St George Girls High School

Student Number:

Teacher:

Mathematics Extension 1

2026 Trial HSC Examination

General Instructions

- Reading time – **10 minutes**
- Working time – **2 hours**
- Write using **black** pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in **Section I**, use the multiple-choice answer sheet provided
- For questions in **Section II**:
  - answer the questions in the writing booklets provided
  - begin each question in a new booklet
  - extra writing booklets are available if needed
  - show relevant mathematical reasoning and/or calculations
  - marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided
- Diagrams are not to scale

Total Marks: 70

Section I – 10 marks (pages 1–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 6–10)

- Attempt Questions 11–15
- Allow about 1 hour 45 minutes for this section

Q1 - 10	/10
Q11	/12
Q12	/12
Q13	/12
Q14	/12
Q15	/12
Total	**/70**
Percentage	**%**

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1. $2\cos 3A \sin A =$

(A) $\sin 4A - \sin 2A$.
(B) $\sin 4A + \sin 2A$.
(C) $\sin 2A - \sin 4A$.
(D) $-\sin 4A - \sin 2A$.

2. Determine $\dfrac{d}{dx}\cos^{-1}\dfrac{x}{2}$.

(A) $\dfrac{1}{\sqrt{4-x^2}}$
(B) $\dfrac{-1}{\sqrt{4-x^2}}$
(C) $\dfrac{1}{2\sqrt{4-x^2}}$
(D) $\dfrac{-1}{2\sqrt{4-x^2}}$

3. What is the magnitude of $\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b}$ if $\underset{\sim}{a} = 3\underset{\sim}{i} - 2\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} + 3\underset{\sim}{j}$?

(A) $\dfrac{-3}{\sqrt{10}}$
(B) $\dfrac{-3}{\sqrt{13}}$
(C) $\dfrac{3}{\sqrt{13}}$
(D) $\dfrac{3}{\sqrt{10}}$

4. State the number of solutions to $\cos\frac{x}{2}\sin\frac{x}{2}=0$ for $0\leq x\leq 2\pi$.

(A) 0
(B) 1
(C) 2
(D) 3

5. In how many ways can six people be seated around a circular table if 2 particular people refuse to sit next to each other?

(A) 72
(B) 120
(C) 480
(D) 720

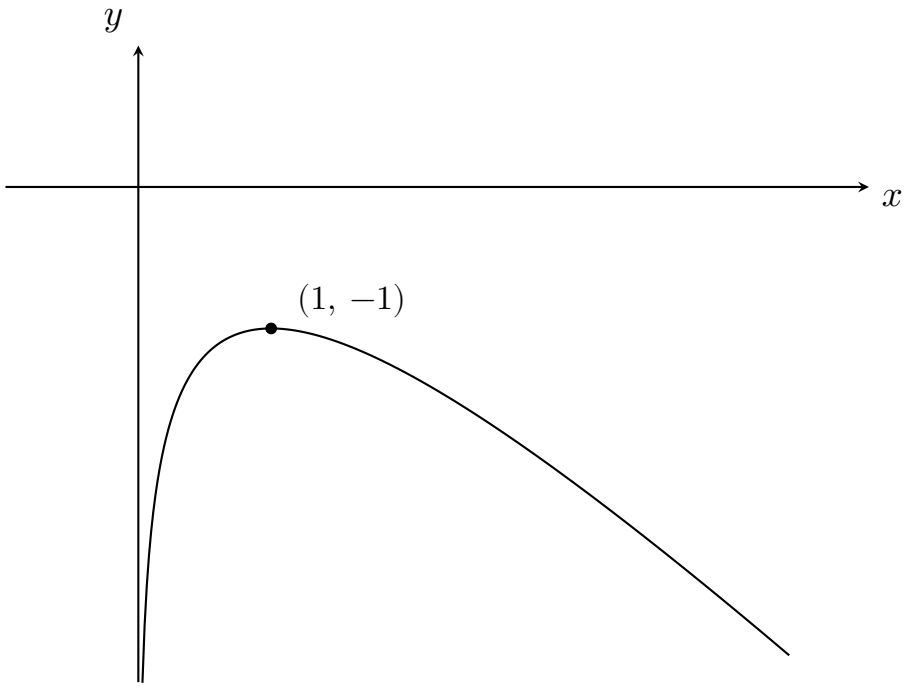
6. Determine $\arcsin\left(\sin\frac{5\pi}{7}\right)$.

(A) $-\frac{5\pi}{7}$
(B) $-\frac{2\pi}{7}$
(C) $\frac{2\pi}{7}$
(D) $\frac{5\pi}{7}$

7. $f(x)$ is a continuous function.
$f(x)=5$ has 3 distinct solutions. $|f(x)|$ will have

(A) at least 3 solutions.
(B) at most 3 solutions.
(C) at most 6 solutions.
(D) at least 6 solutions.

8. Below is the graph of $y = f(x) + g(x)$. It's natural domain is $x > 0$.
What are the possible expressions of $f(x)$ and $g(x)$?



(A) $f(x) = -\ln x$ and $g(x) = x$

(B) $f(x) = -\dfrac{1}{x}$ and $g(x) = x$

(C) $f(x) = \dfrac{1}{x}$ and $g(x) = -x$

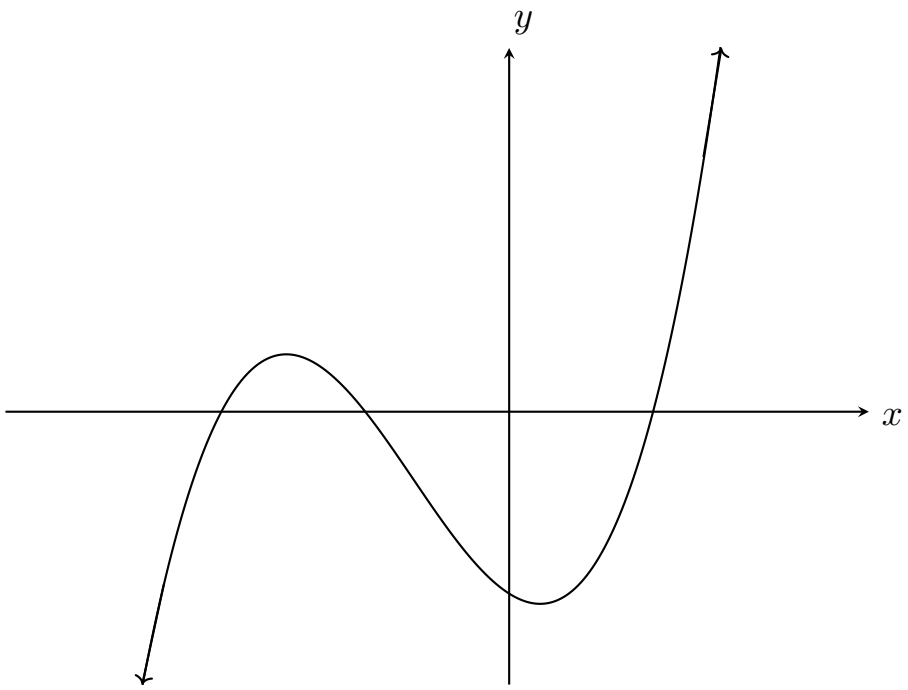
(D) $f(x) = \ln x$ and $g(x) = -x$

9. The graph of $y = x^3 + bx^2 - x + d$ is given below. Assume it is drawn to scale. Which of the following statements are true?

I. $b < 0$

II. $d < 0$

III. $bd < 0$



(A) I only.

(B) II only.

(C) I and III.

(D) II and III.

10. $y = f(x)$ is a curve that passes through $(1, 3)$ with a gradient of 2. By letting $g(x) = f^{-1}(x)$, determine $g'(3)$.

(A) $-\dfrac{1}{2}$

(B) $\dfrac{1}{2}$

(C) 3

(D) $\dfrac{1}{3}$

Section II

60 marks
Attempt Questions 11–15
Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet.
Your answers should include relevant mathematical reasoning and/or calculations.

Question 11 (12 marks) Start this in a NEW WRITING BOOKLET.

(a) Find $\int \cos^2 3x \, dx$. **2**

(b) Consider the two vectors $\boldsymbol{a} = \begin{pmatrix} -3 \\ m \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$. If $\boldsymbol{a}$ is parallel to $\boldsymbol{b}$, what is the value of m? **1**

(c) A drone is set to fly at a velocity of $\begin{pmatrix} 15 \\ 12 \end{pmatrix}$ km/h. **2**

However, there is wind blowing at $\begin{pmatrix} -6 \\ 5 \end{pmatrix}$ km/h.

Calculate the overall speed of the drone. Leave your answer to 1 decimal place.

