



Name: _____

Student Number: _____

2023

**YEAR 12 MATHEMATICS EXTENSION 2
TASK 1**

Mathematics

**General
Instructions**

- Reading time – 10 minutes
- Working time – 90 minutes
- Write using black or blue pen
- Diagrams are not to scale
- NESA Approved calculators may be used
- For questions in Section II, show relevant mathematical reasoning and/or calculations

**Total marks:
50**

Section I – 5 marks (page 1–2)

- Attempt Questions 1–5
- Allow about 10 minutes for this section

Section II – 45 marks (pages 3–6)

- Attempt Questions 6–8
- Allow about 1 hour and 20 minutes for this section

Section I

5 marks

Attempt Questions 1–5

Each question in this section is worth 1 mark

Allow about 10 minutes for this section

Circle the correct answer.

Question 1

Given $\underline{a} = 3\underline{i} - 5\underline{j} + 8\underline{k}$. Which of the following represents $\hat{\underline{a}}$?

- A. $\frac{3}{98}\underline{i} - \frac{5}{98}\underline{j} + \frac{4}{49}\underline{k}$
- B. $\frac{1}{16}\underline{i} - \frac{5}{48}\underline{j} + \frac{1}{46}\underline{k}$
- C. $\frac{\sqrt{3}}{4}\underline{i} - \frac{5\sqrt{3}}{12}\underline{j} + \frac{2\sqrt{3}}{3}\underline{k}$
- D. $\frac{3\sqrt{2}}{14}\underline{i} - \frac{5\sqrt{2}}{14}\underline{j} + \frac{4\sqrt{2}}{7}\underline{k}$

Question 2

What is the vector equation which represent a line with a Cartesian equation $y = 2x - 3$?

- A. $\underline{r}(\lambda) = (\underline{i} - \underline{j}) + \lambda(2\underline{i} + \underline{j})$
- B. $\underline{r}(\lambda) = (\underline{i} - \underline{j}) + \lambda(2\underline{i} + 4\underline{j})$
- C. $\underline{r}(\lambda) = (\underline{i} + 2\underline{j}) - 3\lambda\underline{j}$
- D. $\underline{r}(\lambda) = (\underline{i} + \underline{j}) + \lambda(\underline{i} + 2\underline{j})$

Question 3

Consider two vectors $\underline{u} = -2\underline{i} - \underline{j} + 3\underline{k}$ and $\underline{v} = p\underline{i} + \underline{j} + 2\underline{k}$ where $p \in \mathbb{R}$.

Given $\underline{u}$ is perpendicular to $\underline{v}$, what is the value of p ?

- A. $p = -2$
- B. $p = 3.5$
- C. $p = 3$
- D. $p = 2.5$

Question 4

Consider two lines defined by the vector equations

$$L_1 : \underline{r}(\lambda_1) = \underline{a} + \lambda_1 \underline{b}$$

$$L_2 : \underline{r}(\lambda_2) = \underline{c} + \lambda_2 \underline{d}$$

If L_1 and L_2 are parallel, which statement **must be true**?

- A. $\underline{b} = \underline{d}$
- B. $\underline{b} = k\underline{d}$ where $k \in \mathbb{R}$
- C. $\underline{a} = \underline{c}$
- D. $\underline{a} = k\underline{c}$ where $k \in \mathbb{R}$

Question 5

Consider two points in three-dimensional space, $A(2, -1, 3)$ and $B(4, 1, -1)$, such that AB is a diameter of a sphere. What is the Cartesian equation of this sphere?

- A. $(x - 3)^2 + y^2 + (z - 1)^2 = 6$
- B. $(x - 1)^2 + (y + 2)^2 + (z - 2)^2 = 6$
- C. $(x - 3)^2 + y^2 + (z - 1)^2 = 24$
- D. $(x - 1)^2 + (y + 2)^2 + (z - 2)^2 = 24$

Section II

45 marks

Attempt Questions 6 –8

Allow about 1hrs and 20 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (15 marks)	Marks
(a) Given $\underline{a} = -3\underline{i} + 2\underline{j} + \underline{k}$ and $\underline{b} = -2\underline{i} - \underline{j} + 3\underline{k}$, find:	
(i) $\underline{a} - 2\underline{b}$	1
(ii) $\underline{a} \cdot \underline{b}$	1
(iii) the vector projection of $\underline{a}$ onto $\underline{b}$	2
(iv) the vector projection of $\underline{a}$ perpendicular to $\underline{b}$	1
(v) The angle between $\underline{a}$ and $\underline{b}$	2
(b) A triangle is formed in three-dimensional space with the vertices $A(-1, 2, 3)$, $B = (-3, 7, -1)$ and $C = (1, 7, 7)$	
(i) Show that $\triangle ABC$ is isosceles.	2
(ii) Hence, or otherwise, calculate the <u>exact</u> area of $\triangle ABC$	3
(c) Consider the points $A(-1, 2, -3)$, $B(4, m, -6)$ and $C(-7, -8, n)$. Find the values of m and n such that the points are collinear.	3

End of Question 6

Question 7 (15 marks)**Marks**(a) Consider the points $A(-2, 3, 5)$ and $B(3, 5, -4)$ (i) Find the vector equation of the line, L_1 passing through the points A and B **2**(ii) Hence, find the Cartesian equation of the plane which represents the set of points, S , such that $|\overrightarrow{AS}| = |\overrightarrow{BS}|$. **4**

