BAULKHAM HILLS HS PERSEVERE

2021 **Assessment Task 3**

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 90 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- In Questions 6 – 8, show relevant mathematical reasoning and/or calculations

Total marks: 70

Section I – 5 marks (pages 2 – 3)

- Attempt Questions 1 – 5
- Allow about 15 minutes for this section

Section II – 38 marks (pages 4-6)

- Attempt Questions 6-8
- Allow about 82 minutes for this part

Section I

5 Marks

Attempt Questions 1-5

Allow about 8 minutes for this section

Use the multiple choice answer sheet in your booklet for Questions 1-5.

1. For which of the following values of λ is $\lambda(\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k})$ a unit vector?

   (A) $\frac{1}{2}$

   (B) $\frac{1}{4}$

   (C) $\frac{1}{\sqrt{6}}$

   (D) $\frac{1}{\sqrt{3}}$

2. What is the Cartesian equation of the sphere with centre $\underset{\sim}{c} = -3\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$ that passes through the point $(3, -1, 1)$?

   (A) $(x-3)^2 + (y+1)^2 + (z-2)^2 = 7$

   (B) $(x+3)^2 + (y-1)^2 + (z+2)^2 = 7$

   (C) $(x-3)^2 + (y+1)^2 + (z-2)^2 = 49$

   (D) $(x+3)^2 + (y-1)^2 + (z+2)^2 = 49$

3. If the vectors $\underset{\sim}{u} = -3\underset{\sim}{i} + \underset{\sim}{j} + 2t\underset{\sim}{k}$ and $\underset{\sim}{v} = \begin{pmatrix} 1 \\ t \\ -1 \end{pmatrix}$ are perpendicular, what is the value of t?

   (A) -3

   (B) -2

   (C) $\frac{2}{3}$

   (D) 1

4. A particle is moving along the x −axis and it is initially at $x = 2$ and moving towards the origin. At time t, its velocity is v and its displacement from the origin is x.
If $\frac{dv}{dx} = \frac{-1}{2v}$, which of the following best describes its motion?

(A) Acceleration is decreasing and speed is increasing.

(B) Acceleration is decreasing and speed is decreasing.

(C) Acceleration is constant and speed is increasing.

(D) Acceleration is constant and speed is decreasing.

5. A particle moves in simple harmonic motion between $x = 4$ and $x = -4$. It takes 3 seconds to travel from one extremity to the other. Which of the following is a possible equation for the velocity of the particle?

(A) $v = 4\sin\left(\frac{2\pi t}{3}\right)$

(B) $v = \frac{8\pi}{3}\sin\left(\frac{2\pi t}{3}\right)$

(C) $v = 4\sin\left(\frac{\pi t}{3}\right)$

(D) $v = \frac{4\pi}{3}\sin\left(\frac{\pi t}{3}\right)$

End of Section I

Section II

38 marks
Attempt Questions 6-8
Allow about 82 minutes for this section.

Answer each question on the appropriate page of your answer booklet.
In Questions 6-8, your answers should include relevant mathematical reasoning and/or calculations.

Question 6 (12 marks)

a) Given the points $A = (2,4,1)$, $B = (-4,6,4)$ and the origin O, find:

(i) $\overrightarrow{AB}$ **1**

(ii) The midpoint of AB **1**

(iii) The angle between $\overrightarrow{AB}$ and $\overrightarrow{OB}$, correct to the nearest degree. **3**

b) Sketch the line whose vector equation is $\underset{\sim}{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \end{pmatrix}$, showing its intercepts with the coordinate axes. **2**

c) Find all vectors perpendicular to both $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$. **3**

d) Find the projection of $\underset{\sim}{a} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$ onto $\underset{\sim}{b} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$. **2**

Question 7 (14 marks)

a) A particle moves along the x −axis such that its displacement x metres at time t seconds is given by

$$x = \sqrt{3}\sin 3t - \cos 3t - 3$$

(i) Show that the particle is moving in Simple Harmonic Motion. **2**

(ii) Find the period of the motion. **1**

(iii) Between which two x-values does this particle oscillate? **2**

(iv) Find the first time that the particle reaches its maximum distance from the origin. **2**

b) A ball of mass m kg is thrown vertically upwards with initial speed U m/s. It experiences air resistance of magnitude mkv where k is a positive constant and v m/s is its velocity.

