Name.....

Ext 1 Teacher.....

Northern Beaches Secondary College

Manly Campus

2026 **HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading time – 10 minutes
- Working time – 2 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in section II, show relevant mathematical reasoning and/or calculations

Total marks:

70

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1. Which of the following pairs of vectors are parallel?

A. $2\underline{\boldsymbol{i}} + 3\underline{\boldsymbol{j}}$ and $4\underline{\boldsymbol{i}} - 6\underline{\boldsymbol{j}}$

B. $2\underline{\boldsymbol{i}} + 3\underline{\boldsymbol{j}}$ and $-4\underline{\boldsymbol{i}} + 6\underline{\boldsymbol{j}}$

C. $-2\underline{\boldsymbol{i}} - 3\underline{\boldsymbol{j}}$ and $3\underline{\boldsymbol{i}} - 2\underline{\boldsymbol{j}}$

D. $-2\underline{\boldsymbol{i}} + 3\underline{\boldsymbol{j}}$ and $-4\underline{\boldsymbol{i}} + 6\underline{\boldsymbol{j}}$

2. Which of the following has $x = -1$ as a root of multiplicity exactly two?

A. $P(x) = x^3 - 3x + 2$

B. $P(x) = x^4 + x^3 - 3x^2 - 5x - 2$

C. $P(x) = x^3 - 4x^2 + 5x - 2$

D. $P(x) = x^3 + 4x^2 + 5x + 2$

3. Find the value of n if:

$$7({}^{n}C_{n-1}) = {}^{n+2}C_{n-2}$$

A. 3

B. 4

C. 5

D. 6

4. Given $t = \tan\frac{x}{2}$, which of the following is equivalent to $\frac{\sec x + \tan x}{1 + \sin x}$?

A. $\frac{1+t}{1-t}$

B. $\frac{1-t}{1+t}$

C. $\frac{1}{1-t}$

D. $\frac{1+t^2}{1-t^2}$

5. What is the derivative of $\sin^{-1}\left(\frac{3x}{2}\right)$?

A. $\frac{3}{2\sqrt{4-9x^2}}$

B. $\frac{2}{3\sqrt{4-9x^2}}$

C. $\frac{3}{\sqrt{4-9x^2}}$

D. $\frac{3}{\sqrt{2-3x^2}}$

6. If for $X \sim Bin\,(n, p)$, $E(X) = 10$ and $Var(X) = 8$, then the value of n is:

A. 20

B. 30

C. 40

D. 50

7. How many 4-digit codes can be made from the numbers $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ where the digits in the codes are in ascending order and there are no repeated digits?

A. 3024

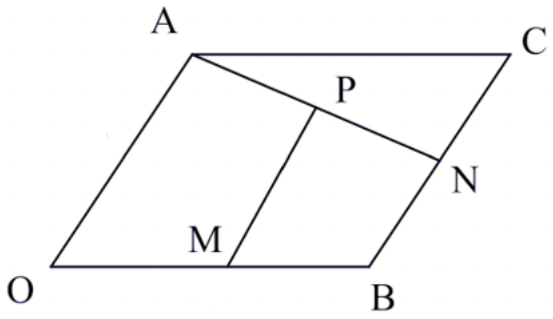
B. 2898

C. 24

D. 126

8. In the diagram below, *OACB* is a parallelogram, where $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.

M is the midpoint of *OB*, *N* is the midpoint of *CB*, and *P* is the midpoint of *AN*.



Which of the following is a correct representation for $\overrightarrow{OP}$?

A. $\frac{3}{4}(\boldsymbol{a}+\boldsymbol{b})$

B. $\frac{3}{4}\boldsymbol{a}+\frac{1}{2}\boldsymbol{b}$

C. $\frac{1}{2}(\boldsymbol{a}+\boldsymbol{b})$

D. $\frac{1}{2}\boldsymbol{a}-\frac{3}{4}\boldsymbol{b}$

9. A particle is moving such that its velocity is given by $v = 6\cos(2t) - 8\sin(2t)$.

What is the correct range of values for the particle's acceleration?

A. $[-20, 20]$

B. $[-10, 10]$

C. $[-8, 8]$

D. $[6, 8]$

10. Laila and Sara are playing a table tennis match. Each rally is independent. The probability that Laila wins a rally is p, and the probability that Sara wins a rally is $1 - p$.

The match is won by the first player to reach 5 points.

What is the probability that Laila wins the match with a final score of 5–3?

A. $\binom{7}{4}p^5(1-p)^3$

B. $\binom{8}{5}p^5(1-p)^3$

C. $\binom{8}{5}p^3(1-p)^5$

D. $\binom{7}{4}p^4(1-p)^3$

Section II

60 marks

Attempt Questions 11–14

Allow about 1 hours and 45 minutes for this section

Answer each question in a writing booklet. Extra writing booklets are available.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

a) Solve $|3-7x| \geq 11$. **2**

b) Given $\boldsymbol{v} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$ and $\boldsymbol{u} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

(i) Find the angle, correct to the nearest degree, between vectors **u** and **v**. **2**

(ii) Find the vector projection of **u** onto **v**. **2**

(iii) Hence or otherwise, find the area of the triangle with vertices at the origin, **u** and **v**. **2**

c) The polynomial $P(x) = 4x^3 - 21x^2 + cx - 4$ has three distinct roots where one root is the reciprocal of the other.

(i) Find all three roots of $P(x) = 0$. **3**

(ii) Hence, find the value of c. **1**

d) Consider the region bounded by the curve $y = \sqrt{x}$, the y-axis and the lines $y = 1$ and $y = 4$.