(d) Solve $\dfrac{1}{x-5} \leq 3$. **3**

(e) Determine the coefficient of x^2 in $\left(1 + \dfrac{1}{x}\right)(x+1)^3$. **2**

(f) $P(x) = (a^2 - 1)x^3 + ax^2 + -7x + 2$ is divisible by $(x - 1)$. **2**
Determine the possible values of a.

Question 12 (12 marks) Start this in a NEW WRITING BOOKLET.

(a) Prove by mathematical induction that for all integers $n \geq 1$: **3**

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

(b) The position vector of a particle at time t seconds is given below. **2**

$$\underset{\sim}{r}(t) = (3-t)\underset{\sim}{i} - 2\sqrt{t}\,\underset{\sim}{j}$$

Determine the velocity vector at $t = 4$.

(c) Graph the curve represented by the pair of parametric equations below for $-1 \leq t \leq 1$. **2**

$$x = 1 + t$$
$$y = -t^2$$

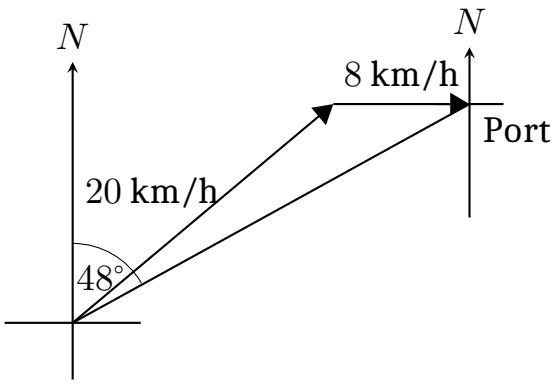
(d) (i) Express $3\cos x + \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$ for $-\dfrac{\pi}{2} \leq \alpha \leq 0$. **3**

(ii) Hence, solve $3\cos x + \sqrt{3}\sin x = 3$ for $0 \leq x \leq 2\pi$. **2**

Question 13 (12 marks) Start this in a NEW WRITING BOOKLET.

(a) By using the substitution $u = 1 - x^2$, find $\int -2x^3\sqrt{1-x^2}\,dx$. **3**

(b) A boat needs to travel to a port that is on a bearing of 048°. **3**
The boat's engine is set to drive at a constant speed of 20 km/h.
There is a current flowing from the West at a constant speed of 8 km/h.



On what bearing, to the nearest degree, should the boat be directed in order to reach the port?

(c) From a height of 1.75 m, a ball was thrown directly upwards at a speed of 25 m/s.
Assume a gravitational acceleration of -10 m/s^2 and no air resistance.

(i) Find an equation to represent the height of the ball from the ground, h, in terms of t seconds after the ball is thrown. **2**

(ii) Hence, when will the ball hit the ground? Answer to the nearest second. **1**

(d) A triangle has two fixed side lengths of length 8 cm and 6 cm. The included angle, θ, is increasing at a constant rate of $\dfrac{\pi}{6}$ radians per second. **3**

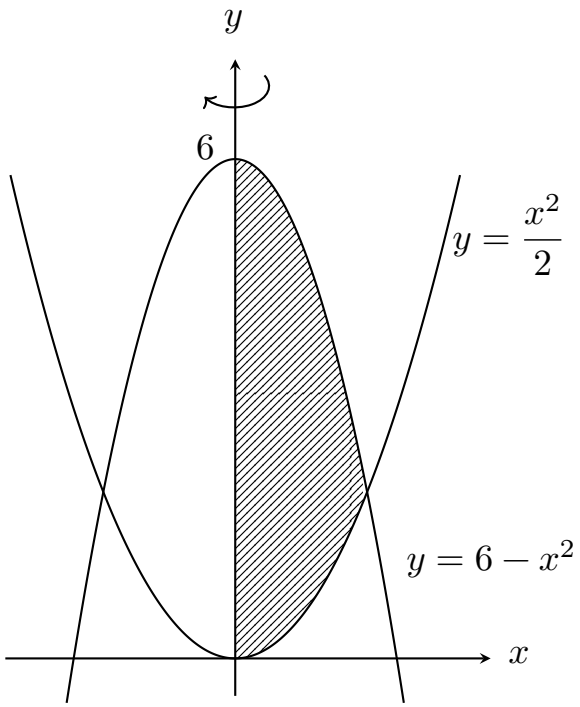
Find the exact rate the area of the triangle is increasing when $\theta = \dfrac{\pi}{3}$.

Question 14 (12 marks) Start this in a NEW WRITING BOOKLET.

(a) Solve for x, given **2**

$$\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}.$$

(b) The region bounded by $y = \dfrac{x^2}{2}$, $y = 6 - x^2$, and $x = 0$ is rotated about the y-axis. **4**



Determine the exact volume formed.

(c) It is known that the product of two consecutive numbers is even. **3**
I.e. $k(k+1) = 2A$, where k and A are integers.

Prove by mathematical induction that for all integers $n \geq 1$,

$$n^3 + 11n \text{ is divisible by } 6.$$

(d) (i) Show that $\binom{n+1}{r+1} = \binom{n}{r} + \binom{n}{r+1}$. **2**

(ii) Hence, find ONE possible set of values for p and q such that **1**

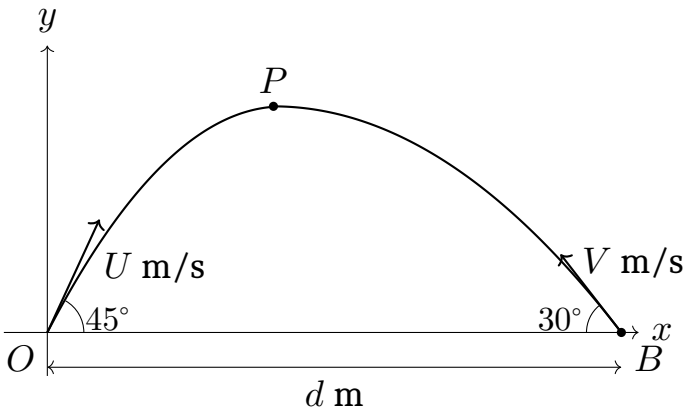
$$\binom{2025}{41} + \binom{2025}{42} + \binom{2026}{43} = \binom{p}{q}.$$

Question 15 (12 marks) Start this in a NEW WRITING BOOKLET.

(a) Two projectiles are fired at the same time to collide at a point P, when they are both at their **maximum height.**

Projectile A is fired from the origin at a speed of U m/s at an angle of 45° from the horizontal.

Projectile B is fired from point B, d metres from O at a speed of V m/s at an angle of 30° from the horizontal, as shown in the diagram below.



The displacement vector equations from O are as follows.
You do NOT need to derive these.

$$\underset{\sim}{r_A}(t) = \left(\frac{U\sqrt{2}}{2}\,t\right)\underset{\sim}{i} + \left(\frac{U\sqrt{2}}{2}\,t - 5t^2\right)\underset{\sim}{j}$$

$$\underset{\sim}{r_B}(t) = \left(d - \frac{V\sqrt{3}}{2}\,t\right)\underset{\sim}{i} + \left(\frac{V}{2}\,t - 5t^2\right)\underset{\sim}{j}$$

(i) Show that the projectile from O reaches its maximum height after **1**

$$t = \frac{U\sqrt{2}}{20}.$$

(ii) Show that at point P, **1**

$$V = U\sqrt{2}.$$

(iii) Hence, determine the distance d between O and B in terms of U. **2**

Question 15 continues on the next page

Question 15 continued

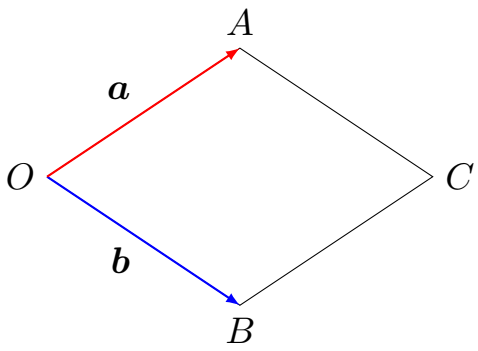
(b) An object has three forces acting on it. The forces are: **4**

$$\mathbf{F}_1 = \begin{pmatrix} -a^2 + 3a \\ a^3 - 6a \end{pmatrix} \quad \mathbf{F}_2 = \begin{pmatrix} k + 4 \\ -10a + k \end{pmatrix} \quad \mathbf{F}_3 = \begin{pmatrix} -3a - 2k \\ -24 \end{pmatrix}$$

If the object is in equilibrium, what are the possible values of a?

(c) The figure $OACB$ below is a rhombus of area 4 square units.