(i) Show that its acceleration is given by $\ddot{x} = -g - kv$, where g is the acceleration due to gravity. **1**

(ii) Show that $t = \frac{1}{k}\log_e\left(\frac{g+kU}{g}\right)$ when the ball reaches its maximum height. **3**

(iii) It can be shown that the maximum height reached by the ball is given by

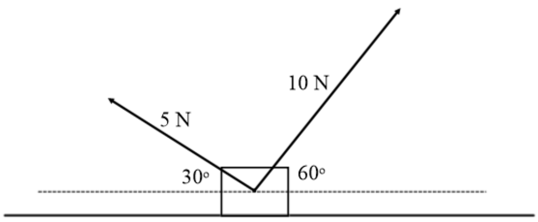
$$h = \frac{U}{k} - \frac{g}{k^2}\log_e\left(\frac{g+kU}{g}\right) \qquad \text{(Do NOT prove this.)}$$

After reaching its maximum height the ball then falls back to the ground reaching a velocity of V m/s just before impact.

Show that $k(U+V) = g\log_e\left(\frac{g+kU}{g-kV}\right)$. **3**

Question 8 (12 marks)

a) A 2 kg mass is initially at rest on a smooth horizontal surface. The mass is acted upon by two constant forces that cause it to move horizontally. The forces applied to the mass are 10 newtons (in a direction 60° upwards from horizontal) and 5 newtons (in a direction 30° upwards from horizontal) as shown in the diagram.



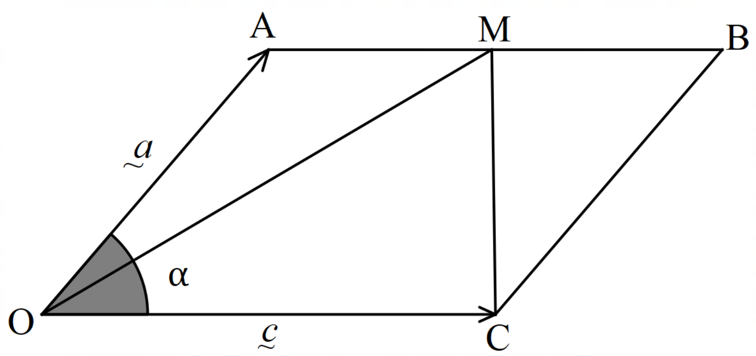
(i) Find the normal reaction force that the surface exerts on the mass, given that the acceleration due to gravity is $g = 10\ \mathrm{m/s^2}$. **2**

(ii) Find the acceleration of the mass after it starts to move. **2**

(iii) Find its velocity after 3 seconds. **1**

b) The diagram below shows a parallelogram $OABC$ where $\angle AOC = \alpha$, $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OC} = \underset{\sim}{c}$, and $\left|\overrightarrow{OC}\right| = 2\left|\overrightarrow{OA}\right|$.

M is a point on AB such that $\overrightarrow{AM} = k\overrightarrow{AB}$ where $0 \le k \le 1$, and $\angle OMC = 90°$.



(i) Use a vector method to show that $|\underset{\sim}{a}|^2(1-2k)\big(2\cos\alpha-(1-2k)\big)=0$. **4**

(ii) Find the values of α such that there are two possible positions for M, satisfying all the given conditions. **3**

