SYDNEY BOYS HIGH SCHOOL

Student Number:

Name:

Class:

2022

YEAR 12
TERM 1
ASSESSMENT TASK 2

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 1.5 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- Marks may **NOT** be awarded for messy or badly arranged work
- For questions in Section II, show ALL relevant mathematical reasoning and/or calculations

Total Marks: 47

Section I – 10 marks (pages 2–5)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 37 marks (pages 6–9)

- Attempt all Questions in Section II
- Allow about 1 hour and 15 minutes for this section

Examiner: AMG

Section I

10 marks

Attempt Questions 1–10
Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–10.

---

1 Given that $z = 1 + i$, what is the value of z^8?

A. –16

B. –8

C. 8

D. 16

2 The polynomial $P(z)$ has real coefficients and $z = 1 + 2i$ is a zero of $P(z)$.

Which quadratic polynomial below must be a factor of $P(z)$?

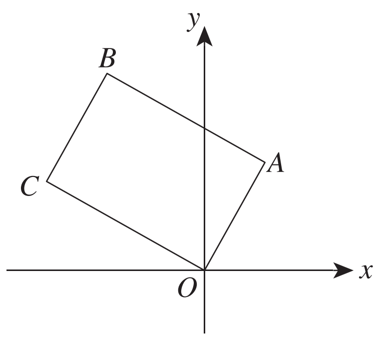
A. $z^2 + 2z + 5$

B. $z^2 - 2z - 3$

C. $z^2 - 2z + 5$

D. $z^2 + 2z - 3$

3 The Argand diagram shows the rectangle $OABC$ where $OC = 2OA$.
Vertex A corresponds to the complex number ω.



Which of the following complex numbers corresponds to vertex B?

A. $(1-2i)\omega$

B. $(1+2i)\omega$

C. $2(1-i)\omega$

D. $2(1+i)\omega$

4 In an Argand diagram, a circle C has diameter AB, where the points A and B represent the numbers $1-5i$ and $3-i$ respectively.

What is the equation of C?

A. $|z-2+3i|=2\sqrt{5}$

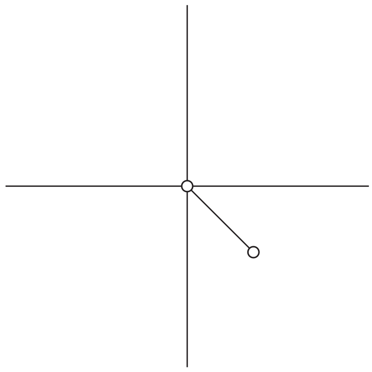
B. $|z-2+3i|=\sqrt{5}$

C. $|z+2-3i|=2\sqrt{5}$

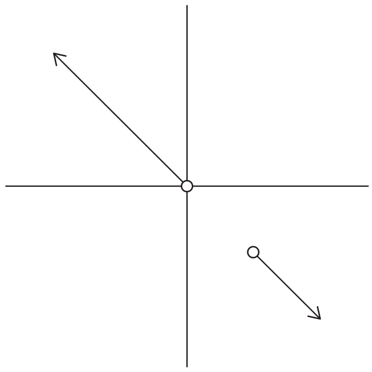
D. $|z+2-3i|=\sqrt{5}$

5 Which diagram best represents the solutions to the equation $\arg\left(\dfrac{z}{z-1+i}\right)=\pi$?

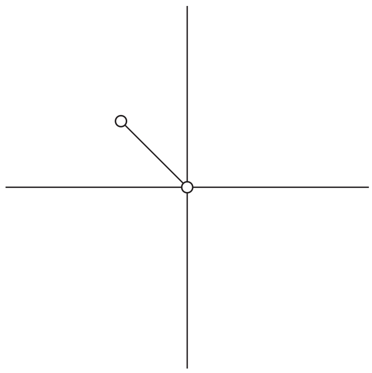
A.



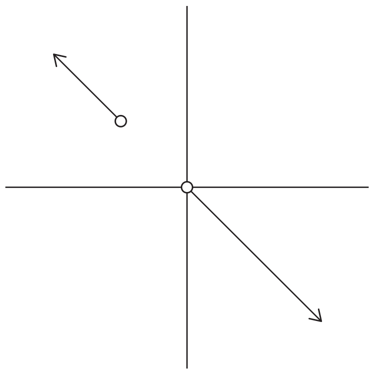
B.