$\overrightarrow{OA}$ is defined as $\boldsymbol{a}$ and $\overrightarrow{OB}$ is defined as $\boldsymbol{b}$.



The information below are given. Do NOT prove them.

- the area of a rhombus is $A = \frac{1}{2}xy$, where x and y are the diagonal lengths
- $(\boldsymbol{u} + \boldsymbol{v}) \cdot (\boldsymbol{u} + \boldsymbol{v}) = |\boldsymbol{u}|^2 + |\boldsymbol{v}|^2 + 2\boldsymbol{u} \cdot \boldsymbol{v}$

(i) Show that **2**

$$|\boldsymbol{a} + \boldsymbol{b}|^2 \, |\boldsymbol{b} - \boldsymbol{a}|^2 = 64.$$

(ii) Hence, show that: **2**

$$|\boldsymbol{a}|^4 - (\boldsymbol{a} \cdot \boldsymbol{b})^2 = 16$$

End of paper

MARKER'S COMMENTS

1. $2\cos 3A \sin A = 2 \cdot \frac{1}{2}[\sin(3A+A) - \sin(3A-A)]$

$= \sin 4A - \sin 2A$

A

2. $\frac{d}{dx}\cos^{-1}\left(\frac{x}{2}\right) = \frac{1}{2} \cdot \frac{-1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$

$= \frac{-1}{\sqrt{4-x^2}}$

B

3. $\text{proj}_{\underset{\sim}{a}} \underset{\sim}{b} = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}|^2} \underset{\sim}{a}$

$= \frac{3-6}{3^2+2^2}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$= -\frac{3}{13}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$

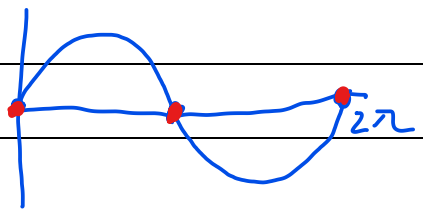
$\therefore |\text{proj}_{\underset{\sim}{a}} \underset{\sim}{b}| = \frac{3}{13}\sqrt{3^2+2^2}$

$= \frac{3}{\sqrt{13}}$

C

4. $\cos\frac{x}{2}\sin\frac{x}{2} = 0$

$\frac{1}{2}\sin x = 0$



$\therefore$ 3 solutions

D

Question 5

Step 1: Find the total number of circular arrangements
For n distinct people around a circular table, the total number of arrangements is given by: $(n-1)!$
For 6 people: Total arrangements $= (6-1)! = 5! = 120$

Step 2: Find the number of arrangements where the 2 people sit together
Treat the 2 people who refuse to sit together as a single block. This leaves 5units to arrange around the circular table: $(5-1)! = 4! = 24$

The 2 people within the block can arrange themselves in: $2! = 2$ ways

So, the number of arrangements where they are together is: $24 \times 2 = 48$

Step 3: Subtract to find the desired arrangements
Number of ways $= 120 - 48 = 72$

Correct Answer: (A)

Question 6

Step 1: Understand the principal range of the arcsine function
The range of $\arcsin(\theta)$is: $\left[-, \frac{\pi}{2}\frac{\pi}{2}\right]$

Step 2: Use trigonometric symmetry
The angle $\frac{5\pi}{7}$ lies in the second quadrant ($\frac{\pi}{2} < \frac{5\pi}{7} < \pi$). Using the identity $\sin(\pi - \theta) = \sin\theta$:

$$\sin\left(\frac{5\pi}{7}\right) = \sin\left(\pi - \frac{5\pi}{7}\right) = \sin\left(\frac{2\pi}{7}\right)$$

Step 3: Evaluate the inverse sine
Since $\frac{2\pi}{7} \in \left[-, \frac{\pi}{2}\frac{\pi}{2}\right]$: $\arcsin\left(\sin\frac{5\pi}{7}\right) = \frac{2\pi}{7}$

Correct Answer: (C)

Question 7

Step 1: Analyse the given equation
The absolute value equation is: $|\ f(x)\ | = 5 \iff f(x) = 5$ or $f(x) = -5$

Step 2: Count the solutions

- We are given that $f(x) = 5$has 3distinct solutions.
- All 3of these solutions satisfy $|\ f(x)\ | = 5$.
- In addition, any solutions to $f(x) = -5$would also satisfy $|\ f(x)\ | = 5$.

Therefore, $|\ f(x)\ | = 5$must have **at least 3 solutions**.

Correct Answer: (A)

Question 8

Step 1: Analyse the graph's key features

1. **Natural Domain:** $x > 0$, and as $x \to 0^+$, $y \to -\infty$.
2. **Stationary Point:** A local maximum occurs at $(1, -1)$.
3. **End Behaviour:** As $x \to \infty$, $y \to -\infty$.

Step 2: Test the given functions
Consider option **(D)**: $y = f(x) + g(x) = \ln x - x$:

- **Domain:** $x > 0$.
- **Value at $x = 1$:**

$$y(1) = \ln(1) - 1 = 0 - 1 = -1$$

- **Derivative:**

$$y' = \frac{1}{x} - 1$$

Setting $y' = 0 \implies x = 1$.

- **Second Derivative:**

$$y'' = -\frac{1}{x^2} < 0 \implies \text{local maximum at } (1, -1)$$

This matches all features of the given graph.

Correct Answer: (D)

Question 9

Step 1: Determine the sign of d
The y-intercept occurs at $x = 0$: $y(0) = d$

From the graph, the curve crosses the y-axis below the origin ($y < 0$), so $d < 0$ (Statement II is true)

Step 2: Determine the sign of b
Find the second derivative of $y = x^3 + bx^2 - x + d$:

$$y' = 3x^2 + 2bx - 1$$

$$y'' = 6x + 2b$$

At $x = 0$, the curve is concave up (just before reaching the local minimum on the right):

$$y''(0) = 2b > 0 \implies b > 0$$

Thus, $b < 0$ is false (Statement I is false).

Step 3: Determine the sign of bd
Since $b > 0$ and $d < 0$: $bd < 0$ (Statement III is true)

Correct Answer: (D)

Question 10

Step 1: Recall the derivative of an inverse function
Given $g(x) = f^{-1}(x)$:

$$g'(y) = \frac{1}{f'(x)} \text{ where } y = f(x)$$

Step 2: Substitute the known values
The point on $f(x)$is $(1, 3)$, which means $f(1) = 3 \implies g(3) = 1$.
The gradient of $f(x)$at $x = 1$is $f'(1) = 2$.

Applying the formula:

$$g'(3) = \frac{1}{f'(1)} = \frac{1}{2}$$

Correct Answer: (B)

12MAX 2026 Trial HSC Exam Q11 MARKER'S COMMENTS

Q11 a) $\int \cos^2 3x \, dx$

$= \int \frac{1}{2} + \frac{1}{2}\cos 6x \, dx$

$= \frac{1}{2}\int 1 + \cos 6x \, dx$

$= \frac{1}{2}\left(x + \frac{\sin 6x}{6}\right) + c$

or $\frac{1}{2}x + \frac{\sin 6x}{12} + c$

1 mark for rewriting the integrand correctly

1 mark for integrating correctly

Overall, this question was well answered. (2)

b) $\underset{\sim}{a} = \begin{pmatrix} -3 \\ m \end{pmatrix}$ $\underset{\sim}{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

Now $\underset{\sim}{a} = k\underset{\sim}{b}$ since $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$

$\begin{pmatrix} -3 \\ m \end{pmatrix} = k\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$\therefore k = -3$

$\therefore m = k \times 4$

$m = -3 \times 4$

$m = -12$ ——— 1 mark

Overall, this question was well answered. (1)

c) Let $\underset{\sim}{w}$ be the velocity wind vector

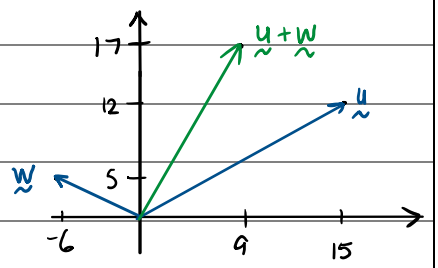
and $\underset{\sim}{u}$ be the velocity vector of the drone

$\underset{\sim}{w} = \begin{pmatrix} -6 \\ 5 \end{pmatrix}$ & $\underset{\sim}{u} = \begin{pmatrix} 15 \\ 12 \end{pmatrix}$

Resulting vector $= \underset{\sim}{u} + \underset{\sim}{w}$

$= \begin{pmatrix} 15 \\ 12 \end{pmatrix} + \begin{pmatrix} -6 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} 9 \\ 17 \end{pmatrix}$ ——— 1 mark for resulting vector