Find the exact volume of the solid of revolution formed when the region is rotated about the y-axis. **3**

End of Question 11

Question 12 (15 marks)

a) (i) Express $3\sin x + \sqrt{3}\cos x$ in the form $R\sin(x+\alpha)$ where $R>0$ and $0<\alpha<\frac{\pi}{2}$. **2**

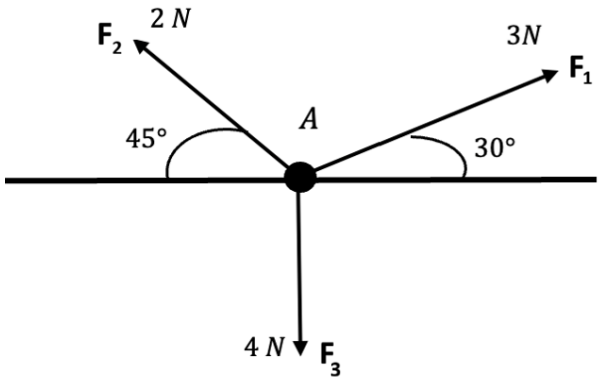
(ii) Hence solve the equation $3\sin x + \sqrt{3}\cos x = \sqrt{6}$ for $0 \le x \le 2\pi$. **2**

b) Differentiate $y = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to x.

Express your answer as a single fraction in its simplest form. **3**

c) The diagram below shows the three forces acting on an object at point A. **3**

By first resolving the forces acting on the object, find the direction of the resultant force. Give your answer to the nearest degree.



d) Using the substitution $u = 1-x^2$, find $\int_0^1 x^3\sqrt{1-x^2}\,dx$ **3**

e) $\underset{\sim}{\mathbf{p}}$ and $\underset{\sim}{\mathbf{q}}$ are unit vectors such that vectors $\underset{\sim}{\mathbf{a}} = \underset{\sim}{\mathbf{p}} + 2\underset{\sim}{\mathbf{q}}$ and $\underset{\sim}{\mathbf{b}} = 5\underset{\sim}{\mathbf{p}} - 4\underset{\sim}{\mathbf{q}}$ are perpendicular to each other. Find the angle between vectors $\underset{\sim}{\mathbf{p}}$ and $\underset{\sim}{\mathbf{q}}$. **2**

End of Question 12

Question 13 (15 marks)

a) A school claims that based on past data, 92% of students submit their homework on time each week. A teacher randomly checks 200 students each week.

(i) Show that the expected value and the standard deviation of the sampling proportion are 0.92 and approximately 0.019, respectively **1**

(ii) Part of a table of $P(Z < z)$ values, where Z is a standard normal variable, is shown. **3**

Z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07
1.0	0.84134	0.84375	0.84614	0.84849	0.85083	0.85314	0.85543	0.85769
1.1	0.86433	0.86650	0.86864	0.87076	0.87286	0.87493	0.87698	0.87900
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	0.89435	0.89617	0.89796
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	0.91149	0.91309	0.91466
1.4	0.91924	0.92073	0.92220	0.92364	0.92507	0.92647	0.92785	0.92922

The school requires the teacher to notify the Head of Department if 90% or fewer students submit their homework on time in a given week.

Using the information above and the standard normal distribution table, find the probability that the teacher will need to report this to the Head of Department at least once during a 20-week semester.

b) (i) Show that $\cos(3\theta) = 4\cos^3\theta - 3\cos\theta$. **1**

(ii) Hence use the substitution $x = \cos\theta$ to show that the equation $8x^3 - 6x + 1 = 0$ has roots $\cos\frac{2\pi}{9}$, $\cos\frac{4\pi}{9}$ and $\cos\frac{8\pi}{9}$. **2**

(iii) Hence, show $\sec\frac{2\pi}{9} + \sec\frac{4\pi}{9} + \sec\frac{8\pi}{9} = 6$. **2**

Question 13 continues on the next page

c) A population of wallabies on an island is being studied. The wallabies are being introduced to the island after becoming locally extinct.
Without environmental constraints, wallaby numbers grow exponentially.
However, the island has a carrying capacity for the wallaby population, and hence the rate of change of wallabies on the island is assumed to follow the logistic equation:

$$\frac{dN}{dt} = kN(P-N)$$

where N is the number of wallabies, t is time in years, P is the carrying capacity of the island and k is a constant.

(i) It is given that $\frac{1}{N(P-N)} = \frac{1}{P}\left(\frac{1}{N} + \frac{1}{P-N}\right)$. (DO NOT PROVE THIS)

Show that $N = \frac{P}{1+Be^{-Pkt}}$, where B is a constant. **3**

(ii) Initially, 500 wallabies are introduced to the island. According to research on wallabies on the mainland, the carrying capacity of the island is estimated to be 5000 wallabies.
Five years after their introduction, the wallaby population on the island is found to be 720.

