1}{2}\left(k+1\right)\left(5k+2\right)$ $= RHS$ $\therefore$ statement true by mathematical induction for all positive integers	3 marks : correct set out of mathematical induction proof 2 marks : mostly correct proof with one mistake 1 mark: little progress towards a correct proof

(b)	$\overrightarrow{CB} = \overrightarrow{CA} + \overrightarrow{AB}$ $= \begin{pmatrix} -4 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ $\overrightarrow{CB} = \begin{pmatrix} -6 \\ 4 \end{pmatrix}$ $\left\|\overrightarrow{CB}\right\| = \sqrt{(-6)^2 + 4^2} = \sqrt{52}$ Unit vector is $\overrightarrow{CB} = \frac{1}{\sqrt{52}} \begin{pmatrix} -6 \\ 4 \end{pmatrix}$	2 marks : correct unit vector with calculation of magnitude shown 1 mark: some progress towards getting unit vector

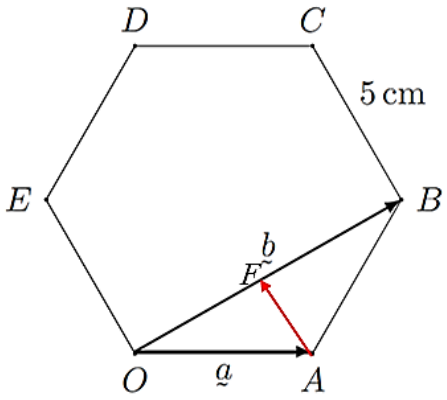
(c) i.	Regular hexagon so $OA = AB$ In ΔOAB, $\angle BOA = 30°$ $\underset{\sim}{a} \bullet \underset{\sim}{b} = \lvert\underset{\sim}{a}\rvert \times \lvert\underset{\sim}{b}\rvert \cos(\angle BOA)$ $\lvert\underset{\sim}{b}\rvert = \sqrt{5^2 + 5^2 - 2(5)(5)\cos 120°}$ $\lvert\underset{\sim}{b}\rvert = \sqrt{25 + 25 + 25} = \sqrt{75} = 5\sqrt{3}$ $\therefore \ \underset{\sim}{a} \bullet \underset{\sim}{b} = 5 \times 5\sqrt{3} \cos(30)$ $= 37.5$	2 marks : correct dot product showing all steps and substitutions 1 mark: one mistake or missing steps in the solutions. Or vector notation not used.

(c) ii.

$|\underset{\sim}{b}| = 5\sqrt{3}, \quad |\underset{\sim}{a}| = 5, \quad \overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}, \quad \underset{\sim}{a} \bullet \underset{\sim}{b} = 37.5$

$\overrightarrow{AD}$ is perpendicular bisector of $\overrightarrow{OB}$

Let $\overrightarrow{AD}$ and $\overrightarrow{OB}$ intersect at F.



$$\overrightarrow{AF} = \frac{1}{2}\underset{\sim}{b} - \underset{\sim}{a}$$

$$\overrightarrow{AD} = k\overrightarrow{AF}$$
$$= k\left(\frac{1}{2}\underset{\sim}{b} - \underset{\sim}{a}\right)$$

$$\left|\overrightarrow{AF}\right| = \sqrt{5^2 - \left(\frac{5\sqrt{3}}{2}\right)^2}$$
$$= 2.5$$

$$\left|\overrightarrow{AD}\right| = 5 + 2 \times 2.5 = 10$$

$$\left|\overrightarrow{AD}\right| = k\left|\overrightarrow{AF}\right|$$
$$10 = k(2.5)$$
$$k = 4$$

$$\therefore \quad \overrightarrow{AD} = 4\overrightarrow{AF}$$
$$= 4\left(\frac{1}{2}\underset{\sim}{b} - \underset{\sim}{a}\right)$$
$$= 2\underset{\sim}{b} - 4\underset{\sim}{a}$$

Markers Comment:
Lots of different methods were used to find the vector. Marks were awarded as long as the vector/s and vector additions were correct.
Some students did not use vector notation and as a result did not get full marks.

3 marks : correct addition of vectors to get the vector $\overrightarrow{AD}$.
2 marks : good progress towards getting $\overrightarrow{AD}$ with a mistake or nut using vector notation
1 mark: limited progress towards a correct method to get the vector $\overrightarrow{AD}$.