End of Examination

<table>
<tr><th>Solution</th><th>Marks</th><th>Comments</th></tr>
<tr><td colspan="3" align="center">SECTION I</td></tr>
<tr><td>1. C -
$$\left|\lambda\left(\underset{\sim}{i}-\underset{\sim}{j}+2\underset{\sim}{k}\right)\right| = 1$$
$$\lambda\sqrt{1^2+1^2+2^2} = 1$$
$$\lambda\sqrt{6} = 1$$
$$\lambda = \frac{1}{\sqrt{6}}$$</td><td>1</td><td>97% of the grade correctly answered this question</td></tr>
<tr><td>2. D – General equation of a sphere is;
$$(x-a)^2+(y-b)^2+(z-c)^2 = r^2$$
where the centre is (a, b, c) and r = radius
Our sphere passes through $(3, -1, 1)$, so;
$$(3+3)^2+(-1-1)^2+(1+2)^2 = r^2$$
$$r^2 = 49$$
$\therefore$ sphere has equation $(x+3)^2+(y-1)^2+(z+2)^2 = 49$</td><td>1</td><td>93% of the grade correctly answered this question</td></tr>
<tr><td>3. A –
$$\underset{\sim}{u}\cdot\underset{\sim}{v} = 0$$
$$\begin{pmatrix}-3\\1\\2t\end{pmatrix}\cdot\begin{pmatrix}1\\t\\-1\end{pmatrix} = 0$$
$$-3+t-2t = 0$$
$$t = -3$$</td><td>1</td><td>100% of the grade correctly answered this question</td></tr>
<tr><td>4. C – when $t = 0$; $x = 2$, $v < 0$ and $\frac{dv}{dx} = -\frac{1}{2v} > 0$
$$\ddot{x} = v\frac{dv}{dx}$$
$$\therefore \ddot{x} = -\frac{1}{2} < 0$$
as velocity and acceleration is in the same direction, speed must be increasing
thus acceleration is constant and speed is increasing</td><td>1</td><td>54% of the grade correctly answered this question</td></tr>
<tr><td>5. D - $-4 \le x \le 4 \Rightarrow$ amplitude is 4 units
$$\frac{T}{2} = 3$$
$$T = 6$$
$$\frac{2\pi}{n} = 6$$
$$n = \frac{\pi}{3}$$
$$\therefore x = 4\cos\frac{\pi t}{3}$$
$$v = \frac{4\pi}{3}\cos\frac{\pi t}{3}$$</td><td>1</td><td>51% of the grade correctly answered this question</td></tr>
</table>

Section II

Question 6.

a) (i) $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \\ 3 \end{pmatrix}$

(1) correct answer.

Avg: 0.99/1

(ii) Midpoint of AB $= \left(\frac{-4+2}{2}, \frac{6+4}{2}, \frac{4+1}{2} \right)$

$= (-1, 5, 2.5)$

(1) correct answer

Avg: 0.77/1
*Quite a few students left their answer in column vector form, which is NOT a point. Midpoint is a point not a vector.
*Some students used vector methods which made it more complicated that necessary.

(iii) $\overrightarrow{AB} \cdot \overrightarrow{OB} = \begin{pmatrix} -6 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 6 \\ 4 \end{pmatrix}$

$= 24 + 12 + 12$

$= 48$

$\overrightarrow{AB} \cdot \overrightarrow{OB} = |\overrightarrow{AB}||\overrightarrow{OB}| \cos\theta$

$= \sqrt{49} \times \sqrt{68} \cos\theta$

$= 7\sqrt{68} \cos\theta$

$7\sqrt{68}\cos\theta = 48$

$\cos\theta = \frac{48}{7\sqrt{68}}$

$\theta \doteqdot 34°$

(3) correct solution
(2) finds two expressions for $\overrightarrow{AB} \cdot \overrightarrow{OB}$
(1) finds one expression for $\overrightarrow{AB} \cdot \overrightarrow{OB}$

Avg: 2.85/3

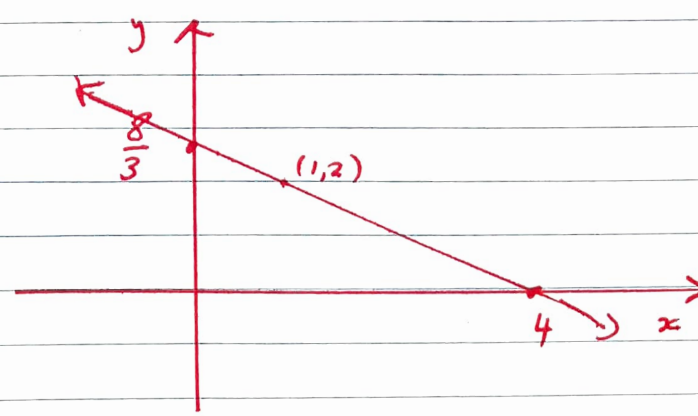
b) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

$\therefore x = 1 + 3\lambda, \quad y = 2 - 2\lambda$

$\lambda = \frac{x-1}{3}$

$y = 2 - \frac{2}{3}(x-1)$

y-int $= 8/3$, x-int $= 4$.