Speed $= |\underset{\sim}{u}+\underset{\sim}{w}| = \sqrt{9^2+17^2}$

$\doteq 19.2$ km/h ——— 1 mark for correct speed (2)

2 marks – Complete and correct solution

1½ marks – If vectors were added incorrectly and speed was correctly calculated. Eg $\begin{pmatrix} 15 \\ 12 \end{pmatrix} + \begin{pmatrix} -6 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ -7 \end{pmatrix}$ $\therefore$ speed $= \sqrt{9^2 + -7^2}$

1 mark – If vectors were subtracted and speed was calculated as $\sqrt{490}$

– For resulting vector $\begin{pmatrix} 9 \\ 17 \end{pmatrix}$

Overall, not well answered

MARKER'S COMMENTS

d) Method 1 Multiplying both sides by the square of the denominator

$$\frac{1}{x-5} \leq 3$$

$(x-5)^2 \times \frac{1}{x-5} \leq 3 \times (x-5)^2$, $x \neq 5$

$x - 5 \leq 3(x-5)^2$ ——— 1 mark

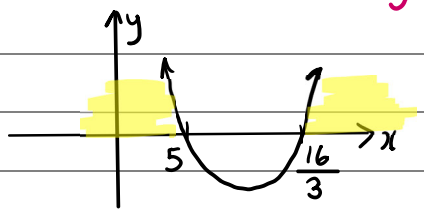
$3(x-5)^2 - (x-5) \geq 0$

$(x-5)(3(x-5) - 1) \geq 0$

$(x-5)(3x - 15 - 1) \geq 0$

$(x-5)(3x - 16) \geq 0$

or $3x^2 - 31x + 80 \geq 0$ } ——— 1 mark for either inequality



$\therefore x < 5$, $x \geq 5\frac{1}{3}$ ——— 1 mark for correct solution (3)

- 85% of the students used this method and for many it was not well answered.

Many minor errors were made throughout students' solution. Some of the errors were:

— wrong inequality sign was applied after step 1
— the values on the x-axis were in the wrong order
— the concavity of the parabola was drawn incorrectly
— $x = 5$ was included in the solutions. (½ mark off)

- 3 marks for correct solution, $x < 5$ or $x \geq \frac{16}{3}$
- $2\frac{1}{2}$ marks – for solution $x \leq 5$ or $x \geq \frac{16}{3}$
  or – for solution $x \leq \frac{16}{3}$, $x > 5$ if drew → 
- 2 marks – for solution $5 < x \leq \frac{16}{3}$ because of incorrect inequality sign or drawing the parabola incorrectly
  – for obtaining the inequality $3x^2 - 31x + 80 \geq 0$
- 1 mark – for multiplying both sides by the square of the denominator correctly.

For those students who use this method, please REVIEW

MARKER'S COMMENTS

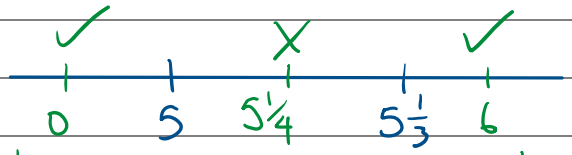
Method 2 – Critical point method

$$\frac{1}{x-5} \leq 3$$

- Discontinous at $x = 5$.
- For equality $\frac{1}{x-5} = 3$

$$1 = 3(x-5)$$
$$1 = 3x - 15$$
$$3x = 16$$
$$x = \frac{16}{3}$$
$$= 5\frac{1}{3}$$

$\therefore$ The critical points are 5 and $5\frac{1}{3}$ 2



$\frac{-1}{5} \leq 3$ ✓

$\frac{1}{\frac{1}{4}} \leq 3$

$4 \not\leq 3$

$\frac{1}{6-5} \leq 3$

$1 \leq 3$

$\therefore x < 5$ or $x \geq 5\frac{1}{3}$ 1 (3)

This was well done for the handful of students who used this method.

MARKER'S COMMENTS

e) $\left(1+\frac{1}{x}\right)(x+1)^3$

$= \left(1+\frac{1}{x}\right)(x^3+3x^2+3x+1)$ —— 1 mark for expanding $(1+x)^3$ correctly

For term in x^2:

$1 \times 3x^2 + \frac{1}{x} \times x^3$

$= 3x^2 + x^2$

$= 4x^2$

$\therefore$ coefficient of x^2 is 4. —— 1 mark for correct answer

(2)

Mostly well attempted. Some minor errors with addition.

Method 2:

A handful of students used the general term to answer this question. A few minor errors were made. Take care if using. Please see solution below

$\left(1+\frac{1}{x}\right)(1+x)^3 = (1+x^{-1})\ {}^3C_r\ 1^{3-r}\ x^r$

$= (1+x^{-1})\ {}^3C_r\ x^r$

$= {}^3C_r\ x^r + {}^3C_r\ x^r \times x^{-1}$

$= {}^3C_r\ x^r + {}^3C_r\ x^{r-1}$ —— 1

For x^2: $r=2$ & $r-1=2$

$\therefore \left(1+\frac{1}{x}\right)(1+x)^3 = {}^3C_2\ x^2 + {}^3C_3\ x^2$

$= 3x^2 + x^2$

$= 4x^2$

$\therefore$ coefficient is 4 —— 1

(2)

OR $\left(1+\frac{1}{x}\right)(1+x)^3 = (1+x^{-1})\left({}^3C_r\ x^{3-r} \cdot 1^r\right)$

$= {}^3C_r\ x^{3-r} + {}^3C_r\ x^{3-r-1}$

For x^2: $r=1$, $3-r=2$

$r=0$, $3-r-1=2$

$\therefore = {}^3C_1\ x^2 + {}^3C_0\ x^2$

$= 3x^2 + x^2$

$= 4x^2$

MARKER'S COMMENTS

f) $P(x) = (a^2-1)x^3 + ax^2 + -7x + 2$

If divisible by $x-1$ then

$P(1) = 0$

$a^2 - 1 + a - 7 + 2 = 0$ } ——— 1 mark

$a^2 + a - 6 = 0$ }

$(a+3)(a-2) = 0$

$\therefore \quad a = -3 \quad \text{or} \quad 2$ ——— 1 mark

(2)

Overall, this part was very well done, However the were a handful of students who need to review factorising a quadratic equation.

Question 12

a) Prove by mathematical induction that for all integers that for all integers $n \geq 1$:

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

Step 1 Prove the statement is true for $n = 1$

LHS $= \frac{1}{n(n+1)}$ RHS $= \frac{n}{n+1}$

$= \frac{1}{1(1+1)}$ $= \frac{1}{1+1}$

$= \frac{1}{2}$ $= \frac{1}{2}$

$\therefore$ LHS = RHS

$\therefore$ Proved that the statement is true for $n = 1$.

Step 2

Assume the statement is true for $n = k$

That is: $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{k(k+1)} = \frac{k}{k+1}$

Step 3 Prove the statement is true for $n = k+1$

RTP: $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+1+1)} = \frac{k+1}{k+1+1}$

That is:

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}$$

$$\text{LHS} = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$$

from step 2 (inductive hypothesis)

$$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$$

link - For using the inductive hypothesis

$$= \frac{k(k+2) + 1}{(k+1)(k+2)}$$

$$= \frac{k^2 + 2k + 1}{(k+1)(k+2)}$$

link for appropriate algebraic manipulation

$$= \frac{(k+1)(k+1)}{(k+1)(k+2)}$$

$$= \frac{k+1}{k+2}$$

$$= \text{RHS}$$

$\therefore$ LHS = RHS **Conclusion** Therefore true for $n = k+1$ when it is true for $n = k$. Since it is true for $n = 1$ then it is also true for $n = 1+1 = 2$, $n = 2+1 = 3$, and so on.

Therefore, it is true for all integers n by the principle of mathematical induction.

Teacher's Comment

Generally, well done.

Some students did not show 'how the inductive step' was used in the proof for $n = k+1$

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$$

from step 2 (inductive hypothesis)

b) The position vector of a particle at time t seconds is given below.

$$\underset{\sim}{r}(t) = (3-t)\,\underset{\sim}{i} - 2\sqrt{t}\,\underset{\sim}{j}$$

Determine the velocity vector at $t=4$.

$$\underset{\sim}{v}(t) = \frac{d}{dt}(3-t)\,\underset{\sim}{i} - \frac{d}{dt}2\sqrt{t}\,\underset{\sim}{j}$$

$$= \frac{d}{dt}(3-t)\,\underset{\sim}{i} - \frac{d}{dt}2t^{1/2}\,\underset{\sim}{j}$$

$$= -\underset{\sim}{i} - 2\times\frac{1}{2}t^{-1/2}\,\underset{\sim}{j}$$

$$= -\underset{\sim}{i} - t^{-1/2}\,\underset{\sim}{j}$$ — link — No ½ mark was awarded.

$$\text{or} = -\underset{\sim}{i} - \frac{1}{\sqrt{t}}\,\underset{\sim}{j}$$

$$\underset{\sim}{v}(4) = -\underset{\sim}{i} - \frac{1}{\sqrt{4}}\,\underset{\sim}{j}$$

$$= -\underset{\sim}{i} - \frac{1}{2}\,\underset{\sim}{j}$$ — link — No ½ mark was awarded.