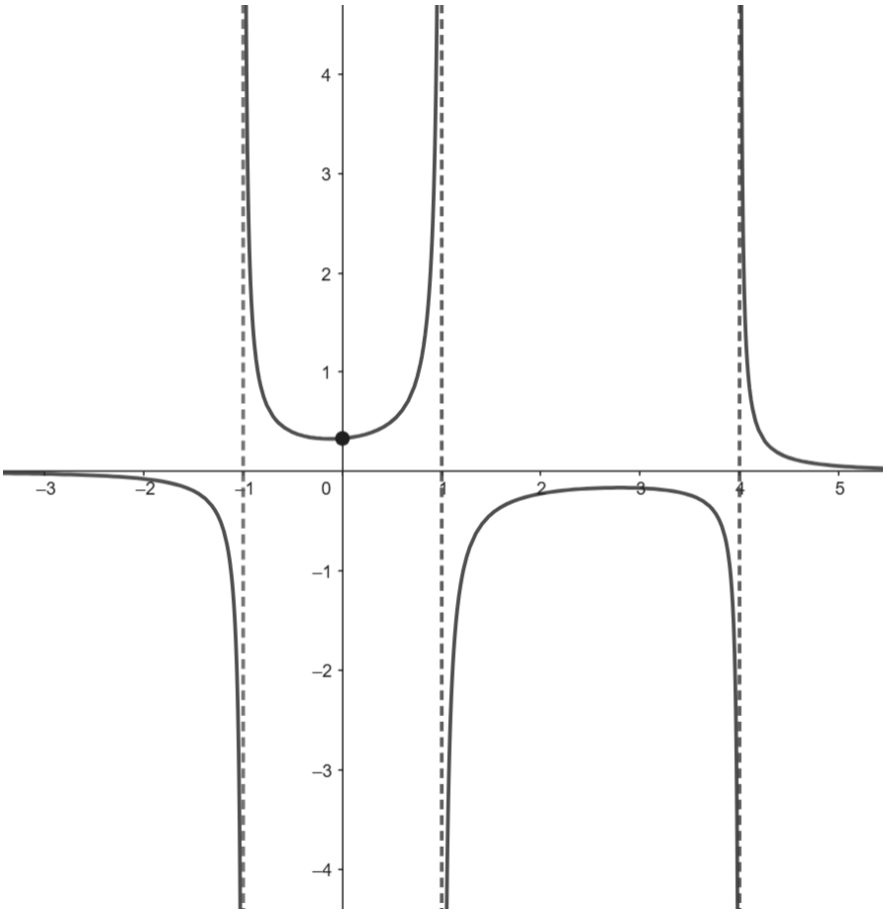
Find the values of P, B and k. **3**

End of Question 13

Question 14 (15 marks)

a) Consider the graph of the function $y = f(x)$ below.
The function has a y-intercept at $y = \frac{1}{3}$. The function also has three vertical asymptotes at $x = -1$, $x = 1$ and $x = 4$.

Using one third of a page, sketch the graph of $y = \sqrt{f(|x|)}$, labelling any asymptotes and intercepts with the coordinate axes. **3**



b) Let $f(n) = 13^{n+1}(6n + 7) - 1$. **3**
Prove using Mathematical Induction that $f(n)$ is divisible by 18 for every positive integer n.

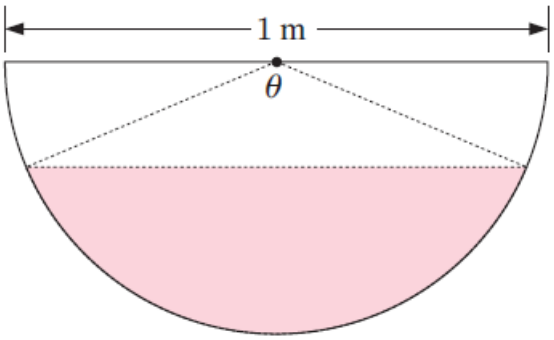
c) Consider the differential equation $\frac{dy}{dx} = (x+1)^2 (y-2)^{\frac{1}{2}}$.
Find the domain of the solution curve that passes through the point (2, 6). **4**

Question 14 continues on the next page

Question 14 (continued)

d) A water trough of length 8 m has a semi-circular cross-section of diameter 1 m.

Water is pumped into the trough at a constant rate of $0.1\ \text{m}^3$ per minute.



(i) Show that the volume of the water in the trough is given by $V = \theta - \sin\theta$ where θ is the angle illustrated (in radians). **2**

(ii) Find the rate at which the water level is rising at the instant the water is 25 cm deep. **3**

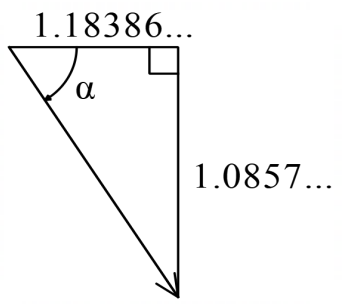
End of Examination

2026 Mathematics Extension 1 Trial marking guidelines		
Q	**Solution**	**Marking allocations**
1	Option D $-4\underline{\boldsymbol{i}} + 6\underline{\boldsymbol{j}} = \mathbf{2}(-2\underline{\boldsymbol{i}} + 3\underline{\boldsymbol{j}})$ Therefore parallel.	D
2	Option A and C P(-1) does not equal 0 Option B actually has as a root of multiplicity **3**, not 2. Option D has root of multiplicity 2.	D
3	Test each option When n = 5 $7\left({}^{5}\mathbf{C}_{4}\right) - \left({}^{7}\mathbf{C}_{3}\right) = 0$	C
4	$= \left(\frac{1+t^2}{1-t^2} + \frac{2t}{1-t^2}\right) \div \left(1 + \frac{2t}{1+t^2}\right)$ $= \frac{1+t^2+2t}{1-t^2} \div \frac{1+t^2+2t}{1+t^2}$ $= \frac{1+t^2}{1-t^2}$	D
5	$y' = \frac{\frac{3}{2}}{\sqrt{1-\frac{9x^2}{4}}}$ from reference sheet $= \frac{3}{\sqrt{4-9x^2}}$	C
6	$E(x) = np$ Var(x)=npq $np = 10$ and $npq = 8$ $\therefore \quad q = \frac{4}{5}$ and $p = \frac{1}{5}$ $\therefore \quad n = 50$	D
7	${}^{9}\mathbf{C}_{4} = 126$ ascending order fixes the order for us, we only need to choose which 4 digits are used.	D

