Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Student Name

2022 **YEAR 11/12**

Mathematics

Extension 2

Task 1 Complex Numbers

December 13

Q	Marks
Section I	
MC 1 - 5	5
Section II	
6	10
7	8
8	9
Total	**32**

General Instructions:

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks:

Section I – 5 marks
- Allow about 10 minutes for this section

Section II - 27 marks
- Allow about 45 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 20% of the course.

Section I **5 Multiple Choice** **Total 5 Marks**

For this part, answer on the multiple-choice answer sheet provided.

1. Let $z = 2 + i$ and $w = 1 - i$. What is the value of $3z + iw$?

   (A) $7 - 4i$

   (B) $5 + 4i$

   (C) $7 + 4i$

   (D) $5 + 4i$

2. The Polynomial $P(z)$ has real coefficients. The complex number α is of the form

   $a + ib$, where a and b are both real, non zero and distinct.

   If $P(a)$, $P'(a)$, $P(b)$, $P'(b)$ and $P(\alpha)$ are all zero,

   what is the minimum degree of $P(z)$?

   (A) 4

   (B) 5

   (C) 6

   (D) 7

3. Which of the following complex numbers is the 6th root of i ?

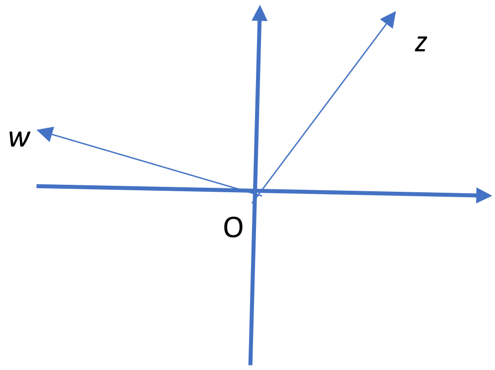
   (A) $-\dfrac{1}{\sqrt{2}} + \dfrac{1}{\sqrt{2}}i$

   (B) $-\dfrac{1}{\sqrt{2}} - \dfrac{1}{\sqrt{2}}i$

   (C) $-\sqrt{2} + \sqrt{2}i$

   (D) $-\sqrt{2} - \sqrt{2}i$

4. The Argand Diagram shows the complex numbers z and w, where z lies in the first quadrant and w lies in the second quadrant.



Which complex number could lie in the 3rd quadrant?

(A) $-w$

(B) $2iz$

(C) $\bar{z}$

(D) $w - z$

5. If w is a complex cube root of unity of the smallest positive argument, which of the following is the value of $\left(1 + \frac{1}{w}\right)^{2020}$?

(A) w

(B) $-w$

(C) 0

(D) 1

Section II **(Total 28 marks)**

There are three questions, use a separate booklet for each question.

Question Six **(Total 10 marks)**

(a) If $z = 1 + i$ and $w = 1 + i\sqrt{3}$

Find in the Cartesian form of $x + iy$

(i) $\bar{z} + w$ (1 mark)

(ii) $\frac{w}{z}$ (1 mark)

(iii) Express z and w in mod arg form. (2 marks)

Hence also express in mod arg form

(iv) zw (1 mark)

(v) w^4 (1 mark)

(b) The locus of a complex number α is given as $|\alpha - 1 - 2i| = 1$

(i) Sketch the locus of α on an Argand Diagram (1 mark)

With reference to the diagram,

(ii) Find the maximum and minium values of $|\alpha|$ (1 mark)

(iii) Show that the minimum value of $\arg(\alpha)$ *is* $\frac{\pi}{2} - 2\tan^{-1}(\frac{1}{2})$ (2 marks)

Question Seven **(Total 8 marks)**

(a) Sketch the region for $\mathrm{Re}(z) \geq |z - \bar{z}|^2$ (2 marks)

(b) Given $P(w) = w^4 - 6w^3 + 15w^2 - 18w + 10$

(i) Show that $1 - i$ is a root of $P(w) = 0$. (1 mark)

(ii) Hence find all the roots of $P(w) = 0$ (2 marks)

(c) Use the fact that $e^{i\theta} = \cos\theta + i\sin\theta$ to show that:

(i) $e^{ni\theta} + e^{-ni\theta} = 2\cos(n\theta)$ and $e^{ni\theta} - e^{-ni\theta} = 2i\sin(n\theta)$ (1 mark)

(ii) Hence express $(e^{i\theta} - e^{-i\theta})^3$ in the form $2i(a\sin 3\theta + b\sin\theta)$

where a and b are integers. (2 marks)

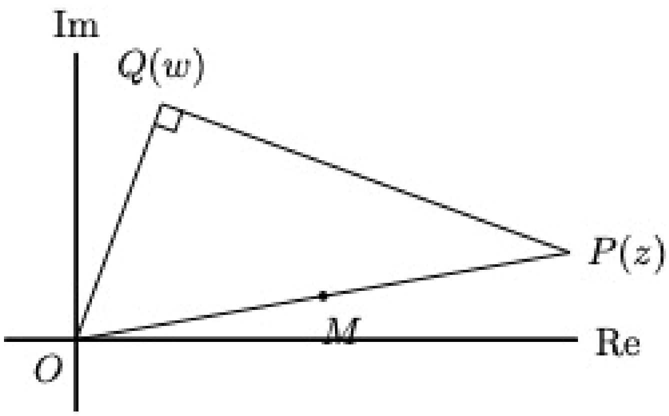
Question Eight **(Total 9 marks)**

(a) Explain why a cubic polynomial with real coefficients must have at least one real root. (2 marks)

(b) The cubic equation $x^3 + bx^2 + cx + d = 0$ has a pure imaginary root.

If the coefficients are real, show that $d = bc$ and $c > 0$ (2 marks)

(c) In the Argand diagram below, ΔOPQ is right angled at Q and $QP = kOQ$ for $k \in R^+$.
M is the midpoint of OP.



(i) If the complex number z is represented by the vector $\overrightarrow{OP}$
and the complex number w is represented by $\overrightarrow{OQ}$

Show that $\overrightarrow{OM} = \frac{1}{2}(1 - ki)w$ (3 marks)

(ii) Express $\overrightarrow{MQ}$ in terms of w, and hence show that

$|\overrightarrow{OM}| = |\overrightarrow{MQ}|$ (2 marks)