Killara High School

Student Number: ____________________

2024 **Task 3 Validation Assessment**

Mathematics Extension 2

General Instructions

- Reading time - 5 minutes
- Working time - 45 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show mathematical reasoning and/or calculations

Total Marks: 27

Section I – 2 marks (Pages 1–1)

- **Attempt Questions 1–2**
- **Allow about 5 minutes for this section**

Section II – 25 marks (Pages 2–3)

- **Attempt Questions 3–4**
- **Allow about 40 minutes for this section**

Question:	1	2	3
Marks:	1	1	13
Awarded:			
Question:	4		Total
Marks:	12		27
Awarded:			

Do not write on this page

Section I

2 marks

Attempt Questions 1–2

Allow about 5 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–2.

---

1. The equations of intersecting lines L and M are given below with respect to a fixed origin O. **1**

$$L : \mathbf{r_1} = 11\mathbf{i}+2\mathbf{j}+17\mathbf{k}+\lambda\left(-2\mathbf{i}+q\mathbf{j}-4\mathbf{k}\right) \text{ and } M : \mathbf{r_2} = -5\mathbf{i}+11\mathbf{j}+\mathbf{k}+\mu\left(p\mathbf{i}+2\mathbf{j}+2\mathbf{k}\right)$$

where λ and μ are parameters and p is a constant.

If L and M are perpendicular, what is the value of p?

A. $-2p+2q=4$

B. $-2p+2q=-4$

C. $-2p+2q=8$

D. $-2p+2q=-8$

2. If $|\mathbf{a}-\mathbf{b}|=|\mathbf{a}|$ and $\mathbf{a}\cdot\mathbf{b}=|\mathbf{a}|^{\mathbf{2}}$ which one of the following is necessarily true? **1**

A. $|\mathbf{a}|=\mathbf{1}$

B. $\mathbf{b}=\mathbf{0}$

C. $2|\mathbf{a}|^{\mathbf{2}}=|\mathbf{b}|^{\mathbf{2}}$

D. $|\mathbf{a}|=|\mathbf{b}|$ or $\hat{\mathbf{a}}\cdot\hat{\mathbf{b}}=\dfrac{1}{\sqrt{2}}$

Section II

25 marks
Attempt Questions 3–4
Allow about 40 minutes for this section.
Answer each question in the appropriate writing booklet.
All necessary working should be shown in every question.

Question 3 (13 Marks) Use a SEPARATE writing booklet.

(a) Let $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ be vectors such that each vector has magnitude 1 and each vector is perpendicular to the other two. Show that if $\mathbf{r} = a\mathbf{u} + b\mathbf{v} + c\mathbf{w}$, then we must have $a = \mathbf{r} \cdot \mathbf{u}, b = \mathbf{r} \cdot \mathbf{v}$ and $c = \mathbf{r} \cdot \mathbf{w}$. **2**

(b) Consider the three vectors $\underset{\sim}{u} = \underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k}$, $\underset{\sim}{v} = a\underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{w} = -3\underset{\sim}{i} - \underset{\sim}{j} + a\underset{\sim}{k}$ where $a \in \mathbb{R}$.

(i) Find the cosine of the angle between $\underset{\sim}{u}$ and $\underset{\sim}{v}$ in terms of a. **1**

(ii) Hence, find the value of a such that $\underset{\sim}{u}$ is perpendicular to $\underset{\sim}{v}$. **1**

(c) Consider the points $A(2, -2, 0)$, $B(1, 5, 2)$ and $P(3, 3, 1)$. Find the projection of $\overrightarrow{AP}$ onto $\overrightarrow{AB}$ in simplest form. **2**

(d) Find a parametric representation for the line through $(1, -1, 3)$ and $(2, 3, 1)$. **2**

(e) Consider the point $P(1, 4, 4)$ and the line $\mathbf{r} = (1, 0, 2) + \lambda(2, 2, 0)$.

(i) Find the shortest distance between the point P and the line $\mathbf{r}$. **2**

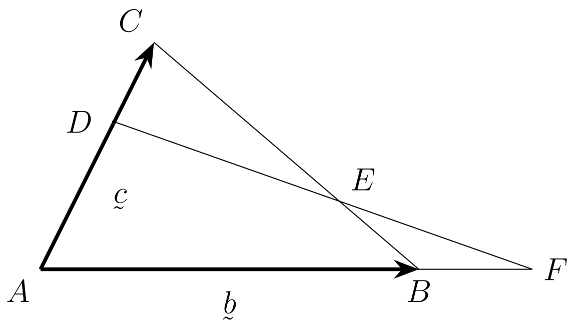
(ii) Find the point P' by reflecting P across the line $\mathbf{r}$. **2**

(iii) Find two values of λ such that the two resulting points on line $\mathbf{r}$, P and P' form a square. **1**

End of Question 3

Question 4 (12 Marks) Use a SEPARATE writing booklet.

(a) In $\triangle ABC$ below, D is the point on AC such that $AD : DC = 2 : 1$. E is the point on BC such that $BE : EC = 1 : 2$.



When DE is extended, it meets the extension of AB at F. Let $\overrightarrow{AB} = \underset{\sim}{b}$ and $\overrightarrow{AC} = \underset{\sim}{c}$.

(i) Show that $\overrightarrow{DE} = \frac{2}{3}\underset{\sim}{b} - \frac{1}{3}\underset{\sim}{c}$. **2**

(ii) Show that $AF : BF = 4 : 1$. **3**

Hint: You may assume that $\overrightarrow{DF}$ is a scalar multiple of $\overrightarrow{DE}$, and $\overrightarrow{AF}$ is a scalar multiple of $\overrightarrow{AB}$.

(b) Given $A(3, -2, 1), B(1, -3, 5)$ and $C(2, 1, z)$, find all values of z so that $\triangle ABC$ is right angled. **2**

(c) If $\underset{\sim}{a} = 7\underset{\sim}{i} + \lambda\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = -6\underset{\sim}{i} - (2-\lambda)\underset{\sim}{j} - \underset{\sim}{k}$ where $\lambda \in \mathbb{R}^-$ and vector $\underset{\sim}{a} + \underset{\sim}{b}$ is be perpendicular to vector $\underset{\sim}{b}$, then find λ. **2**

(d) (i) Find the cartesian equation of a sphere with a diameter formed by a segment of the line described by the vector equation $\underset{\sim}{r} = \underset{\sim}{0} + \lambda(1, 1, 1)$ and with positive x, y and z axes as tangents to the surface of the sphere. **2**

(ii) Find the range of λ such that, for a sphere with centre (c, c, c), λ describes the line segment that is a diameter of the sphere. **1**

END OF PAPER