Year 12	Mathematics Extension 1	Ass Task 4 2026 HSC
Question 14	Solutions and Marking Guidelines	TRIAL HSC

Outcomes Addressed in this Question

ME12-4 uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution

Question	Solutions	Marking Guidelines
(a)	$\frac{dy}{dx} = xy$ $\int \frac{dy}{y} = \int x dx$ $\ln y = \frac{x^2}{2} + C$ $y = e^{\frac{x^2}{2}+C}$ $= Ae^{\frac{x^2}{2}}$	**2 marks:** correct solution (last and second last line here both acceptable) **1 mark:** separates the variables, or equivalent merit
(b)(i)	If $T = A + Be^{kt}$. $\frac{dT}{dt} = kBe^{kt}$. Also, $k(T-A) = k\left(A + Be^{kt} - A\right)$ $= k\left(Be^{kt}\right)$ $= kBe^{kt}$ $= \frac{dT}{dt}$. $\therefore \frac{dT}{dt} = k(T-A)$.	**1 mark:** correct solution

Question 14 continued…

(b)(ii)

$A = 20$ is given in the question.

When $t = 6$, $T = 80$.

$$\therefore \quad 80 = 20 + Be^{6k}$$
$$Be^{6k} = 60. \qquad \text{— ①}$$

When $t = 8$, $T = 50$.

$$\therefore \quad 50 = 20 + Be^{8k}$$
$$Be^{8k} = 30. \qquad \text{— ②}$$

Divide ② by ①:

$$\frac{Be^{8k}}{Be^{6k}} = \frac{30}{60}$$
$$e^{2k} = \frac{1}{2}$$
$$2k = \ln\frac{1}{2}$$
$$k = \frac{1}{2}\ln\frac{1}{2}$$
$$= -\frac{1}{2}\ln 2, \text{ as required.}$$

Substitute into ①: $Be^{6\left(-\frac{1}{2}\ln 2\right)} = 60$

$$Be^{-3\ln 2} = 60$$
$$B = 60e^{3\ln 2}$$
$$= 60e^{\ln 8}$$
$$= 60 \times 8$$
$$= 480.$$

3 marks: correct solution

2 marks: obtains $k = -\frac{\ln 2}{2}$, or equivalent merit.

1 mark: sets up two equations from the given data

(b)(iii)

Now $T = 20 + 480e^{\left(-\frac{1}{2}\ln 2\right)t}$

If $t = 0$, $T = 20 + 480e^{0}$

$= 500.$

$\therefore$ the initial temperature is 500°C.

1 mark: correct solution

Question 14 continued…

(c)

$$\text{Area} = \frac{1}{2}r^2\theta = 10$$

$$\therefore \quad \theta = \frac{20}{r^2}$$

$$\therefore \quad \frac{d\theta}{dr} = -\frac{40}{r^3}$$

$$\frac{d\theta}{dt} = 2 \text{ (given)}$$

$$\therefore \quad \frac{dr}{dt} = \frac{dr}{d\theta} \times \frac{d\theta}{dt}$$

$$= -\frac{r^3}{40} \times 2$$

$$= -\frac{r^3}{20}$$

When $r = 4$,

$$\frac{dr}{dt} = -\frac{4^3}{20}$$

$$= -\frac{16}{5}$$

$$= -3.2 \text{ cm s}^{-1}$$

(That is radius is *decreasing* at a rate of 3.2 cm s^{-1}.)

3 marks:
Correct solution

2 marks:
Finds $\frac{dr}{dt}$ in terms of r,
or equivalent merit

1 mark:
Finds $\frac{d\theta}{dr}$.
Or…
Attempts to use the chain rule with data from question,
or equivalent merit

Note: the area of a sector formula, $A = \frac{1}{2}r^2$, is given on the formula sheet.
However, an *extraordinarily large number of students* attempted to derive this formula by using $A = \frac{\theta}{360} \times \pi r^2$ … mixing radians and degrees!

And… calculus only works in radians

2026 Yr12 HSC Assessment Task 4

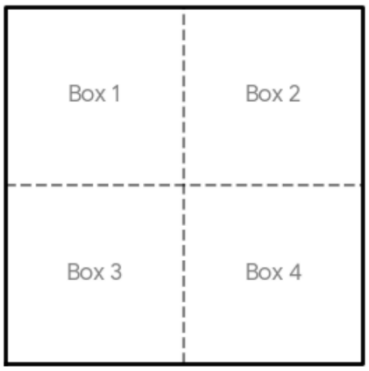
Question 15 Solutions and Marking Guidelines

Outcomes Addressed in this Question

uses concepts of permutations and combinations to solve problems involving counting or ordering and applies appropriate statistical processes to present, analyse and interpret data

Solutions	Marking Guidelines

a)



Dividing the square into four equal one by one squares we can see by the Pigeonhole Principle that there must be a square that has two points. The largest distance in the square is the diagonal, which is $\sqrt{2}$. Hence, there must be at least two points that have a maximum distance of $\sqrt{2}$.