(2) correct graph
(1) a correct intercept.

Avg: 1.96/2

c) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = 0 \quad \therefore 2x - y + z = 0$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} = 0 \quad \therefore 3x + y + 4z = 0$

Solving simultaneously,

$5x + 5z = 0$

$z = -x$.

$2x - y - x = 0$

$x = y$

$\therefore x = y = -z$.

The vectors required are in the form

$\lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ where $\lambda \in \mathbb{R}$ $(\lambda \neq 0)$

(3) correct solution

(2) obtains $x = y = -z$.

(1) obtains simultaneous equations and attempts to solve

Avg: 2.38/3

*Some students found a vector (eg ***i***+***j***-***k***) but not **all** vectors

*Some did not specify lambda is a real number (and non -zero)

d) $\text{proj}_{\underset{\sim}{b}} \underset{\sim}{a} = \left(\dfrac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{\underset{\sim}{b} \cdot \underset{\sim}{b}} \right) \underset{\sim}{b}$

$= \dfrac{-4+2-3}{1+1+1} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$= -\dfrac{5}{3} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

or $\dfrac{5}{3} \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

or any equivalent.

(2) correct solution

(1) significant progress.

Avg:1.9/2

Overall , the average marks show that Q6 was answered well by the majority of students apart from the issues mentioned above

Question 7.

a) (i)

$x = \sqrt{3}\sin 3t - \cos 3t - 3$

$\dot{x} = 3\sqrt{3}\cos 3t + 3\sin 3t$

$\ddot{x} = -9\sqrt{3}\sin 3t + 9\cos 3t$

$= -9(\sqrt{3}\sin 3t - \cos 3t)$

$= -9(x+3)$

$= -n^2(x+3)$ with $n = 3$

$\therefore$ Particle moves in Simple Harmonic Motion

(2) correctly shown
(1) Finds/attempts to find velocity and acceleration

(ii) $T = \dfrac{2\pi}{n} = \dfrac{2\pi}{3}$ seconds.

(1) correct answer

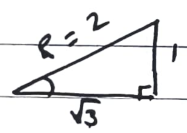
(iii) $x + 3 = \sqrt{3}\sin 3t - \cos 3t$

$= R\sin(3t - \alpha)$

$= R\sin 3t\cos\alpha - R\cos 3t\sin\alpha$

$\cancel{R}\cos\alpha = \dfrac{\sqrt{3}}{R}$

$\cancel{R}\sin\alpha = \dfrac{1}{R}$



$R = 2$

$\alpha = \dfrac{\pi}{6}$

$\therefore x = \underbrace{2\sin\left(3t - \frac{\pi}{6}\right)}_{\substack{\text{max } 2 \\ \text{min } -2}} - 3$

$\therefore -5 \le x \le -1$

$\therefore$ Oscillates between $x = -5$ and $x = 1$.

(2) correct soln
(1) significant progress

(iv) When $x = -5$

$-5 = 2\sin\left(3t - \frac{\pi}{6}\right) - 3$

$-1 = \sin\left(3t - \frac{\pi}{6}\right)$

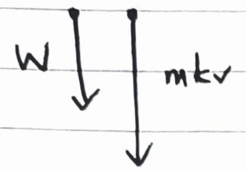
$3t - \dfrac{\pi}{6} = \dfrac{3\pi}{2}$ (first occasion)

$3t = \dfrac{3\pi}{2} + \dfrac{\pi}{6} = \dfrac{5\pi}{3}$

$t = \dfrac{5\pi}{9}$ seconds.