C.

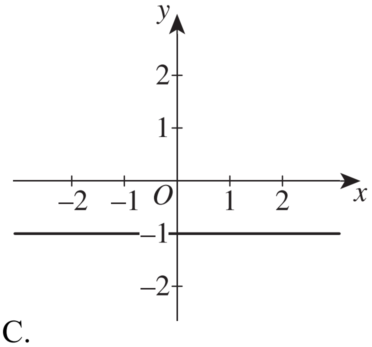


D.

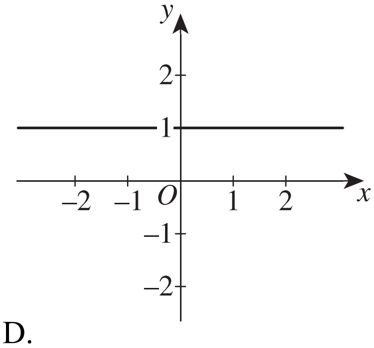


6 Which of the following diagram shows the subset of the complex plane satisfied by the equation $i\bar{z} - iz = 2$?

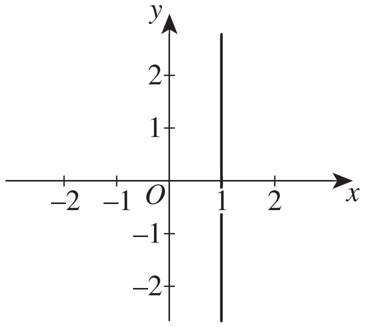
A.



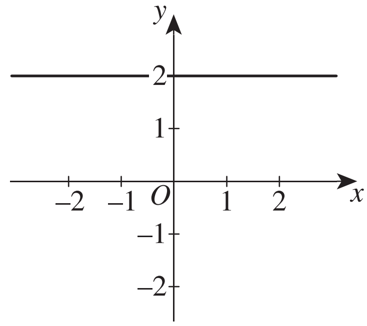
B.



C.

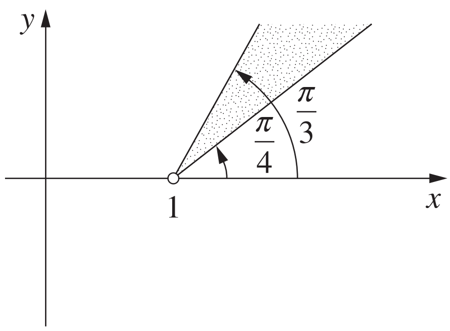


D.

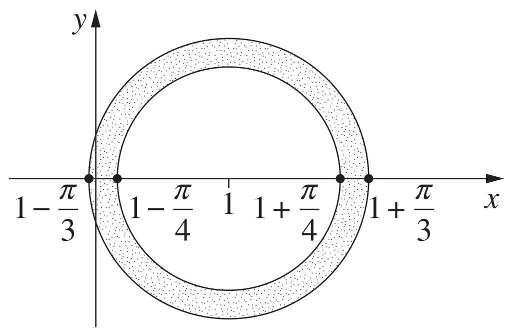


7 Which region on the Argand diagram is defined by $\frac{\pi}{4} \leq \left| \operatorname{Re}(z) - 1 \right| \leq \frac{\pi}{3}$?

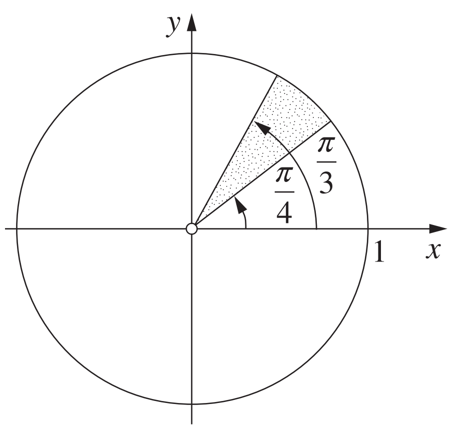
A.