1 mark
correctly establishes "pigeons" and "holes"

2 marks
clearly reasons why 5th point must be no more than root 2 away from another point

b)

$$E(X) = np \qquad \sigma = \sqrt{np(1-p)}$$
$$= 100(0.3) \qquad = \sqrt{100(0.3)(1-0.3)}$$
$$= 30 \qquad = \sqrt{21}$$

1 mark
finds either expect value or standard deviation correctly

2 marks
finds bother expect value and standard deviation correctly

c)

The question asks to find $P\left(0.09 < \hat{P} < 0.11\right)$.

Since we can assume a normal approximation, we will use z-scores to find the probability.

$$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.1(1-0.1)}{1000}} = \sqrt{0.00009}$$

$$z_{0.09} = \frac{0.09-0.1}{\sqrt{0.00009}} = -1.05 \quad \text{and} \quad z_{0.09} = \frac{0.11-0.1}{\sqrt{0.00009}} = 1.05$$

$$P\left(0.09 < \hat{P} < 0.11\right) = P\left(\hat{P} < 0.11\right) - P\left(\hat{P} < 0.09\right)$$
$$= P\left(z < 1.05\right) - P\left(z < -1.05\right)$$
$$= P\left(z < 1.05\right) - \left[1 - P\left(z < 1.05\right)\right]$$
$$=$$
$$= 0.73$$

1 mark
uses formula to calculate standard deviation

2 marks
finds a relevant z-score

3 marks
finds correct probability

d i)

$$(1+x)^{2n} = {}^{2n}C_0(1)^{2n}(x)^0 + {}^{2n}C_1(1)^{2n-1}(x)^1 + {}^{2n}C_2(1)^{2n-2}(x)^2 + \ldots + {}^{2n}C_{2n-1}(1)^1(x)^{2n-1} + {}^{2n}C_{2n}(1)^0(x)^{2n}$$
$$= {}^{2n}C_0 x + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \ldots + {}^{2n}C_{2n-1}x^{2n-1} + {}^{2n}C_{2n}x^{2n}$$

$$(1-x)^{2n} = {}^{2n}C_0(1)^{2n}(-x)^0 + {}^{2n}C_1(1)^{2n-1}(-x)^1 + {}^{2n}C_2(1)^{2n-2}(-x)^2 + \ldots + {}^{2n}C_{2n-1}(1)^1(-x)^{2n-1} + {}^{2n}C_{2n}(1)^0(-x)^{2n}$$
$$= {}^{2n}C_0 - {}^{2n}C_1 x + {}^{2n}C_2 x^2 - \ldots - {}^{2n}C_{2n-1}x^{2n-1} + {}^{2n}C_{2n}x^{2n}$$

$$(1+x)^{2n} + (1-x)^{2n} = \left[{}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \ldots + {}^{2n}C_{2n-1}x^{2n-1} + {}^{2n}C_{2n}x^{2n}\right]$$
$$+\left[{}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 - \ldots - {}^{2n}C_{2n-1}x^{2n-1} + {}^{2n}C_{2n}x^{2n}\right]$$

Collecting like terms, the odd powered terms cancel and the even powered terms are doubled.

$$(1+x)^{2n} + (1-x)^{2n} = 2\,{}^{2n}C_0 + 2\,{}^{2n}C_2 x^2 + \ldots + 2\,{}^{2n}C_{2n}x^{2n}$$
$$= 2\left[{}^{2n}C_0 + {}^{2n}C_2 x^2 + \ldots + {}^{2n}C_{2n}x^{2n}\right]$$

1 mark
correctly expands both Binomial expressions

2 marks
clearly shows how like terms are simplified to get correct final expression

ii)

Let $n = 10$ and $x = 1$

$$(1+1)^{2(10)} + (1-1)^{2(10)} = 2\left[{}^{2(10)}C_0 + {}^{2(10)}C_2(1)^2 + \ldots + {}^{2(10)}C_{2(10)}(1)^{2(10)}\right]$$
$$2^{20} + 0^{20} = 2\left[{}^{20}C_0 + {}^{20}C_2 + \ldots + {}^{20}C_{20}\right]$$
$${}^{20}C_0 + {}^{20}C_2 + \ldots + {}^{20}C_{20} = 2^{20} \div 2$$
$$= 2^{19}$$
$$= 524288$$

