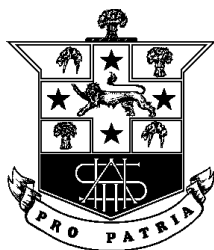


STUDENT NAME: _____

TEACHER: _____



HURLSTONE
AGRICULTURAL
HIGH
SCHOOL

2020

Year 12
TASK ONE

Mathematics Extension 2

Examiners

- Mr G Huxley, Mr G Rawson

Total marks:
33

Section I – 3 marks (MC booklet)

- Attempt Questions 1 – 3
- Allow about $3\frac{7}{11}$ minutes for this section

Section II – 30 marks (2 separate booklets)

- Attempt Questions 4 – 5
- Allow about $36\frac{4}{11}$ minutes for this section

General Instructions

- Reading time – 3 minutes
- Working time – 40 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided inside this cover page
- A multiple choice answer sheet is provided at the back of this cover page
- In Questions 4 – 5 show relevant mathematical reasoning and/or calculations
- Write your name at the top of each booklet. Answer the questions in the space provided. These spaces provide guidance for the expected length of response
- Extra writing space is provided at the back of each question booklet. If you use this space, clearly indicate which part of the question you are answering

Section I

3 marks

Attempt Questions 1–3

Use the multiple choice answer sheet provided on page ?? for Questions 1–3.

1. Let $z = 7 + 10i$. What is the value of $\overline{iz}$?
 - A. $-10 + 7i$
 - B. $-10 - 7i$
 - C. $10 + 7i$
 - D. $10 - 7i$

2. If ω is a complex cubed root of unity, then which of the following is **NOT** correct?
 - A. $\omega^2 = -\omega - 1$
 - B. ω^2 is the other complex root
 - C. $\omega^6 = \omega^2 + \omega$
 - D. $\bar{\omega}$ is also a root

3. The complex number z satisfies $|z - 1| = 1$

What is the greatest distance that z can be from the point I on the Argand diagram?

 - A. 1
 - B. $\sqrt{5}$
 - C. $2\sqrt{2}$
 - D. $\sqrt{2} + 1$

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SECTION II**30 marks****Attempt Questions 4 – 5****Allow about 36.36364 minutes for this section.**

In Questions 4 – 5, your responses should include relevant mathematical reasoning and/or calculations.

- **Write your name at the top of each booklet.** Answer the questions in the space provided. These spaces provide guidance for the expected length of response.
- Extra writing space is provided at the back of each question booklet. If you use this space, clearly indicate which part of the question you are answering.

Question 4 (15 marks)(a) Let $z = 2 - i\sqrt{3}$ and $w = 1 - i\sqrt{3}$ (i) Find $z + \bar{w}$ **1**

(ii) Express w in modulus-argument form **2**

(iii) Write w^9 in its simplest form **2**

Question 4 continues on page 2

Question 4 continued...

(b) (i) Find the square roots of $3 + 4i$

3

(ii) Hence, or otherwise, solve the equation $z^2 + iz - 1 - i = 0$

2

Question 4 continues on page 3

TEACHER: _____

2

[illegible]

Page | 3

Question 4(c) continued...

- (ii) Hence, or otherwise, find all solutions of $\cos \alpha + \cos 2\alpha + \cos 3\alpha = 0$, for $0 \leq \alpha \leq 2\pi$

3

[illegible]

End of Question 4

TEACHER: _____

If you use this space, clearly indicate which part you are answering

[illegible]

[illegible]

STUDENT NAME: _____

TEACHER: _____

- **Write your name at the top of each booklet.** Answer the questions in the space provided. These spaces provide guidance for the expected length of response.
- Extra writing space is provided at the back of each question booklet. If you use this space, clearly indicate which part of the question you are answering.

Question 5 (15 marks)

- (a) Sketch the region in the complex plane where the inequalities

2

$$|z + 2| \geq 2 \quad \text{and} \quad |z - i| \leq 1$$

both hold.

On your diagram, indicate relevant points on the x and y axes only.

Question 5 continues on page 2

Question 5 continued...

(b) Let $z = 1 - \cos 2\theta + i \sin 2\theta$, where $0 < \theta \leq \pi$

(i) Show that $|z| = 2 \sin \theta$

2

(ii) Show that $\arg(z) = \frac{\pi}{2} - \theta$

2

Question 5 continues on page 3

STUDENT NAME: _____

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Question 5 continued...

- (c) (i) The complex numbers 0 , u and v form the vertices of an equilateral triangle in the Argand diagram. 2

Show that $u^2 + v^2 = uv$

- (ii) Give an example of two non-zero complex numbers u and v , so that 0 , u and v form the vertices of an equilateral triangle. 2

You may give your answer in any relevant form.

Question 5 continues on page 4

Question 5 continued...

(d) Given that $\omega = e^{\frac{2\pi i}{7}}$,

(i) Write down the modulus and argument of ω^4 and ω^5 1

(ii) Find the value of 2

$$(\omega + \omega^6)(\omega^2 + \omega^5) + (\omega^2 + \omega^5)(\omega^3 + \omega^4) + (\omega^3 + \omega^4)(\omega + \omega^6)$$

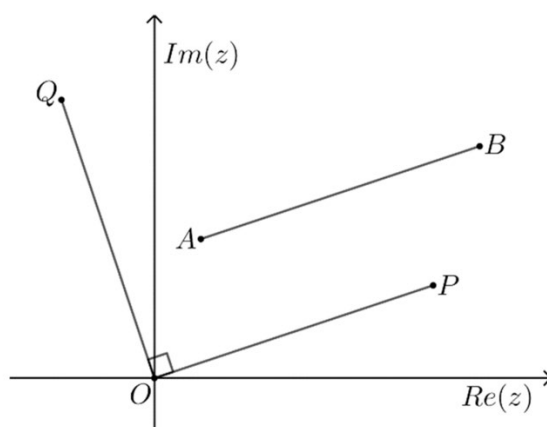
Question 5 continues on page 5

STUDENT NAME: _____

TEACHER: _____

Question 5 continued...

- (e) In the Argand diagram below, intervals AB , OP and OQ are equal in length, OP is parallel to AB and $\angle POQ = \frac{\pi}{2}$. If A and B represent the complex numbers $1 + 3i$ and $7 + 5i$ respectively, find the complex number which is represented by Q .

2**End of Question 5**

Extra writing space

If you use this space, clearly indicate which part you are answering.

[illegible]

TEACHER: _____

[illegible]

Handwriting practice lines consisting of multiple sets of three horizontal lines (top, middle, and bottom lines) for letter formation.

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