



2024 Assessment Task 2

Mathematics Extension 2

**General
Instructions**

- Reading time - 5 minutes
- Working time - 70 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show mathematical reasoning and/or calculations

**Total Marks:
49**

Section I – 3 marks (Pages 1–2)

- Attempt Questions 1–3
- Allow about 5 minutes for this section

Section II – 46 marks (Pages 3–6)

- Attempt Questions 4–7
- Allow about 65 minutes for this section

Question:	1	2	3	4	5	6	7	Total
Marks:	1	1	1	12	11	13	10	49
Awarded:								

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Section I

3 marks

Attempt Questions 1–3

Allow about 5 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–3.

1. Consider the following statement:

1

“If a student completes their homework, then they will do better in their examination.”

Which of the following is the converse of this statement?

- A. If a student completes their homework, then they will not do better in their examination.
- B. If a student does not complete their homework, then they will not do better in their examination.
- C. If a student does not do better in their examination, then they did not do their homework.
- D. If a student does do better in their examination, then they did do their homework.

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2. A mathematics student, reading an old textbook, found the following scrawled in the margin:

1

$$\forall x \in (\mathbb{R} - \mathbb{Q}) \nexists p, q \in \mathbb{Q} : |p - x| = |q - x|$$

What does it mean?

- A. All rational numbers are different distances from two irrational numbers.
- B. For all irrational numbers, there can be found only 2 numbers from which it is the same distance.
- C. For all irrational numbers, you cannot find another pair of numbers equidistant.
- D. There are no two rational numbers spaced the same distance from any chosen irrational number.

3. Using a suitable substitution, $\int_0^{-5} \frac{1}{2 + \sqrt{1 - x}} dx$ is equivalent to

1

- A. $\int_1^6 \frac{1}{2 + \sqrt{u}} du$
- B. $\int_{\sqrt{6}}^1 \frac{2u}{u + 2} du$
- C. $\int_1^6 \frac{2u}{u + 2} du$
- D. $\int_1^{\sqrt{6}} \frac{2u}{u + 2} du$

Section II

46 marks

Attempt Questions 4–7

Allow about 65 minutes for this section.

Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

Question 4 (12 Marks)

Use a SEPARATE writing booklet.

- (a) Prove that if $x \in \mathbb{Z}$ and $7x + 9$ is even, then x is odd. 2
- (b) Is “All A’s are not B’s” logically equivalent to “All B’s are not A’s”? Justify your answer. 2
- (c) Consider $n + 2$ distinct points on the circumference of a circle. If consecutive points are joined by line segments creating a polygon with $n + 2$ sides, prove by induction that the angle sum of the polygon is $180n^\circ$. 2
- (d) Evaluate $\int_0^{\frac{\pi}{6}} x \cos x dx$ 2
- (e) (i) Show that $\int_0^{na} f(x) dx = n \int_0^a f(nx) dx$ 1
- (ii) Give a geometric interpretation of your result in part (i). 1
- (f) By using the substitution $u^2 = 1 - x^2$ show that 2

$$\int_0^1 5x (1 - x^2)^{\frac{3}{2}} dx = 1$$

End of Question 4

Question 5 (11 Marks)

Use a SEPARATE writing booklet.

- (a) Find $\int \frac{3}{(x^2 + 1)(x^2 + 4)} dx$. **2**
- (b) Prove that $\sqrt{2m}$ is irrational, whenever m is odd. **2**
- (c) Prove that the product of any four consecutive positive integers is a multiple of 24. **2**
- (d) Find $\int \frac{4t + 1}{t^2 + 6t + 10} dt$. **2**
- (e) Integrate the following using the substitution $x = e^u$ **3**

$$\int \frac{1}{\ln(x)} + \ln(\ln(x)) dx$$

End of Question 5

Question 6 (13 Marks)

Use a SEPARATE writing booklet.

- (a) Evaluate $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^3}{(\sin x)^3 + (\cos x)^3} dx$ given that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$. **2**
- (b) Find $\int \frac{1}{5 \sin x - 12 \cos x + 13} dx$ **3**
- (c) Prove $\frac{x^2}{y^2} + \frac{y^2}{z^2} + \frac{z^2}{x^2} \geq \frac{1}{yz} \sqrt{x^2 y^2 + z^4} + \frac{1}{xz} \sqrt{y^2 z^2 + x^4}$ for $x, y, z > 0$. **3**
- (d) Prove that the inequality $n^2 < 3^n$ holds for every natural number. **2**
- (e) (i) For real numbers a, b , prove that $||a| - |b|| \leq |a - b|$. **2**
- (ii) Hence, prove $|a - b| \geq ||a - c| - |b - c||$. **1**

End of Question 6

Question 7 (10 Marks)

Use a SEPARATE writing booklet.

- (a) (i) Show the recursive formula for $\int \cos^n x dx$ is given by **2**

$$I_n = \frac{1}{n} \sin x \cos^{n-1} x + \frac{n-1}{n} I_{n-2} \text{ and } I_0 = \int dx = x$$

- (ii) Hence integrate $\int_0^\pi \cos^8 x dx$. **2**

- (b) Evaluate $\int_1^{\log_2 6} \frac{2^{2x+1} - 9 \times 2^x - 6}{4^x - 2^x - 6} dx$. **3**

- (c) Show that for any positive integers m and n , 2 is between $\frac{m}{n}$ and $\frac{m+2n}{m}$, inclusive. **3**

END OF PAPER