1 mark
correctly determines value of expression

Year 12	Mathematics Extension 1	Assessment Task 4 2026
	Solutions and Marking Guidelines	
	Outcomes Addressed in this Question	
	ME12-4 uses calculus in the solution of applied problems, including differential equations and volumes of solids of revolution	

Question 16	**Solutions**	**Marking Guidelines**
(a)	$\sin^2 x = \frac{1-\cos 2x}{2}$ $f(x) = \int \frac{1-\cos 2x}{2}\,dx$ $f(x) = \frac{x}{2} - \frac{\sin 2x}{4} + C$ At $\left(\frac{\pi}{4}, \frac{\pi}{8}\right)$ $\frac{\pi}{8} = \frac{\frac{\pi}{4}}{2} - \frac{\sin\left(2\times\frac{\pi}{4}\right)}{4} + C$ $\frac{\pi}{8} = \frac{\pi}{8} - \frac{1}{4} + C$ $C = \frac{1}{4}$ $\therefore f(x) = \frac{x}{2} - \frac{1}{4}\sin 2x + \frac{1}{4}$	**3 marks** Correctly integrates the function AND substitutes the relevant points to determine the constant of integration AND writes the correct function *f(x)* **2 marks** Two criterions above OR makes significant progress towards the correct solution. **1 mark** One correct criterion above OR makes some progress towards the correct solution.
(b)	$y = \ln(x+1)$ $e^y = x+1$ $x = e^y - 1$ When $x = 0, y = 0$ When $x = 3, y = \ln 4$ $V = \pi \int_0^{\ln 4} (e^y - 1)^2\,dy$ $V = \pi \int_0^{\ln 4} (e^{2y} - 2e^y + 1)dy$ $V = \pi\left[\frac{1}{2}e^{2y} - 2e^y + y\right]_0^{\ln 4}$ $V = \pi\left[\frac{1}{2}e^{2(\ln 4)} - 2e^{\ln 4} + \ln 4 - \left(\frac{1}{2}e^{2(0)} - 2e^0 + 0\right)\right]$ $V = \pi\left(\ln 4 + \frac{3}{2}\right)u^3$ Must change the limits of integration. You cannot use the limits at *x=0* to *x=3*. Some students forgot to include the π.	**3 marks** Re-writes the function in terms of *x* and correctly integrates the function using the appropriate volume formula AND determines the correct solution in exact form. **2 marks** Two criterions above OR makes significant progress towards the correct solution. **1 mark** One correct criterion above OR makes some progress towards the correct solution.

(c)

(i)

$$\text{Let } y = \left(\tan^{-1}\left(\frac{x}{3}\right)\right)^2$$

$$\frac{dy}{dx} = 2\tan^{-1}\left(\frac{x}{3}\right) \times \frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{3}\right)\right)$$

$$= 2\tan^{-1}\left(\frac{x}{3}\right) \times \frac{1/3}{1+\left(\frac{x}{3}\right)^2}$$

$$= 2\tan^{-1}\left(\frac{x}{3}\right) \times \frac{3}{x^2+9}$$

$$\frac{dy}{dx} = \frac{6\tan^{-1}\left(\frac{x}{3}\right)}{x^2+9}$$

Students must be explicit in demonstrating their solution. Skipping steps is unadvised which could have led to simple transcription errors.

2 marks
Uses the chain rule to determine the correct solution AND shows the solution through relevant mathematical calculations.

1 mark
Uses the chain rule to differentiate, however, contains a singular error OR makes some progress towards the correct solution.

(ii)

$$\frac{1}{\pi}\int_0^{\sqrt{3}} \frac{\tan^{-1}\left(\frac{x}{3}\right)}{x^2+9}\,dx = \frac{1}{6\pi}\left[\left(\tan^{-1}\left(\frac{x}{3}\right)\right)^2\right]_0^{\sqrt{3}}$$

$$= \frac{1}{6\pi}\left[\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)^2 - \tan^{-1}\left(\frac{0}{2}\right)^2\right]$$

$$= \frac{1}{6\pi}\left(\left(\frac{\pi}{6}\right)^2 - 0\right)$$

$$= \frac{\pi}{216}$$

2 marks
Writes the equivalent integration step using part (i) or otherwise AND determines the correct solution.

1 mark
Writes the equivalent integration step with a singular mistake OR makes some progress towards the correct solution.