8	$\underset{\sim}{\mathbf{a}}+\overrightarrow{\mathrm{AN}}=\underset{\sim}{\mathbf{b}}+\frac{1}{2}\underset{\sim}{\mathbf{a}}$ $\overrightarrow{\mathrm{AN}}=\underset{\sim}{\mathbf{b}}-\frac{1}{2}\underset{\sim}{\mathbf{a}}$ $\overrightarrow{\mathrm{AP}}=\frac{1}{2}\left(\underset{\sim}{\mathbf{b}}-\frac{1}{2}\underset{\sim}{\mathbf{a}}\right)$ $\therefore \quad \overrightarrow{\mathrm{OP}}=\underset{\sim}{\mathbf{a}}+\frac{1}{2}\left(\underset{\sim}{\mathbf{b}}-\frac{1}{2}\underset{\sim}{\mathbf{a}}\right)$ $=\underset{\sim}{\mathbf{a}}+\frac{1}{2}\underset{\sim}{\mathbf{a}}-\frac{1}{4}\underset{\sim}{\mathbf{b}}$ $=\frac{3}{4}\underset{\sim}{\mathbf{a}}+\frac{1}{2}\underset{\sim}{\mathbf{b}}$	B
9	$v=6\cos(2t)-8\sin(2t)$ $\frac{dv}{dt}=-12\sin(2t)-16\cos(2t)$ $=-20\sin\left(2t+\tan^{-1}\left(\frac{16}{20}\right)\right)$ $\therefore$ range is [-20, 20]	A
10	In 7 games Laila must win 4 and lose 3. Which is $^{7}\mathbf{C}_{4}\,p^{4}(1-p)^{3}$ Then Laila must win the 8th game so the score will be 5-3 So $^{7}\mathbf{C}_{4}\,p^{4}(1-p)^{3}\times p \quad ^{7}\mathbf{C}_{4}\,p^{4}(1-p)^{3}\times p$ $=^{7}\mathbf{C}_{4}\,p^{5}(1-p)^{3} \quad =^{7}\mathbf{C}_{4}\,p^{5}(1-p)^{3}$	A
11a	$3-7x\geq 11$ $x\leq -\frac{8}{7}$ *OR* $3-7x\leq -11$ $x\geq 2$	2 marks for correct working and solutions. 1 mark for correct working but uses incorrect inequality signs OR only states one correct solution.
11b i	$\underset{\sim}{\mathbf{u}}\cdot\underset{\sim}{\mathbf{v}}=\lvert\underset{\sim}{\mathbf{u}}\rvert\,\lvert\underset{\sim}{\mathbf{v}}\rvert\cos\theta$ $14=10\times\sqrt{29}\times\cos\theta$ $\cos\theta=\frac{14}{10\sqrt{29}}$ $\theta\approx 75°$	2 marks correct working and solution. 1 mark correctly use of formula or equivalent merit.

11b ii	$\text{proj}_{\underset{\sim}{v}}\underset{\sim}{u} = \frac{\underset{\sim}{u}.\underset{\sim}{v}^2}{\underset{\sim}{v}}\ \underset{\sim}{v}$ $\frac{7}{50}(6i-8j)$ $=0.84i-1.12j$	2 marks for correct working and solution 1 mark for correct use of formula or equivalent merit.
11b iii	$A=\frac{1}{2}\lvert\underset{\sim}{u}\rvert\ \lvert\underset{\sim}{v}\rvert\ \sin\theta$ $=\frac{1}{2}\times 10\times\sqrt{29}\ \times\sin 75°$ $A\approx 26$ units squared	2 marks for correct working and solution 1 mark for correctly using the formula or find the orthogonal vector or equivalent merit
11c i	let the roots be $\alpha, \frac{1}{\alpha}, \gamma$ Product of roots: $\alpha\times\frac{1}{\alpha}\times\gamma=-\frac{-4}{4}$ $\gamma=1$ sum of roots: $\alpha+\frac{1}{\alpha}+1=\frac{21}{4}$ $4\alpha^2-17\alpha+4=0$ $(4\alpha-1)(\alpha-4)=0$ $\alpha=\frac{1}{4}$ or $\alpha=4$ $\therefore$ roots are 1, 4 and $\frac{1}{4}$	3 marks for correct working and solution 2 marks for one correct root and forms the quadratic equation 1 mark for one correct root, then makes no further progress
11c ii		1 mark for correct answer
11d	$V=\pi\int_a^b x^2\,dy$ $V=\pi\int_1^4 y^4\,dy$ $V=\pi\left[\frac{1}{5}y^5\right]_1^4$ $V=\frac{4^5}{5}\pi-\frac{\pi}{5}$ $V=\frac{1\,023\pi}{5}$ units cubed	3 marks for correct answer and solution. 2 marks for correct integral or equivalent merit. 1 mark for correct set-up of integral