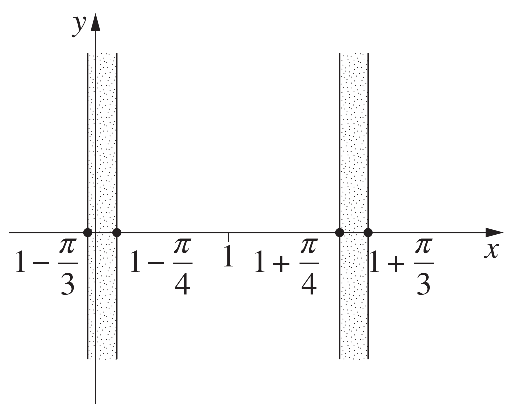
B.



C.



D.



8 How many distinct roots are there of the equation $\left(z^4-1\right)\left(z^2-3iz-2\right)=0$ where $z \in \mathbb{C}$?

A. 3

B. 4

C. 5

D. 6

9 Which of the following curves represents the solution of the equation $\left|\, z-1 \,\right| = \text{Re}\left(z\right)+1$?

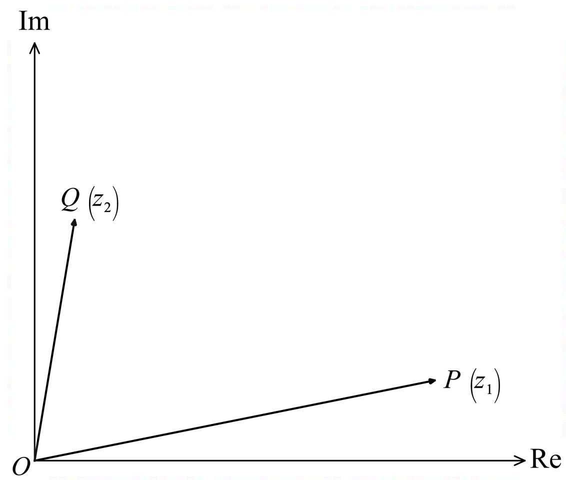
A. A straight line

B. A parabola

C. A circle

D. A semi-circle

10 The points P and Q in the first quadrant represent the complex numbers z_1 and z_2 respectively, as shown in the diagram below.
Which statement about the complex number $z_2 - z_1$ is true?



A. It is represented by $\overrightarrow{QP}$. B

B. Its principal argument lies in the range $\left(\dfrac{\pi}{2}, \pi\right)$.

C. Its real part is positive

D. Its modulus is greater than $\left|\, z_1 + z_2 \,\right|$

Section II

37 marks
Attempt Questions 11-13
Allow 1 hours and 15 minutes for this section

In Questions 11-13, your responses should include ALL relevant mathematical reasoning and/or calculations.

Question 11 (12 marks) Start a NEW Answer Booklet

(a) The complex number z is such that $|z| = 1$ and $\arg z = \frac{\pi}{3}$.

Plot each of the following complex numbers on the same half-page Argand diagram.

(i) z **1**

(ii) $u = z^2$ **1**

(iii) $v = z^2 - \overline{z}$ **1**

(b) Find the equation, in Cartesian form, of the locus of the point z if $|z + 2i| = |z + 4|$. **2**

(c) A point, P, which moves in the complex plane, is represented by the equation

$$|z + 4 + 3i| = 5.$$

(i) Find the value of $\arg z$ when P is in the position that maximises $|z|$. **2**

(ii) Find the modulus of z when $\arg z = \pi - \tan^{-1}\left(\frac{1}{3}\right)$. **2**

(d) Use the principle of mathematical induction to prove that **3**

$$\frac{d^n}{dx^n}\left(\ln(1-x)\right) = -\frac{(n-1)!}{(1-x)^n} \text{ for } n \geq 1.$$

Question 12 (12 marks) Start a NEW Answer Booklet

(a) Write down the roots of $z^6 + (1-8i)z^3 - 8i = 0$, where $z \in \mathbb{C}$. **3**

(b) Sketch and describe the region of the complex plane defined by **2**

$$\arg(z+1)^2 = \frac{\pi}{2}$$

(c) Solve the equation $x^4 + x^2 + 6x + 4 = 0$, for $x \in \mathbb{C}$, given that it has a rational root of multiplicity 2. **2**

(d) (i) Use de Moivre's theorem to prove that $\cos^3\theta = \frac{1}{4}\cos 3\theta + \frac{3}{4}\cos\theta$. **3**