If magnitude of the velocity vector was found then ½ a mark was deducted.

CFPE was awarded to students who had attempted to find the velocity vector either via differentiation or integration?

Teacher's Comments

Quite well done. However, a few students integrated rather than differentiating the given expression for the position vector. A few students attempted to find the magnitude of the velocity vector when the question is clearly asking students to determine the velocity vector.

12c) Graph the curve represented by the pair of parametric equations below for $-1 \leq t \leq 1$

$$x = 1 + t$$

$$y = -t^2$$

$x = 1 + t$

$t = x - 1$

$y = -t^2$

$= -(x-1)^2$ — 1 mk (Not required to show → not part of the question)

At $t = -1$

$x = 1 + t$

$= 1 + (-1)$

$= 0$

$y = -t^2$

$= -(-1)^2$

$= -1$

$\therefore (0, -1)$

At $t = 1$

$x = 1 + t$

$= 1 + 1$

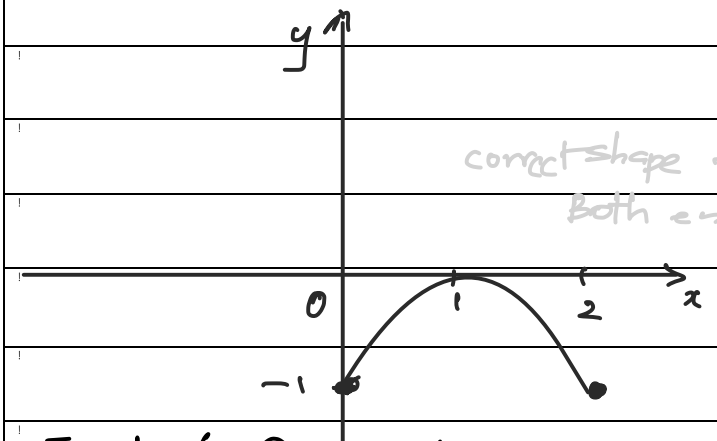
$= 2$

$y = -t^2$

$= -(1)^2$

$= -1$

$\therefore (2, -1)$



correct shape – ½ mk
Both end-points correct – ½ mk
If a student just sketches the graph (bald answer) then award 1 mk for the correct shape and ½ mark each for the correct end-points.

Teacher's Comments

Generally, well attempted. However, the majority of students forgot to consider the given domain.

d) Express $3\cos x + \sqrt{3}\sin x$ in the form $R\cos(x+\alpha)$ for $-\frac{\pi}{2} \leq \alpha \leq 2\pi$.

$3\cos x + \sqrt{3}\sin x = R\cos(x+\alpha)$

$= R[\cos x \cos\alpha - \sin x \sin\alpha]$

$= R\cos x \cos\alpha - R\sin x \sin\alpha$

If we compare the coefficients of $\cos x$ and $\sin x$ on both sides of the equation, we obtain:

$R\cos\alpha = 3$ ---①

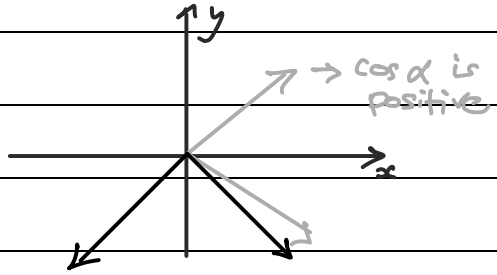
$-R\sin\alpha = \sqrt{3}$ ---②

② ÷ ①

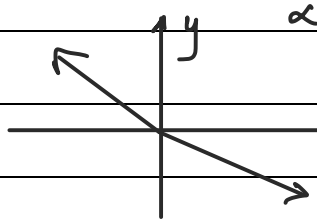
$-\tan\alpha = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$\therefore \tan\alpha = \frac{-1}{\sqrt{3}}$

$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6$ (Related angle)

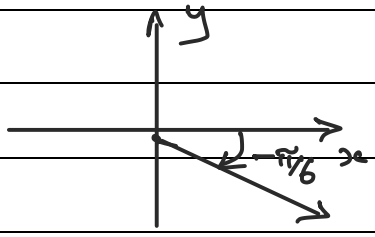


∴ overlap is quadrant 4



But $-\frac{\pi}{2} \leq \alpha \leq 0$

∴ α lies in the 4th quadrant



Recognising $\alpha = \pi/6$ is the related angle → ½ mk

Recognising $\alpha = -\frac{11\pi}{6}$ → 1 mk (As the student recognises the quadrant in which the angle lies)

Recognising that $\alpha = -\pi/6$ – (½ mks (Student considers the domain as well as the quadrant in which α lies).

Squaring and adding the 2 equations above gives:

$R^2 \cos^2\alpha = 9$

$R^2 \sin^2\alpha = 3$

$\therefore \quad R^2\cos^2\alpha + R^2\sin^2\alpha = 12$

$\therefore R^2(\sin^2\alpha + \cos^2\alpha) = 12$

$R = \pm\sqrt{12}$ But $R > 0$ by convention

$= 2\sqrt{3}$ — 1 mk

Writing this in the required form gives us:

$\therefore \quad 3\cos x + \sqrt{3}\sin x = 2\sqrt{3}\cos(x - \pi/6)$

— ½ mk

Teacher's Comments

Poorly attempted. Common error was not recognising that $\alpha = -\pi/6$. Some students were 'tricked' by the fact that they had to leave their answer in the form $R\cos(x + \alpha)$

CFE from part (i)

ii) Hence, solve $3\cos x + \sqrt{3}\sin x = 3$ for $0 \le x \le 2\pi$.

$$2\sqrt{3}\cos\left(x - \pi/6\right) = 3$$

$$\cos\left(x - \pi/6\right) = 3 \div 2\sqrt{3}$$

$$= 3 \times \frac{1}{2\sqrt{3}}$$

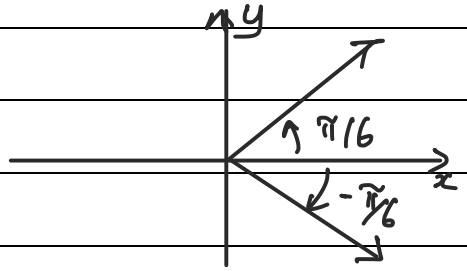
$$= \frac{3}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3\sqrt{3}}{6}$$

$$= \frac{\sqrt{3}}{2}$$

$$\therefore \cos\left(x - \pi/6\right) = \frac{\sqrt{3}}{2}$$

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6} \quad \text{(Related angle)}$$



$0 \le x \le 2\pi$

$-\pi/6 \le x - \pi/6 \le \frac{11\pi}{6}$

If students had simplified their question then they were penalised for doing so.

$\therefore x - \frac{\pi}{6} = -\pi/6, \pi/6, \frac{11\pi}{6}$ — ½mk, ½mk, ½mk

$\therefore x = -\pi/6 + \pi/6, \frac{\pi}{6} + \pi/6, \frac{11\pi}{6} + \pi/6$

$= 0, \frac{\pi}{3}, 2\pi$

½mk for ALL the correct final answers

Teacher's Comments

Poorly attempted. Students need to re-visit the concepts of considering the domain and the appropriate quadrants.

Question 13

(a)

We want to evaluate the integral:

$$\int -2x^3\sqrt{1-x^2}\,dx$$

Using the substitution:

$$u = 1 - x^2 \implies du = -2x\,dx$$

Rearranging for x^2:

$$x^2 = 1 - u$$

Rewrite the integrand by splitting $-2x^3$ into $x^2(-2x)$:

$$\int x^2\sqrt{1-x^2}(-2x\,dx)$$

Substitute u and du into the integral:

$$\int (1-u)\sqrt{u}\,du = \int (u^{1/2} - u^{3/2})\,du$$

Integrate term-by-term:

$$= \frac{u^{3/2}}{\frac{3}{2}} - \frac{u^{5/2}}{\frac{5}{2}} + C$$

$$= \frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2} + C$$

Substitute back $u = 1 - x^2$:

$$= \frac{2}{3}(1-x^2)^{3/2} - \frac{2}{5}(1-x^2)^{5/2} + C$$

(Alternatively, factored form: $\frac{2}{15}(1-x^2)^{3/2}(3x^2+2) + C$)

(b)

Let:

- $\vec{v}_{\text{boat}}$ be the boat's velocity vector with speed $|\,\vec{v}_{\text{boat}}\,| = 20$ km/h.
- $\vec{v}_{\text{current}}$ be the current's velocity vector with speed $|\,\vec{v}_{\text{current}}\,| = 8$ km/h flowing from the West towards the East (bearing 90°).
- $\vec{v}_{\text{resultant}}$ be the resultant velocity along the required bearing of 48°.