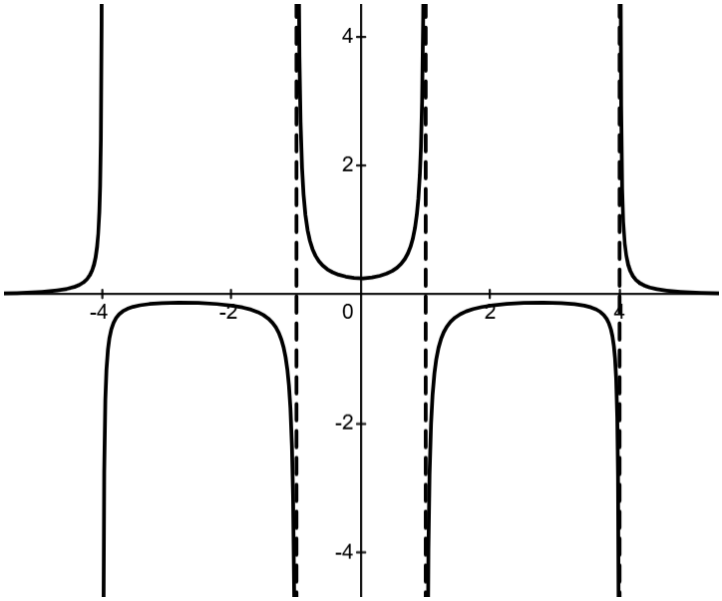
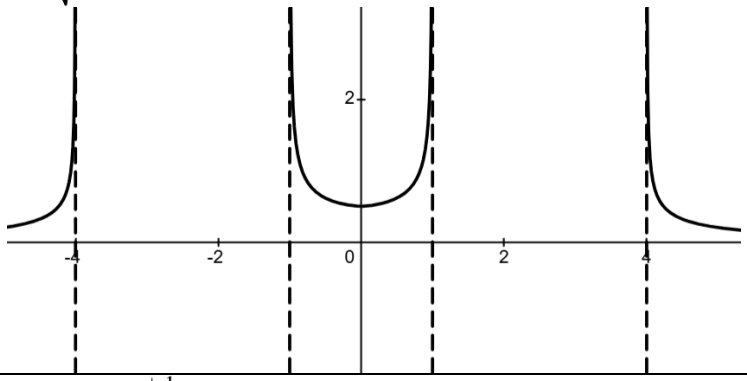
12ai	$R=\sqrt{3^2+\left(\sqrt{3}\right)^2}=\sqrt{12}=2\sqrt{3}$ $\alpha=\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)=\frac{\pi}{6}$ $\therefore\ 3\sin x+\sqrt{3}\cos x\equiv 2\sqrt{3}\sin\left(x+\frac{\pi}{6}\right)$	2 marks 1 mark for R, 1 mark for α
12aii	$2\sqrt{3}\sin\left(x+\frac{\pi}{6}\right)=\sqrt{6}$ $\sin\left(x+\frac{\pi}{6}\right)=\frac{1}{\sqrt{2}},\quad \left(x+\frac{\pi}{6}\right)\in\left[\frac{\pi}{3},\frac{7\pi}{3}\right]$ $x+\frac{\pi}{6}=\frac{\pi}{4},\frac{3\pi}{4}$ $x=\frac{\pi}{12},\frac{7\pi}{12}$	2 marks 1 mark for one solution / 1 mark for correct process with one error
12b	$\frac{d}{dx}\left[\tan^{-1}f(x)\right]=\frac{f'(x)}{1+[f(x)]^2}$ $f(x)=x(1-x^2)^{\frac{1}{2}}$ $f'(x)=\frac{(1-x^2)^{\frac{1}{2}}-x\times\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)}{1-x^2}$ $f'(x)=\frac{(1-x^2)^{\frac{1}{2}}+x^2(1-x^2)^{-\frac{1}{2}}}{1-x^2}$ $f'(x)=\frac{1-x^2+x^2}{(1-x^2)^{\frac{3}{2}}}$ $f'(x)=\frac{1}{(1-x^2)^{\frac{3}{2}}}$ $\frac{d}{dx}\left[\tan^{-1}\frac{x}{\sqrt{1-x^2}}\right]$ $=\frac{1}{(1-x^2)^{\frac{3}{2}}}\times\frac{1}{1+\frac{x^2}{1-x^2}}$ $=\frac{1}{(1-x^2)^{\frac{3}{2}}}\times\frac{1}{\frac{1-x^2+x^2}{1-x^2}}$ $=\frac{1}{(1-x^2)^{\frac{3}{2}}}\times(1-x^2)$ $=\frac{1}{\sqrt{1-x^2}}$	3 marks for correct solution 2 marks for obtaining a correct expression for the derivative such as: $\frac{1}{(1-x^2)^{\frac{3}{2}}}\times\frac{1}{1+\frac{x^2}{1-x^2}}$ 1 mark for finding $\frac{1}{1+[f(x)]^2}$ or $f'(x)$ correctly, where $f(x)=\frac{x}{\sqrt{1-x^2}}$