(b) Consider the lines

$$L_1 : x = -3 + \lambda, \quad y = 2 + 2\lambda, \quad z = 5 + 3\lambda$$

$$L_2 : x = -5 + \mu, \quad y = -2 - 5\mu, \quad z = -1 + 2\mu$$

(i) Find the point of intersection between the two lines **2**(ii) Find a vector equation of a line, L_3 , such that L_1, L_2 and L_3 are concurrent and L_3 is perpendicular to L_1 and L_2 . **3**(c) Consider the surface of sphere defined by the position vector $\underline{r}$ such that

$$|\underline{r} - (\underline{i} + 2\underline{j} - 3\underline{k})| = 3$$

(i) Find the cartesian equation of the sphere. **1**(ii) Consider the line L_1 defined by the position vector $\underline{s}(\lambda)$ such that **3**

$$\underline{s}(\lambda) = 2\underline{i} + (4 - 3\lambda)\underline{j} + (\lambda - 5)\underline{k}$$

Find the coordinates of the point of intersections between the sphere and the line

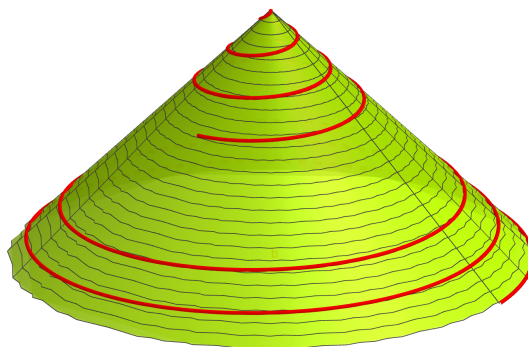
End of Question 7

Question 8 (15 marks)**Marks**

- (a) The Kambala Maths department is creating a Mathematical Christmas Tree. The tree itself is modelled on the equation

$$x^2 + y^2 = z^2 - 10\pi z + 25\pi^2$$

where $0 \leq z \leq 5\pi$ and represented by the green surface below.



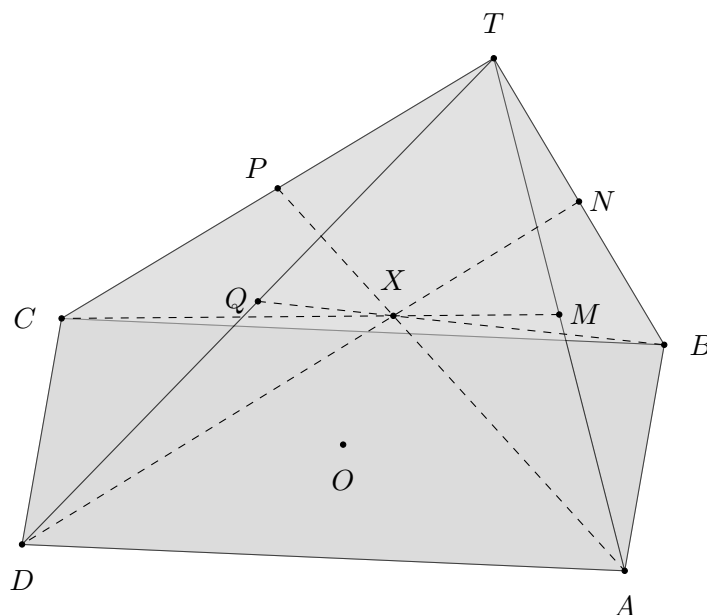
- (i) Ms Archey needs the tree to be stored in a rectangular box. What are the minimum dimensions of the box to fit the tree? 2
- (ii) Mr Greasley hangs red tinsel that spirals around the tree from the bottom of the tree to the peak (**Note: Not all the tinsel is shown in the image above**). The tinsel travels along a path defined by the vector equation. 2

$$\underline{r}(t) = f(t) \cos(\pi t) \underline{i} + f(t) \sin(\pi t) \underline{j} + t \underline{k}$$

Find a definition of $f(t)$ and show that all the tinsel along this path will lie on the surface of the tree.

- (iii) Mr Siu wants to place a star at the top of the Christmas tree and decorate the tinsel with baubles. He starts by placing a bauble at the bottom of the tinsel, he then places a bauble **on the tinsel** every quarter rotation around the tree until he reaches the star. How many baubles does he need to buy? 3

- (b) Consider a **rectangular** based pyramid $ABCDT$ shown below. M , N , P and Q are the midpoints of AT , BT , CT and DT respectively. O is the center of the base of the pyramid (Note: T does not have to be directly above O as it is not a right pyramid).



Let $\overrightarrow{AT} = \underline{r}$, $\overrightarrow{AB} = \underline{p}$ and $\overrightarrow{AD} = \underline{q}$

- (i) Find an expression for $\overrightarrow{AP}$ 1
- (ii) Assume that the lines AP and BQ intersect at a point X . Show that $\overrightarrow{OX} = \frac{1}{3}\underline{r} - \frac{1}{6}\underline{p} - \frac{1}{6}\underline{q}$ 3
- (iii) Hence, show that the line segments AP , BQ , DN are concurrent 2
- (iv) Calculate the value of $\frac{|\overrightarrow{OX}|}{|\overrightarrow{OT}|}$ 2

End of Question 8

End of Task