From vector addition, the velocities form a triangle:

$$\vec{v}_{\text{resultant}} = \vec{v}_{\text{boat}} + \vec{v}_{\text{current}}$$

1. Find the angle opposite the boat's velocity vector:
    - The resultant path makes an angle of 48°with North.
    - The East direction makes an angle of 90°with North.
    - The angle between the resultant direction and the East direction is:

$$90° - 48° = 42°$$

2. Use the Sine Rule to find the angle θ between the boat's heading and the resultant heading:

$$\frac{\sin\theta}{8} = \frac{\sin 42°}{20}$$

$$\sin\theta = \frac{8\sin 42°}{20} = 0.4\sin 42° \approx 0.26766$$

$$\theta = \arcsin(0.26766) \approx 15.52°$$

3. Calculate the required bearing:
   Since the current is pushing the boat to the East, the boat must steer into the current (more towards the North):

$$\text{Bearing} = 48° - 15.52° = 32.48° \approx 32°$$

Answer: The boat should be directed on a bearing of 32°

(c)

(i) Find an equation for the height $h(t)$:

The acceleration due to gravity is:

$$\ddot{h}(t) = -10\ \text{m/s}^2$$

Integrate to find the velocity $v(t) = \dot{h}(t)$:

$$v(t) = -10t + C_1$$

Given the initial velocity $v(0) = 25$ m/s:

$$C_1 = 25 \implies v(t) = -10t + 25$$

Integrate to find the height $h(t)$:

$$h(t) = -5t^2 + 25t + C_2$$

Given the initial height $h(0) = 1.75$ m:

$$C_2 = 1.75$$

Therefore, the equation is:

$$h(t) = -5t^2 + 25t + 1.75$$

(ii) When will the ball hit the ground?

The ball hits the ground when $h(t) = 0$:

$$-5t^2 + 25t + 1.75 = 0$$

Divide the entire equation by -5:

$$t^2 - 5t - 0.35 = 0$$

Using the quadratic formula $t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$:

$$t = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-0.35)}}{2(1)}$$

$$t = \frac{5 \pm \sqrt{25 + 1.4}}{2} = \frac{5 \pm \sqrt{26.4}}{2}$$

Since $t > 0$:

$$t = \frac{5 + 5.138}{2} \approx 5.07 \text{ seconds}$$

To the nearest second:

$$t \approx 5 \text{ seconds}$$

(d)

The area $A$of a triangle with side lengths $a = 8$ cm, $b = 6$ cm, and included angle $\theta$is:

$$A = \frac{1}{2}ab \sin\theta = \frac{1}{2}(8)(6)\sin\theta = 24\sin\theta$$

Differentiating with respect to θ:

$$\frac{dA}{d\theta} = 24\cos\theta$$

Using the Chain Rule to find the rate of change of area with respect to time t:

$$\frac{dA}{dt} = \frac{dA}{d\theta} \cdot \frac{d\theta}{dt}$$

Given $\frac{d\theta}{dt} = \frac{\pi}{6}$ rad/sand $\theta = \frac{\pi}{3}$:

$$\frac{dA}{dt} = \left(24\cos\left(\frac{\pi}{3}\right)\right) \cdot \frac{\pi}{6}$$

$$\frac{dA}{dt} = \left(24 \cdot \frac{1}{2}\right) \cdot \frac{\pi}{6} = 12 \cdot \frac{\pi}{6} = 2\pi \text{ cm}^2/\text{s}$$

Answer: The exact rate of increase of the area is 2π cm^2/s.

QUESTION 14

METHOD 1

$$\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$$

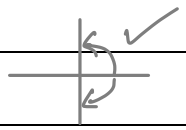
$$+$$

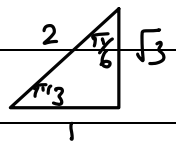
$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \quad \text{(identity)}$$

(1) mark uses identity and either substitutes or adds.

$$2\sin^{-1}x = \frac{\pi}{6} + \frac{\pi}{2}$$

$$2\sin^{-1}x = \frac{4\pi}{6}$$





$$\sin^{-1}x = \frac{\pi}{3}$$

$$x = \sin\frac{\pi}{3}$$

$$x = \frac{\sqrt{3}}{2}$$

(1) correctly finds x.

Method 2

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

$$\therefore \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$$

$$\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$$

$$\sin^{-1}x - \left(\frac{\pi}{2} - \sin^{-1}x\right) = \frac{\pi}{6}$$

$$\sin^{-1}x - \frac{\pi}{2} + \sin^{-1}x = \frac{\pi}{6}$$

$$2\sin^{-1}x = \frac{\pi}{6} + \frac{\pi}{2}$$

$$2\sin^{-1}x = \frac{4\pi}{6}$$

$$\sin^{-1}x = \frac{\pi}{3}$$

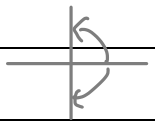
$$x = \sin\frac{\pi}{3}$$

$$x = \frac{\sqrt{3}}{2}$$

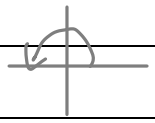
Q14 continued

METHOD 2

a) $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

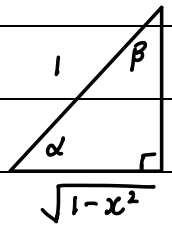
let $\alpha = \sin^{-1}x$ 

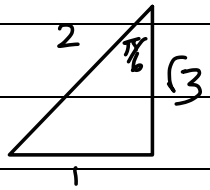
$\therefore \sin\alpha = \frac{x}{1}$

let $\beta = \cos^{-1}x$ 

$\cos\beta = \frac{x}{1}$

Overlap quad 1 $\therefore$ $x > 0$





$\therefore \alpha - \beta = \frac{\pi}{6}$

$\sin(\alpha-\beta) = \sin\frac{\pi}{6}$

(½) Apply strategy of taking sin of both sides

$\sin\alpha\cos\beta - \cos\alpha\sin\beta = \frac{1}{2}$

(½) Executing sin expansion correctly

$x \times x - \sqrt{1-x^2} \times \sqrt{1-x^2} = \frac{1}{2}$

$x^2 - (1-x^2) = \frac{1}{2}$

$x^2 - 1 + x^2 = \frac{1}{2}$

(1) Using the triangles to correctly find x (½ mark for justifying $x>0$)

$2x^2 = \frac{3}{2}$

$x^2 = \frac{3}{4}$

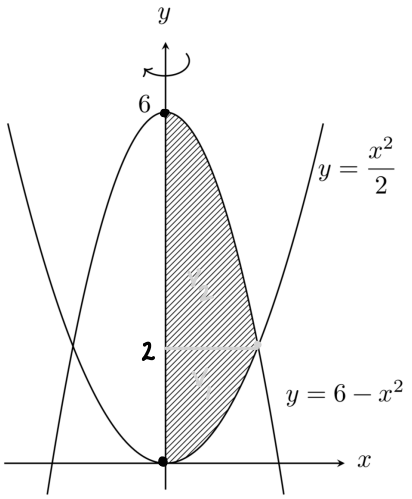
$x = \pm\frac{\sqrt{3}}{2}$ $\quad x > 0$

$x = \frac{\sqrt{3}}{2}$

NOTE: Review of inverse trig functions is advised. Many students had difficulty. For a 2 mark question, working out to obtain answer is required.

Q14 continued...

b)



$y = \frac{x^2}{2}$

$x^2 = 2y$

(½) finds an expression for x^2 for either function.