	Another option is to first recognise that $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)=\sin^{-1}x$!	
12c	$F_1+F_2+F_3$ $=\begin{pmatrix}3\cos 30°\\3\sin 30°\end{pmatrix}+\begin{pmatrix}2\cos 135°\\2\sin 135°\end{pmatrix}+\begin{pmatrix}0\\-4\end{pmatrix}$ $=\begin{pmatrix}1.183862649...\\-1.085786438...\end{pmatrix}$  $\alpha=\tan^{-1}\frac{1.085786...}{1.18...}\approx 43°$ Therefore, the direction of the resultant vector is 43° below the positive x-direction, or 317° measured anticlockwise from the positive x-direction.	3 marks: 1st mark for correctly writing either F_1 or F_2 as a vector 2nd mark for writing $F_1+F_2+F_3$ as a vector 3rd mark for finding the direction of that vector OR 1 mark for adding the horizontal components, 1 mark for adding the vertical components, and 1 mark for finding the direction as an angle **Note**: the answer did not have to be in exact form. Consider using decimals instead of mucking around with surds/ fractions.
12d	$u=1-x^2$ $du=-2xdx$ When $x=0$, $u=1$ When $x=1$, $u=0$ $x^2=1-u$ $I=\int_0^1 x^2\sqrt{1-x^2}\,.\,xdx$ $I=-\frac{1}{2}\int_0^1 x^2\sqrt{1-x^2}\,(-2x)dx$ $I=-\frac{1}{2}\int_1^0 (1-u)u^{\frac{1}{2}}\,\mathrm{du}$	3 marks: 1st mark for correctly doing TWO of the following things: • Converting $\int_0^1$ to $\int_1^0$ • Converting dx into du using $du=-2x\,dx$ • Converting $\sqrt{1-x^2}$ into $\sqrt{u}$ 2nd mark for expressing the entire integral in terms of u 3rd mark for integrating and evaluating **Note**: So many careless errors…

	$I=\frac{1}{2}\int_0^1 \left(u^{\frac{1}{2}}-u^{\frac{3}{2}}\right)du$ $I=\frac{1}{2}\left[\frac{2}{3}u^{\frac{3}{2}}-\frac{2}{5}u^{\frac{5}{2}}\right]_0^1$ $I=\frac{1}{2}\left(\frac{2}{3}-\frac{2}{5}-(0)\right)$ $I=\frac{2}{15}$	
12e	$\cos\theta=\frac{\underset{\sim}{\mathbf{p}}\cdot\underset{\sim}{\mathbf{q}}}{\lvert\underset{\sim}{\mathbf{p}}\rvert\,\lvert\underset{\sim}{\mathbf{q}}\rvert}=\frac{\underset{\sim}{\mathbf{p}}\cdot\underset{\sim}{\mathbf{q}}}{1\times 1}=\underset{\sim}{\mathbf{p}}\cdot\underset{\sim}{\mathbf{q}}$ $\underset{\sim}{\mathbf{a}}\cdot\underset{\sim}{\mathbf{b}}=0$ $\left(\underset{\sim}{\mathbf{p}}+2\underset{\sim}{\mathbf{q}}\right)\cdot\left(5\underset{\sim}{\mathbf{p}}-4\underset{\sim}{\mathbf{q}}\right)=0$ $5\lvert\underset{\sim}{\mathbf{p}}\rvert^2-4\left(\underset{\sim}{\mathbf{p}}\cdot\underset{\sim}{\mathbf{q}}\right)+10\left(\underset{\sim}{\mathbf{p}}\cdot\underset{\sim}{\mathbf{q}}\right)-8\lvert\underset{\sim}{\mathbf{q}}\rvert^2=0$ $5(1)^2+6\left(\underset{\sim}{\mathbf{p}}\cdot\underset{\sim}{\mathbf{q}}\right)-8(1)^2=0$ $6\left(\underset{\sim}{\mathbf{p}}.\underset{\sim}{\mathbf{q}}\right)-3=0$ $\underset{\sim}{\mathbf{p}}.\underset{\sim}{\mathbf{q}}=\frac{1}{2}$ $\therefore\quad \cos\theta=\frac{1}{2}\ (0\le\theta<\pi)$ $\theta=\frac{\pi}{3}$	2 marks for correct solution 1st mark for correctly expanding $\left(\underset{\sim}{\mathbf{p}}+2\underset{\sim}{\mathbf{q}}\right)\cdot\left(5\underset{\sim}{\mathbf{p}}-4\underset{\sim}{\mathbf{q}}\right)$, 2nd mark for finding the angle **Note: p** and **q** are vectors, not scalars!
13a i)	$\mathrm{E}(\hat{p})=\mathrm{p}$ $=0.92$ Standard deviation $=\sqrt{\frac{p(1-p)}{n}}$ $=\sqrt{\frac{0.92(1-0.92)}{200}}$ $=0.019$(3 dp)	1 mark for correct working for "show"
13a(ii)	$P(\hat{p}\le 0.9)=\left(z\le\frac{0.9-0.92}{0.019}\right)$ $=\mathrm{P}(z\le -1.05)$ $=1-0.8531$ $=0.1469$ Let X be the number of weeks the teacher reports. $X\sim Bin(20, 0.1469)$ $P(X\ge 1)=1-P(X=0)$ $=1-(1-0.1469)^{20}$ $=$ 0.958313..... ≈ 0.96	1 mark for correct z-score or equivalent 1 mark for finding $P(\hat{p}\le 0.9)$ or equivalent 1 mark for correct solution
13b i)	*LHS:* $cos(3\theta)=cos(2\theta+\theta)$ $=\cos 2\theta cos\theta-sin2\theta\theta sin\theta$	1 mark for a correct solution.