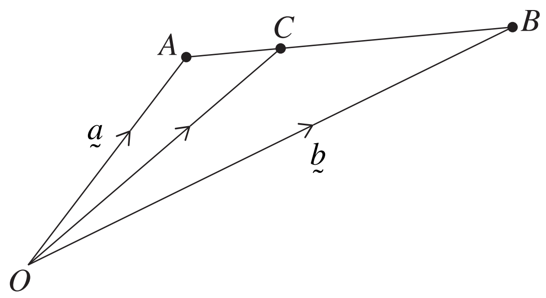
(ii) Hence, find the smallest positive solution of **2**

$$4\cos^3\theta - 3\cos\theta = 1.$$

Question 13 (13 marks) Start a NEW Answer Booklet

(a) The point C divides the interval AB so that $AC : CB = n : m$. **2**

The position vectors of A and B are $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively, as shown in the diagram.



Prove that $\overrightarrow{OC} = \dfrac{m}{m+n}\underset{\sim}{a} + \dfrac{n}{m+n}\underset{\sim}{b}$.

(b) The Bernstein polynomial $B_{k,n}(t)$ of order k and degree n is defined by

$$B_{k,n}(t) = \frac{n!}{(n-k)!k!}t^k(1-t)^{n-k} \text{ for } 0 \le k \le n.$$

(i) Write down the polynomials $B_{0,2}(t)$, $B_{1,2}(t)$, and $B_{2,2}(t)$. **2**

(ii) The three fixed complex numbers α, β, and γ are represented on the Argand diagram by the vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$, and $\overrightarrow{OC}$ respectively. **3**

Three other complex numbers p, q, and r are represented by the vectors $\overrightarrow{OP}$, $\overrightarrow{OQ}$, and $\overrightarrow{OR}$ respectively.

The point P divides the interval AB in the ratio $t : 1-t$.

The point Q divides the interval BC in the ratio $t : 1-t$.

Likewise the point R divides the interval PQ in the ratio $t : 1-t$.

Using part (a), or otherwise, show that

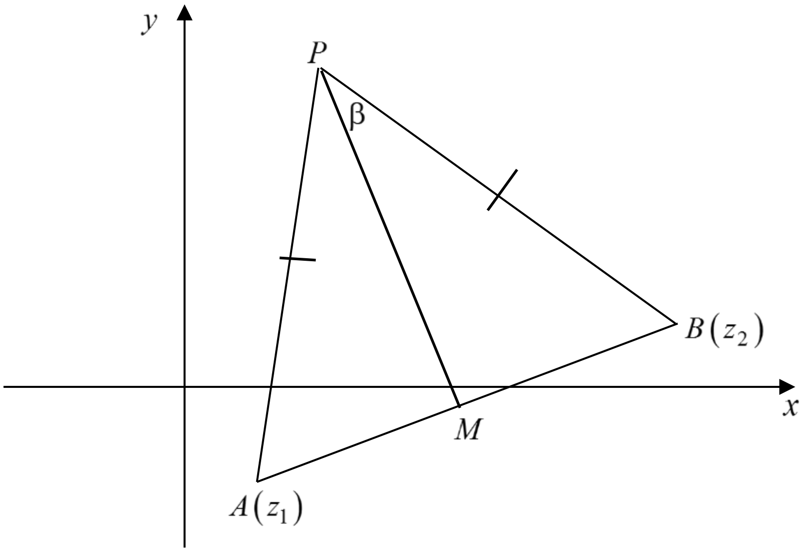
$$r = \alpha B_{0,2}(t) + \beta B_{1,2}(t) + \gamma B_{2,2}(t).$$

(iii) Given that $\alpha = 1+i$, $\beta = 2+3i$, and $\gamma = 3+i$, find the Cartesian equation of the path R moves on as t varies. **2**

Question 13 continues on page 9

Question 13 (continued)

(c) In the following Argand diagram, ABP is an isosceles triangle with $AP = BP$.
A and B represent complex numbers z_1 and z_2 respectively, and M is the midpoint of AB.



(i) Write in terms of z_1 and z_2 an expression for the complex number represented by AM. **1**

(ii) You are given that $\angle BPM = \beta$. **2**
Show that $\overrightarrow{MP}$ represents the complex number $\frac{1}{2}i\left(z_2 - z_1\right)\cot\beta$.

(iii) Hence find an expression for the complex number represented by the point P. **1**
Write your answer in the form $az_1 + bz_2$, where a and b are complex numbers that depend only on β.

End of paper

BLANK PAGE