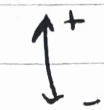
(2) correct solution
(1) identify a relevant condition such as max distance occurs at an 'amplitude'
(1) obtain t=2pi/9

b) (i) Forces :



Initial velocity: $U \uparrow$

Direction :



$$m\ddot{x} = -mg - mkv$$

$$\therefore \ddot{x} = -g - kv.$$

(1) correctly shown

(ii) $$\frac{dv}{dt} = -(g+kv)$$

$$\frac{dt}{dv} = \frac{-1}{g+kv}$$

$$t = \int \frac{-1}{g+kv} \cdot dv$$

$$= -\frac{1}{k} \ln (g+kv) + c$$

$\left.\begin{matrix} t=0 \\ v=U \end{matrix}\right]$ $$0 = -\frac{1}{k} \ln (g+kU) + c$$

$$c = \frac{1}{k} \ln (g+kU)$$

$$\therefore t = -\frac{1}{k} \ln (g+kv) + \frac{1}{k} \ln (g+kU)$$

$$= \frac{1}{k} \ln \frac{g+kU}{g+kv}$$

but at maximum height, $v = 0$

$$\therefore t = \frac{1}{k} \ln \left(\frac{g+kU}{g} \right)$$

(3) correct solution
(2) integrates correctly
(1) attempts to integrate to find t

(iii) $\ddot{x} = g - kv = v \cdot \frac{dv}{dx}$

$$\frac{dv}{dx} = \frac{g - kv}{v}$$

$$\frac{dx}{dv} = \frac{v}{g - kv}$$

$$\int_0^h dx = \int_0^V \frac{v}{g-kv} \cdot dv$$

$$h = \int_0^V \frac{v}{g-kv} \cdot dv$$

$$= -\frac{1}{k} \int_0^V \frac{(g-kv) - g}{g - kv} \cdot dv$$

$$= -\frac{1}{k} \int_0^V 1 - \frac{g}{g-kv} \cdot dv$$

$$= -\frac{1}{k} \left[v + \frac{g}{k} \ln (g - kv) \right]_0^V$$

$$= -\frac{1}{k} \left(V + \frac{g}{k} \ln (g - kV) - (0 + \frac{g}{k} \ln g) \right)$$

$$= -\frac{1}{k} \left(V + \frac{g}{k} \ln \frac{g - kV}{g} \right)$$

but $h = \frac{U}{k} - \frac{g}{k^2} \ln \frac{g + kU}{g}$ ..... given

$$\therefore \frac{U}{k} - \frac{g}{k^2} \ln \frac{g+kU}{g} = -\frac{V}{k} - \frac{g}{k^2} \ln \frac{g - kV}{g}$$

$$\therefore \frac{U}{k} + \frac{V}{k} = \frac{g}{k^2} \left(\ln \frac{g+kU}{g} - \ln \frac{g-kV}{g} \right)$$

Multiplying by k^2:

$$k(U+V) = g \ln\left(\frac{g+kU}{g} \div \frac{g-kV}{g}\right)$$

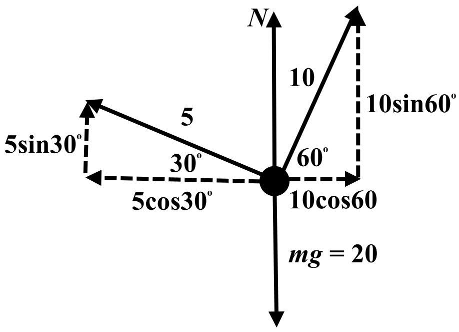
$$\therefore k(U+V) = g \ln \frac{g+kU}{g-kV}$$

(3) correct solution
(2)correct integration of v/(g-kv) with correct bounds
(1) attempt to integrate v/(g-kv)

MARKER'S COMMENTS FOR Q7

(a) (i) Very well done by most students. Few students did not realise how to show Simple Harmonic Motion. Students need to use $\ddot{x}$ and show $\ddot{x} = -n^2(x-c)$, where c is the centre of motion. This question appears frequently in the HSC and students who did not get full marks are encouraged to revise this. Students received 1 mark for finding $\dot{x}$ and $\ddot{x}$.
(ii) Very well done.
(iii) Very well done by most students. Few students did not find the correct α or did not answer what the question was asking.
(iv) Well done by most. Some students did not realise they need to find t for $x = -5$ and not $x = -1$. Distance from the origin means distance from $x = 0$, not the centre of motion. Unfortunately, a few students had the centre of motion to be at $x = 0$ and could not access the second mark. Common mistake of $x = \frac{2\pi}{9}$ received 1 mark.