	$= (2cos^2\theta - 1)cos\theta - 2sin^2\theta cos\theta$ $= 2cos^3\theta - cos\theta - 2(1 - cos^2\theta)cos\theta$ $= 2cos^3\theta - cos\theta - 2cos\theta + 2cos^3\theta$ $= 4cos^3\theta - 3cos\theta$	
13b(ii)	Let $x = cos\theta$ $8cos^3\theta - 6cos\theta + 1 = 0$ $2(4cos^3\theta - 3cos\theta) + 1 = 0$ $2cos(3\theta) + 1 = 0$ (from part (i)) $cos(3\theta) = -\frac{1}{2}$ (2^{nd} ,3^{rd} , 6^{th} quadrants) $3\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$ $\theta = \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{8\pi}{9}$	1 mark for $cos(3\theta) = -\frac{1}{2}$ 1 mark for correct evaluation of θ Remember to express the roots as $cos\frac{2\pi}{9}, cos\frac{4\pi}{9}, cos\frac{8\pi}{9}$
13b(iii)	LHS: $=\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ $= \frac{\alpha\beta + \beta\gamma + \alpha\gamma}{\alpha\beta\gamma}$ (1 mark) $= \frac{\frac{-6}{8}}{\frac{-1}{8}}$ $= 6$ (1 mark) = RHS OR LHS: $sec\frac{2\pi}{9} + sec\frac{4\pi}{9} + sec\frac{8\pi}{9}$ $= \frac{1}{cos\frac{2\pi}{9}} + \frac{1}{cos\frac{4\pi}{9}} + \frac{1}{cos\frac{8\pi}{9}}$ $= \frac{cos\frac{4\pi}{9}cos\frac{8\pi}{9} + cos\frac{2\pi}{9}cos\frac{4\pi}{9} + cos\frac{8\pi}{9}cos\frac{2\pi}{9}}{cos\frac{2\pi}{9}cos\frac{4\pi}{9}ccos\frac{8\pi}{9}}$ (1 mark) $= \frac{\frac{-6}{8}}{\frac{-1}{8}}$ (From sum of roots 2 at a time and product of roots) $= 6$ (1 mark) = RHS	Need to use "hence" (ie part ii)
13c(i)	$\frac{dN}{dt} = kN(P - N)$ $\int \frac{dN}{dt} = \int kN(P - N)$ $\int \frac{dt}{dN} = \int \frac{1}{kN(P - N)}$ $k\int dt = \int \left(\frac{1}{P}\left(\frac{1}{N} + \frac{1}{P-N}\right)\right) dN$ 1 mark for correct integrals $kt = \frac{1}{P}(ln\lvert N\rvert - ln\lvert P - N)\rvert)$ $kt + c = \frac{1}{P}\left(ln\left(\left\lvert\frac{N}{P-N}\right\rvert\right)\right)$ $Pkt + c = ln\left\lvert\frac{N}{P-N}\right\rvert$ $e^{pkt+c} = \frac{N}{P-N}$ 1 mark for expressing as a correct exponential Let B = $\pm e^c$ $Be^{pkt} = \frac{N}{P-N}$ $N = (P - N)Be^{pkt}$	3 marks for a completely correct solution 1 mark for correct integrals 1 mark for correct exponential equation

	$N + NBe^{Pkt} = PBe^{Pkt}$ $N(1 + Be^{Pkt}) = PBe^{Pkt}$ $N = \frac{PBe^{Pkt}}{1 + Be^{Pkt}}$ $N = \frac{PBe^{Pkt}}{Be^{Pkt}\left(1+\frac{1}{B}e^{-Pkt}\right)}$ $N = \frac{P}{1+Be^{-Pkt}}$ as required. Alternate method: Let A = $\pm e^{c}$ $Ae^{pkt} = \frac{N}{P-N}$ $N = (P - N)Ae^{pkt}$ $N + NAe^{Pkt} = PAe^{Pkt}$ $N(1 + Ae^{Pkt}) = PAe^{Pkt}$ $N = \frac{PAe^{Pkt}}{1 + Ae^{Pkt}}$ $N = \frac{PAe^{Pkt}}{1+Ae^{Pkt}} \div \frac{Ae^{Pkt}}{Ae^{Pkt}}$ $N = \frac{P}{\frac{1}{A}e^{-Pkt}+1}$ where B = $\frac{1}{A}$ $N = \frac{P}{Be^{-Pkt}+1}$	
13c(ii)	t =0, P= 5000, N=500 $500 = \frac{5000}{1 + Be^{0}}$ $500 = \frac{5000}{1 + B}$ $500 + 500B = 5000$ $B = 9$ $N = \frac{P}{1 + 9e^{-Pkt}}$ $720 = \frac{5000}{1 + 9e^{-25000k}}$ $720 + 6480e^{-25000k} = 5000$ $6480e^{-25000k} = 4280$ $e^{-25000k} = \frac{107}{162}$ $\ln e^{-25000k} = ln\frac{107}{162}$ $-25000k = ln\frac{107}{162}$ $k = \frac{ln\frac{107}{162}}{-25000}$ $k = 0.0000165907$ ….	1 mark for correct P and B 1 mark for significant progress towards finding k 1 mark for correct value of k.

14a	$y = f(\|x\|)$  $y = \sqrt{f(\|x\|)}$  	3 marks: correct answer – must show correct y-intercept and correct asymptotes 2 marks: correct graph of $y = f(\|x\|)$ with correct y-intercept OR correct graph of $y = \sqrt{f(\|x\|)}$ with no or incorrect y-intercept 1 mark: correct graph of $y = \sqrt{f(x)}$ OR Correct graph of LHS or RHS of $y = f(\|x\|)$ - must clearly show asymptotic behaviour of the curve and correct y-intercept OR finds correct y-intercept of $y = f(\|x\|)$ and shows correct shape of $y = f(\|x\|)$ with correct/incorrect asymptotes
14b	$f(n) = 13^{n+1}(6n+7) - 1$ $S(1)$: $f(1) = 13^2(6 \times 1 + 7) - 1 = 2196 = 18 \times 122$ $\therefore S(1)$ is true Assume $S(k)$: $f(k) = 13^{k+1}(6k+7) - 1$ i.e. $13^{k+1}(6k+7) - 1 = 18M$; $M \in \mathbb{Z}^+$ $RTP \quad S(k+1) : 13^{k+2}[6(k+1)+7] - 1 = 18N, \ N \in \mathbb{Z}^+$ i.e. $13^{k+2}(6k+13) - 1 = 18N$	3 marks: correct answer 2 marks: finds correct value of S(1). Must show $2196 = 18 \times 122$, AND states correct forms of S(k) and $S(k+1)$ as multiples of 18. 1 mark: finds S(1) as $2196 = 18 \times 122$ (not just "2196 is divisible by 18") OR correct statement of S(k) or S(k+1)