$y = 6 - x^2$

$x^2 = 6 - y$

Intersection of $y = \frac{x^2}{2}$ and $y = 6 - x^2$

$\frac{x^2}{2} = 6 - x^2$

$x^2 = 12 - 2x^2$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

(1) mark to find the point of intersection

when $x = 2$, $y = \frac{2^2}{2}$

$= 2$

$\therefore V = \pi \int_0^2 2y \, dy + \pi \int_2^6 (6 - y) \, dy$

(1) mark for correct process (addition of

$= \pi \left[y^2 \right]_0^2 + \pi \left[6y - \frac{y^2}{2} \right]_2^6$

$= \pi \left[2^2 - 0^2 \right] + \pi \left[6(6) - \frac{6^2}{2} - \left(6(2) - \frac{2^2}{2} \right) \right]$

$= 4\pi + \pi(36 - 18 - 10)$

$= 4\pi + 8\pi$

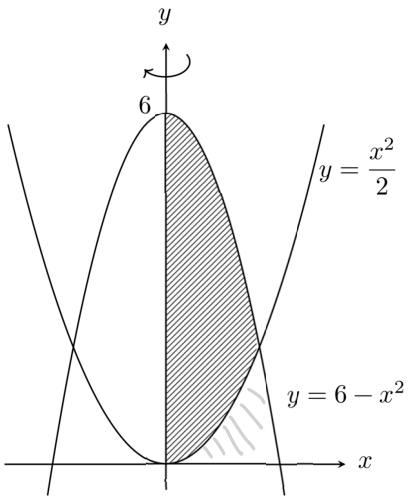
$= 12\pi \ u^3$

(1) correctly evaluates Volume.

No CFPE as many of the ways to find the volume were completely incorrect and made much easier.

NOTE: 1 mark out of 4 awarded in total if the process was incorrect, however, demonstrated understanding of Volume formula.

METHOD 2



2 marks out of 4 awarded if student found point of intersection AND demonstrated correct use of volume formula.

$$V = \pi\int_0^6 (6-y)\,dy - \left[\pi\int_0^2 (6-y)\,dy - \pi\int_0^2 2y\,dy\right]$$

Same marking scheme.

$$= 18\pi - \pi\left[6y - \frac{y^2}{2} - y^2\right]_0^2$$

$$= 18\pi - \pi\left[12 - 2 - 4 - 0\right]$$

$$= 18\pi - 6\pi$$

$$= 12\pi$$

NOTE: This question was very poorly done. Many just found the volume from 6 to zero. Students did not find the point of intersection the split the volume as part was with one curve and part was with another. When students did split, the had the wrong bounds. Please review.

Q14 continued...

c) Step 1 - Base Case - Prove true for $n=1$

$n^3 + 11n$

$n=1 \quad = (1)^3 + 11(1)$

$= 1 + 11$

$= 12$

$= 6(2)$ which is divisible by 6

$\therefore$ True for $n=1$

(1) Correctly proves true for $n=1$

Step 2 - Inductive Hypothesis - Assume true for $n=k$

$k^3 + 11k = 6M$ where $M \in \mathbb{Z}$

Step 3 - Inductive step - Prove true for $n=k+1$

ie Prove $(k+1)^3 + 11(k+1)$ is divisible by 6.

(½) Correct assumption (must have a mention of $M \in \mathbb{Z}$ in both Step 2 and 3 if two pronumerals introduced) AND correct statement of what is being proved.

$(k+1)^3 + 11(k+1)$

$= k^3 + 3k^2 + 3k + 1 + 11k + 11$

$= k^3 + 11k + 3k^2 + 3k + 12$

$= 6M + 12 + 3k(k+1)$ Note $k(k+1) = 2A$ $A, k \in \mathbb{Z}$ (given)

$= 6M + 12 + 3 \times 2A$

$= 6M + 12 + 6A$

$= 6(M + 2 + A)$ $M, A \in \mathbb{Z}$

which is divisible by 6

(½) Use of assumption anywhere in proof.

(1) Correct use of given information and correctly proved result.

$\therefore$ True for $n=k+1$ when true for $n=k$.

$\therefore$ By the principle of mathematical induction, true for all integer $n \geq 1$.

(½ mark deducted for no conclusion or incorrect highlighted statement)

NOTE: This was well attempted. Some setting out issues, must define $6M$ for $M \in \mathbb{Z}$ and you need to state what you are proving. Many successfully proved result.

Q14 continued . . .

d) (i)

$RHS = \binom{n}{r} + \binom{n}{r+1}$

$RHS = {}^{n}C_{r} + {}^{n}C_{r+1}$

$$= \frac{n!}{r!(n-r)!} + \frac{n!}{(r+1)!(n-r-1)!}$$

$$= \frac{n!(r+1)}{(r+1)r!(n-r)!} + \frac{n!(n-r)}{(r+1)!(n-r)(n-r-1)!}$$

$$= \frac{n!(r+1+n-r)}{(r+1)!(n-r)!}$$

$$= \frac{n!(n+1)}{(r+1)!(n-r)!}$$

$$= \frac{(n+1)!}{(r+1)!(n-r)!}$$

$$= \frac{(n+1)!}{(r+1)![n+1-(r+1)]!}$$

$$= {}^{n+1}C_{r+1}$$

$= LHS$

(1/2) Correctly writes a 'combination' in factorial notation (any of them)

(1) Correctly makes a common denominator from unrolling techniques. (Some 1/2 marks if some working was incorrect)

(1/2) Correctly proves result with justification for the highlighted part of the proof or some mention of $LHS = \frac{(n+1)!}{(r+1)!(n-r)!}$

$LHS = \binom{n+1}{r+1}$

$LHS = {}^{n+1}C_{r+1}$

$$= \frac{(n+1)!}{(r+1)![n+1-(r+1)]!}$$

$$= \frac{(n+1)!}{(r+1)!(n-r)!}$$

NOTE: Many errors for ${}^{n+1}C_{r+1} = \frac{(n+1)!}{(r+1)![n+1-(r+1)]!}$ ↑ negative not well dealt with.

Also be clear $\frac{(n+1)!}{(r+1)!(n-r)!}$ → do not go straight there

$$= \frac{(n+1)!}{(r+1)![n+1-(r+1)]!} = {}^{n+1}C_{r+1}$$

Q14 d) continued

(ii)

$$\binom{n+1}{r+1} = \binom{n}{r} + \binom{n}{r+1}$$

$$\binom{2025}{41} + \binom{2025}{42} + \binom{2026}{43} = \binom{p}{q}$$

$$\binom{2025}{41} + \binom{2025}{42} + \binom{2026}{43}$$

$$= \binom{2025+1}{42} + \binom{2026}{43}$$

$$= \binom{2026}{42} + \binom{2026}{43}$$

$$= \binom{2026+1}{43}$$

$$= \binom{2027}{43}$$

$$= \binom{p}{q}$$

$\frac{1}{2}$ $\frac{1}{2}$

$\therefore p = 2027 \quad q = 43$

Q15.

a(i)

Method 1

If $\underset{\sim}{r}_A(t) = \frac{U\sqrt{2}}{2} t\underset{\sim}{i} + \left(\frac{U\sqrt{2}}{2} t - 5t^2\right)\underset{\sim}{j}$

then $\underset{\sim}{v}_A(t) = \frac{U\sqrt{2}}{2}\underset{\sim}{i} + \left(\frac{U\sqrt{2}}{2} - 10t\right)\underset{\sim}{j}$

Projectile A reaches its maximum height when its vertical velocity, $\dot{y}_A = 0$

$\therefore \quad \frac{U\sqrt{2}}{2} - 10t = 0$

$$10t = \frac{U\sqrt{2}}{2}$$

$$t = \frac{U\sqrt{2}}{20} \text{ seconds}$$

1 for fully correct proof, explaining where equation comes from.

Method 2

Find time of flight of Projectile A by finding when $y_A = 0$, then half.

$$\frac{U\sqrt{2}}{2} t - 5t^2 = 0$$

$$t\left(\frac{U\sqrt{2}}{2} - 5t\right) = 0$$

Made with Goodnotes

Q15

a (i) cont'd. $5t = \frac{U\sqrt{2}}{2}$

$t = \frac{U\sqrt{2}}{10}$ — time of whole flight

∴ Projectile reaches max height at

$t = \frac{U\sqrt{2}}{10} \div 2$

$t = \frac{U\sqrt{2}}{20}$ seconds.

Method 3

Find maximum of y_A by using the vertex formula $x = \frac{-b}{2a}$.

$y_A = \frac{U\sqrt{2}}{2} t - 5t^2$

$= -5t^2 + \frac{U\sqrt{2}}{2} t$

$t = \frac{-b}{2a} = -\frac{U\sqrt{2}}{2} \div 2(-5)$

$= \frac{U\sqrt{2}}{20}$ seconds.

Made with Goodnotes

Q15

a)(ii)

Method 1 At point P, the y component of each projectile is equal.

i.e. $y_A = y_B$

$$\therefore \frac{U\sqrt{2}}{2}t - 5t^2 = \frac{V}{2}t - 5t^2 \quad \text{— 1}$$

$$\therefore \frac{U\sqrt{2}}{2}t = \frac{V}{2}t$$

$$\therefore V = U\sqrt{2}$$

Method 2 At point P, we know the time is $t = \frac{U\sqrt{2}}{20}$.