(b) (i) Very well done by all students. Better solutions had a diagram.
(ii) Well done by most. Most students knew to use dv/dt and successfully integrated for t for 3 marks. Attempting to integrate $\frac{-1}{g+kv}$ gets 1 mark and correct integration with correct limits or C receives the second mark.
(iii) Most difficult question in Question 7. Many students made multiple attempts before arriving at their final submission. Students received 1 mark for attempting to integrate $\frac{v}{g-kv}$, 2 marks for the correct integration with correct limits or C and 3 marks if they were able to successfully arrive at the final result after applying log rules and make $k(U+V)$ the subject. Many students struggled and lost marks in the three various components. Common mistakes include having the limits the wrong way around, fudging the final answer and making two mistakes instead of going back to check for the original error, thus losing two marks instead of one, unable to integrate correctly and careless errors with negatives and when applying log rules. Better solutions included a force diagram and derived the new equation of motion, used definite integral instead of indefinite which proved to be more difficult, and clear steps in each line of working as it is a 'show' question. Always remember, if a marker can't read it, they can't mark it.

Solution	Marks	Comments
QUESTION 8		
8(a) (i)  Resolving the vertical forces vertical forces = 0 $N + 5\sin 30° + 10\sin 60° - 20 = 0$ $N = 20 - 5\left(\frac{1}{2}\right) - 10\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{35}{2} - 5\sqrt{3}$ $= \frac{35 - 10\sqrt{3}}{2}$ Newtons	**2**	**2 marks** • Correct solution **1 mark** • Attempts to resolve vertical forces ***NOTE:*** Several students only considered force due to gravity and the Normal force. Resolving the two oblique forces must have been considered in order to gain the first mark. Those who drew a force diagram tended to be more successful in this question. **Recommendation: ALWAYS draw a force diagram** Leave your answers as exact values, unless asked to do otherwise. Rounding answers that are then used in further parts can create rounding errors.
8(a) (ii) Resolving the horizontal forces horizontal forces = $m\ddot{x}$ $2\ddot{x} = 10\cos 60° - 5\cos 30°$ $= 10\left(\frac{1}{2}\right) - 5\left(\frac{\sqrt{3}}{2}\right)$ $= \frac{10 - 5\sqrt{3}}{2}$ $\ddot{x} = \frac{10 - 5\sqrt{3}}{4}$ m/s^2	**2**	**2 marks** • Correct solution **1 mark** • Attempts to resolve horizontal forces ***NOTE:*** Students did better in part (ii) Interestingly, students who ignored the vertical components of the oblique forces in part (i) were happy to use the horizontal forces in part (ii)
8(a) (iii) $\frac{dv}{dt} = \frac{10 - 5\sqrt{3}}{4}$ $\int_0^v dv = \frac{10 - 5\sqrt{3}}{4}\int_0^3 dt$ $v = \frac{10 - 5\sqrt{3}}{4} \times 3$ $= \frac{30 - 15\sqrt{3}}{4}$ m/s ***OR*** as acceleration is constant $\ddot{x} = \frac{v}{t}$ $v = \ddot{x} \times t$ $= \frac{10 - 5\sqrt{3}}{4} \times 3$ $= \frac{30 - 15\sqrt{3}}{4}\frac{m}{s}$	**1**	**1 mark** • Correct answer ***NOTE:*** DO NOT use formulas from Physics, they are not recognised in this course, if you want to use them then you would have to derive them first. It is also a dangerous habit to get into as in this course we can get questions with variable acceleration and the Physics formulae assume constant acceleration