	$f(k+1)=13^{k+2}(6k+13)-1$ From assumption, $-1=18M-13^{k+1}(6k+7)$ $f(k+1)=13^{k+2}(6k+13)+18M-13^{k+1}(6k+7)$ $=18M-13^{k+1}[13(6k+13)-(6k+7)]$ $=18M-13^{k+1}[78k+169-6k-7]$ $=18M-13^{k+1}(72k+162)$ $=18M-13^{k+1}\times 18(4k+9)$ $=18\left(M-13^{k+1}(4k+9)\right)$ is divisible by 18 Hence true by induction for all $n \in \mathbb{Z}^+$	
14c	$\frac{dy}{dx}=(x+1)^2(y-2)^{\frac{1}{2}}$ $\int (y-1)^{-\frac{1}{2}}dy=\int (x+1)^2\,dx$ $2(y-1)^{\frac{1}{2}}=\frac{(x+1)^3}{3}+C$ $2\sqrt{y-2}=\frac{1}{3}(x+1)^3+C$ sub $x=2, y=6$: $4=9+C$ $C=-5$ $\therefore \quad 2\sqrt{y-2}=\frac{1}{3}(x+1)^3-5$ $\sqrt{y-2}\geq 0 \Rightarrow \frac{1}{3}(x+1)^3-5\geq 0$ $(x+1)^3\geq 15$ $x\geq \sqrt[3]{15}-1$	4 marks: correct answer 3 marks: correct primitive functions, evaluates a constant and attempts to find domain 2 marks: finds two correct primitive functions 1 mark: separates variables <u>and</u> finds one correct primitive function

14di	$V = Ah$ $H = 8 \Rightarrow V = 8A$ $A = \frac{1}{2} r^2(\theta - \sin\theta)$ $r = \frac{1}{2}$ $\Rightarrow A = \frac{1}{2}\left(\frac{1}{2}\right)^2(\theta - \sin\theta)$ $A = \frac{1}{8}(\theta - \sin\theta)$ $\Rightarrow V = 8 \times \frac{1}{8}(\theta - \sin\theta)$ $V = \theta - \sin\theta$	2 marks: correct answer 1 mark: finds correct <u>area</u> of shaded segment OR finds a correct area of one (or more) relevant component parts

14dii	$V=\theta-\sin\theta \Rightarrow \frac{dV}{d\theta}=1-\cos\theta$ $\frac{dV}{dt}=\frac{dV}{d\theta}\times\frac{d\theta}{dt}$ $0.1=(1-\cos\theta)\frac{d\theta}{dt}$ $\frac{d\theta}{dt}=\frac{0.1}{1-\cos\theta}$ $\frac{dh}{dt}=\frac{dh}{d\theta}\times\frac{d\theta}{dt}$ $\cos\left(\frac{\theta}{2}\right)=\frac{0.5-h}{0.5}=1-2h$ $h=\frac{1}{2}\left(1-\cos\frac{\theta}{2}\right)=\frac{1}{2}-\frac{1}{2}\cos\left(\frac{\theta}{2}\right)$ $\frac{dh}{d\theta}=-\frac{1}{2}\times-\frac{1}{2}\sin\frac{\theta}{2}=\frac{1}{4}\sin\frac{\theta}{2}$ $\frac{dh}{dt}=\frac{1}{4}\sin\frac{\theta}{2}\times\frac{0.1}{1-\cos\theta}$ $=\frac{1}{4}\sin\frac{\theta}{2}\times\frac{0.1}{1-\left(1-2\sin^2\left(\frac{\theta}{2}\right)\right)}$ $=\frac{1}{4}\sin\frac{\theta}{2}\times\frac{0.1}{2\sin^2\left(\frac{\theta}{2}\right)}$ $=\frac{0.1}{4}\times\frac{1}{2}\times\frac{1}{\sin\frac{\theta}{2}}$ $\frac{dh}{dt}=\frac{1}{80\sin\frac{\theta}{2}}$ For $h=25\text{cm}=0.25m,\ 0.5-h=0.25$ $\frac{\theta}{2}=\cos^{-1}\left(\frac{0.25}{0.5}\right)=\cos^{-1}\frac{1}{2}=\frac{\pi}{3}$ $\sin\frac{\theta}{2}=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$ $\frac{dh}{dt}=\frac{1}{80\times\frac{\sqrt{3}}{2}}=\frac{1}{40\sqrt{3}}=\frac{\sqrt{3}}{120}$ m/min	3 marks: correct answer 2 marks: finds correct expressions for any two of: $\frac{dV}{d\theta}, \frac{d\theta}{dt}, \frac{dh}{dt}, \frac{dh}{d\theta}$ or h 1 mark: finds a correct expression for any one of: $\frac{dV}{d\theta}, \frac{d\theta}{dt}, \frac{dh}{dt}, \frac{dh}{d\theta}$ or h