At Point P, the vertical velocity of projectile B is also zero.

i.e. $\dot{y}_B = 0$

$$v_B(t) = -\frac{V\sqrt{3}}{2}\underset{\sim}{i} - \left(\frac{V}{2} - 10t\right)\underset{\sim}{j}$$

$$\therefore \frac{V}{2} - 10t = 0 \quad \text{— } \tfrac{1}{2}$$

Made with Goodnotes

Q15

a) ii) cont'd

$$10t = \frac{V}{2}$$

$$t = \frac{V}{20}$$

$$\therefore \quad t = \frac{V}{20} = \frac{U\sqrt{2}}{20} \quad \text{(from part (i))} \quad \frac{1}{2}$$

$$\therefore \quad V = U\sqrt{2}.$$

Part ii) was fairly well done.
For Method 1, many students made this a lengthy process, by substituting t, rather than just cancelling as shown in solution.

Made with Goodnotes

Q15

a)(iii) To determine d, we can use the fact that the horizontal position of both projectiles is equal at point P.

i.e. $x_A = x_B$ — 1 for using this

$$\therefore \quad \frac{V\sqrt{2}}{2}t = d - \frac{V\sqrt{3}}{2}t$$

$$d = \frac{V\sqrt{2}\,t}{2} + \frac{V\sqrt{3}}{2}t$$

at Point P, $t = \frac{V\sqrt{2}}{20}$ and $v = V\sqrt{2}$

So substituting these:

$$d = \frac{V\sqrt{2}}{2}\left(\frac{V\sqrt{2}}{20}\right) + \frac{V\sqrt{2}\times\sqrt{3}}{2}\left(\frac{V\sqrt{2}}{20}\right)$$

$$d = \frac{2V^2}{40} + \frac{V\sqrt{6}\times V\sqrt{2}}{40}$$

$$d = \frac{2V^2 + V^2\sqrt{12}}{40}$$

$$d = \frac{V^2(2+2\sqrt{3})}{40}$$

$$d = \frac{V^2(1+\sqrt{3})}{20}$$

1 for anything equivalent.

A number of students thought that projectile A would land where projectile B was projected from, and so used tried to use the range of A.

Made with Goodnotes

Q15

(b) If 3 forces are acting on a object, and the object is in equilibrium, then $F_1 + F_2 + F_3 = 0$ ← no movement.

So adding the x-components:

$-a^2 + 3a + k + 4 - 3a - 2k = 0$ (1) – 1 mark.

and y components:

$a^3 - 6a - 10a + k - 24 = 0$ (2)

Unfortunately a number of students thought the forces were equal to each other.

Simplifying the equations and then eliminating k:

(1) $-a^2 - k + 4 = 0$

$k = 4 - a^2$

(2) $a^3 - 16a + k - 24 = 0$

SUB $k = 4 - a^2$

$a^3 - 16a + 4 - a^2 - 24 = 0$

$a^3 - a^2 - 16a - 20 = 0$ – 1 (needs to be a cubic equation)

Unfortunately many students simplified equation (1) incorrectly.

Made with Goodnotes

Q15 b) cont'd

To find a, the cubic equation needs to be factorised an solved.

$a^3 - a^2 - 16a - 20 = 0$

Use factors of -20 to look for zeroes.

$F(-2) = 0$
$F(5) = 0$

] calculator can be used.

$\therefore (a+2)(a-5)$ is a factor — 1

$= a^2 - 3a - 10$

$$\begin{array}{r} a + 2 \\ a^2 - 3a - 10 \overline{)\ a^3 - a^2 - 16a - 20} \\ a^3 - 3a^2 - 10a \\ \hline 2a^2 - 6a - 20 \\ 2a^2 - 6a - 20 \\ \hline 0 \end{array}$$

$\therefore$ Equation factorises to

$(a+2)^2(a-5) = 0$ — 1 for confirming 3rd root is multiple, and then answering.

$\therefore a = -2$ or 5.

A number of students need to revise factoring polynomials.

Made with Goodnotes

Q15 b) cont'd

Alternate method to division

If $P(a) = a^3 - a^2 - 16a - 20$

$P'(a) = 3a^2 - 2a - 16$

$= 3a^2 + 6a - 8a - 16$

$= 3a(a+2) - 8(a+2)$

$= (3a - 8)(a+2)$

$\therefore P'(-2) = 0$

$\therefore (a+2)$ is a root of multiplicity 2.

$\therefore P(a) = (a+2)^2(a-5)$

$\therefore$ Equation is $(a+2)^2(a-5) = 0$

$\therefore a = -2$ or 5.

Made with Goodnotes

Q15

c)(i) You need to identify what the diagonals of the rhombus are in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. The diagonals must be referred to and linked to these vectors.

$\overrightarrow{OC} = \underset{\sim}{a} + \underset{\sim}{b}$ or $|\overrightarrow{OC}| = |\underset{\sim}{a} + \underset{\sim}{b}|$

$\overrightarrow{AB} = \underset{\sim}{b} - \underset{\sim}{a}$ $|\overrightarrow{AB}| = |\underset{\sim}{b} - \underset{\sim}{a}|$

1 mark

Given that the area of the rhombus $4u^2$,

$$\frac{1}{2} \times |\overrightarrow{OC}| \times |\overrightarrow{AB}| = 4 \quad -\frac{1}{2}$$

$$\frac{1}{2} \times |\underset{\sim}{a} + \underset{\sim}{b}| \times |\underset{\sim}{b} - \underset{\sim}{a}| = 4$$

$$|\underset{\sim}{a} + \underset{\sim}{b}| \times |\underset{\sim}{b} - \underset{\sim}{a}| = 8 \quad -\frac{1}{2}$$

Squaring both sides:

$$|\underset{\sim}{a} + \underset{\sim}{b}|^2 |\underset{\sim}{b} - \underset{\sim}{a}|^2 = 64$$

- Many students failed to link $\underset{\sim}{a} + \underset{\sim}{b}$ and $\underset{\sim}{b} - \underset{\sim}{a}$ to the diagonals
- Many students failed to add the magnitude symbols when appropriate. They need to understand when they are describing a vector versus a magnitude.

Made with Goodnotes

Q15

c)(ii)

Some students did not recognise that the word "hence" infers they should use the result above.

$$|\underset{\sim}{a}+\underset{\sim}{b}|^2|\underset{\sim}{b}-\underset{\sim}{a}|^2 = 64$$

Looking at the first part, and since $|\underset{\sim}{u}|^2 = \underset{\sim}{u}\cdot\underset{\sim}{u}$

$$\begin{aligned}|\underset{\sim}{a}+\underset{\sim}{b}|^2 &= (\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}+\underset{\sim}{b}) \\ &= \underset{\sim}{a}\cdot\underset{\sim}{a} + \underset{\sim}{a}\cdot\underset{\sim}{b} + \underset{\sim}{b}\cdot\underset{\sim}{a} + \underset{\sim}{b}\cdot\underset{\sim}{b} \\ &= |\underset{\sim}{a}|^2 + 2\underset{\sim}{a}\cdot\underset{\sim}{b} + |\underset{\sim}{b}|^2 \\ &= 2|\underset{\sim}{a}|^2 + 2\underset{\sim}{a}\cdot\underset{\sim}{b} \quad \text{— 1}\end{aligned}$$

Since $|\underset{\sim}{a}| = |\underset{\sim}{b}|$ (side lengths of rhombus equal)

Similarly: $|\underset{\sim}{b}-\underset{\sim}{a}|^2 = 2|\underset{\sim}{a}|^2 - 2\underset{\sim}{a}\cdot\underset{\sim}{b}$

$\therefore$ $|\underset{\sim}{a}+\underset{\sim}{b}|^2|\underset{\sim}{b}-\underset{\sim}{a}|^2 = 64$

Becomes $(2|\underset{\sim}{a}|^2+2\underset{\sim}{a}\cdot\underset{\sim}{b})(2|\underset{\sim}{a}|^2-2\underset{\sim}{a}\cdot\underset{\sim}{b}) = 64$

$$\left[2|\underset{\sim}{a}|^2\right]^2 - \left[2\underset{\sim}{a}\cdot\underset{\sim}{b}\right]^2 = 64$$

$$4|\underset{\sim}{a}|^4 - 4(\underset{\sim}{a}\cdot\underset{\sim}{b})^2 = 64 \quad \text{— 1}$$

Divide by 4: $|\underset{\sim}{a}|^4 - (\underset{\sim}{a}\cdot\underset{\sim}{b})^2 = 16$

Made with Goodnotes