Solution	Marks	Comments
8(b) (i) OM⊥MC $\overrightarrow{OM}\cdot\overrightarrow{MC}=0$ $(\underset{\sim}{a}+k\underset{\sim}{c})\cdot[(1-k)\underset{\sim}{c}-\underset{\sim}{a}]=0$ $(1-k)\underset{\sim}{a}\cdot\underset{\sim}{c}-\underset{\sim}{a}\cdot\underset{\sim}{a}+k(1-k)\underset{\sim}{c}\cdot\underset{\sim}{c}-k\,\underset{\sim}{a}\cdot\underset{\sim}{c}=0$ $(1-2k)\underset{\sim}{a}\cdot\underset{\sim}{c}-\lvert\underset{\sim}{a}\rvert^2+k(1-k)\lvert\underset{\sim}{c}\rvert^2=0$ $(1-2k)\underset{\sim}{a}\cdot\underset{\sim}{c}-\lvert\underset{\sim}{a}\rvert^2+4k(1-k)\lvert\underset{\sim}{a}\rvert^2=0$ $(1-2k)\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{c}\rvert\cos\alpha-(4k^2-4k+1)\lvert\underset{\sim}{a}\rvert^2=0$ $2(1-2k)\lvert\underset{\sim}{a}\rvert^2\cos\alpha-(2k-1)^2\lvert\underset{\sim}{a}\rvert^2=0$ $\lvert\underset{\sim}{a}\rvert^2(1-2k)[2\cos\alpha-(1-2k)]=0$	4	**4 marks** • Correct solution **3 marks** • Uses $\lvert\underset{\sim}{c}\rvert=2\lvert\underset{\sim}{a}\rvert$ and $\underset{\sim}{a}\cdot\underset{\sim}{c}=\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{c}\rvert\cos\alpha$ in the solving of the below equation **2 marks** • Obtains $(1-2k)\underset{\sim}{a}\cdot\underset{\sim}{c}-\lvert\underset{\sim}{a}\rvert^2+k(1-k)\lvert\underset{\sim}{c}\rvert^2=0$ or equivalent • Uses $\lvert\underset{\sim}{c}\rvert=2\lvert\underset{\sim}{a}\rvert$ and $\underset{\sim}{a}\cdot\underset{\sim}{c}=\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{c}\rvert\cos\alpha$ **1 mark** • Obtains $\overrightarrow{OM}$ and $\overrightarrow{MC}$ in terms of $\underset{\sim}{a}$, $\underset{\sim}{c}$ and k. ***NOTE***: Don't just write down new vectors without defining what they are. Take more care with the formation of your characters, if it can't be read, it can't be marked. Use correct vector notation, there is a difference between a vector and a pronumeral and you need the correct notation to distinguish between them. $\lvert\underset{\sim}{a}\rvert=2\lvert\underset{\sim}{c}\rvert$ does NOT mean $\underset{\sim}{a}=2\underset{\sim}{c}$
8(b) (ii) $1-2k=0$ or $1-2k=2\cos\alpha$ $k=\frac{1}{2}$ but when $k=\frac{1}{2}$ M is midpoint $\therefore$ only one possible position for M $k=\frac{1-2\cos\alpha}{2}$ but $0\le k\le 1$ $\therefore 0\le\frac{1-2\cos a}{2}\le 1$ $0\le 1-2\cos\alpha\le 2$ $-1\le -2\cos\alpha\le 1$ $\frac{1}{2}\ge\cos\alpha\ge-\frac{1}{2}$ $-\frac{1}{2}\le\cos\alpha\le\frac{1}{2}$ $\frac{\pi}{3}\le\alpha\le\frac{2\pi}{3}$ However when $\alpha=\frac{\pi}{2}$, $k=\frac{1}{2}$ and as mentioned above, this will only produce one possible position for M Thus $\frac{\pi}{3}\le\alpha\le\frac{2\pi}{3}$, but $\alpha\ne\frac{\pi}{2}$ (or $\frac{\pi}{3}\le\alpha<\frac{\pi}{2}\cup\frac{\pi}{2}<\alpha\le\frac{2\pi}{3}$)	3	**3 marks** • Correct solution **2 marks** • Obtains $\frac{\pi}{3}\le\alpha\le\frac{2\pi}{3}$ but does not consider $\alpha\ne\frac{\pi}{2}$ or the possibility that $k=\frac{1}{2}$ **1 mark** • Obtains the two possibilities of $1-2k=0$ or $1-2k=2\cos\alpha$ • Explains why $k\ne\frac{1}{2}$ or $\alpha\ne\frac{\pi}{2}$ ***NOTE***: The option of $1-2k=0$ actually means that α could be any real value, it is only other conditions in the question that stops it producing any viable answers. Several students chose to use the discriminant in an attempt to show that there are two possible values for M. However, this generally lead nowhere. Use of the condition $0\le k\le 1$ was missing in a lot of solutions, leading to inaccurate answers. Look at what you are writing down; statements like; • $\frac{2\pi}{3}\le\alpha\le\frac{\pi}{3}$ • or α being larger than π, even though it represents an angle in a parallelogram make no sense.