CRANBROOK SCHOOL

Student Number: _____

2025 **YEAR 12 TASK 1**

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 90 Minutes
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 50

Section I – 5 marks

- Attempt Questions 1–5
- Allow about 15 minutes for this section

Section II – 45 marks

- Attempt Questions 6–8
- Allow about 75 minutes for this section

Section I

5 marks
Attempt Questions 1–5

Use the multiple-choice answer sheet for Questions 1–5.

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1 Which of the following is a unit vector parallel to $\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$?

A. $\begin{bmatrix} 3\sqrt{11} \\ -\sqrt{11} \\ \sqrt{11} \end{bmatrix}$

B. $\begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}$

C. $\frac{1}{11}\begin{bmatrix} 3\sqrt{11} \\ -\sqrt{11} \\ \sqrt{11} \end{bmatrix}$

D. $\frac{1}{3\sqrt{2}}\begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}$

2 It is known that two vectors, $\underset{\sim}{a}$ and $\underset{\sim}{b}$, are not parallel and the vector projection of $\underset{\sim}{a}$ onto $\underset{\sim}{b}$ has a greater magnitude than $\underset{\sim}{b}$ but is in the opposite direction of $\underset{\sim}{b}$.

Given $\underset{\sim}{b} = 3\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$, which of the following is a possible value for $\underset{\sim}{a}$?

A. $\underset{\sim}{i} + 2\underset{\sim}{j} - 2\underset{\sim}{k}$

B. $\underset{\sim}{i} - 2\underset{\sim}{j} - 2\underset{\sim}{k}$

C. $\underset{\sim}{i} + 3\underset{\sim}{j} - 3\underset{\sim}{k}$

D. $\underset{\sim}{i} - 3\underset{\sim}{j} - 3\underset{\sim}{k}$

3 Which of the following vectors is parallel to the line $\frac{3-x}{4} = \frac{2+y}{3} = z-1$?

A. $\begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$

C. $\begin{bmatrix} -8 \\ 6 \\ 2 \end{bmatrix}$

B. $\begin{bmatrix} 6 \\ -4 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} 8 \\ 6 \\ 2 \end{bmatrix}$

4 The rectangle *ABCD* is placed in a three dimensional space where $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OC} = \underset{\sim}{c}$ and $\overrightarrow{AB} = \underset{\sim}{b}$.

Which of the following must ALWAYS be true.

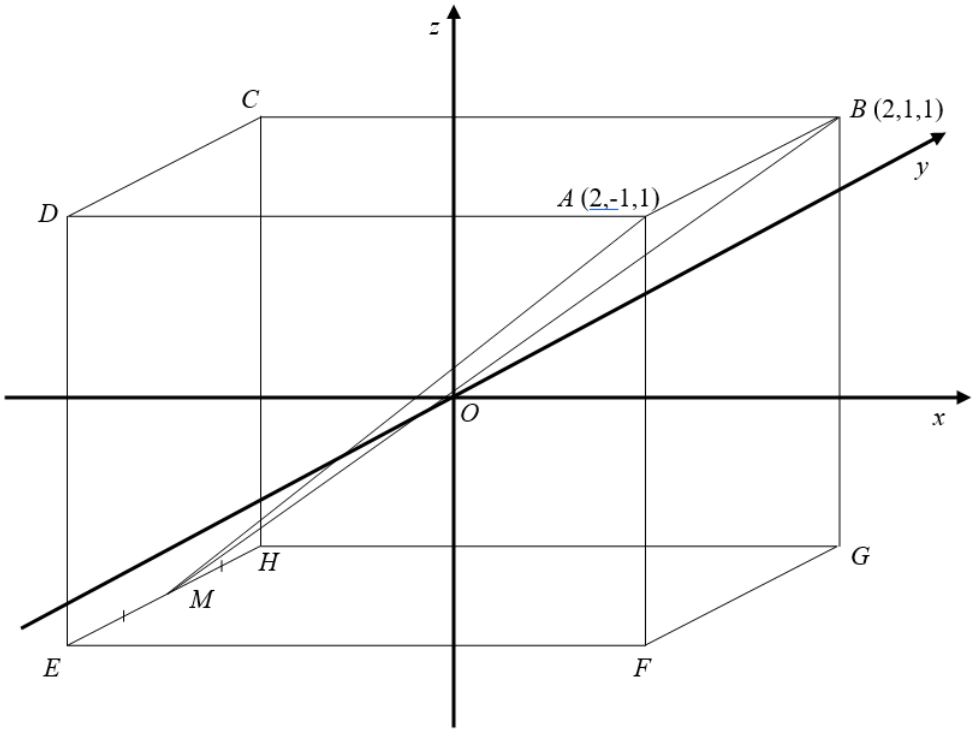
A. $|\underset{\sim}{c} - \underset{\sim}{a}| = |\underset{\sim}{c} - \underset{\sim}{a} - 2\underset{\sim}{b}|$

B. $(\underset{\sim}{c} - \underset{\sim}{a}) \cdot (\underset{\sim}{c} - \underset{\sim}{a} - 2\underset{\sim}{b}) > 0$

C. $(\underset{\sim}{c} - \underset{\sim}{a}) \cdot (\underset{\sim}{c} - \underset{\sim}{a} - 2\underset{\sim}{b}) < 0$

D. $(\underset{\sim}{c} - \underset{\sim}{a}) \cdot (\underset{\sim}{c} - \underset{\sim}{a} - 2\underset{\sim}{b}) = 0$

5 A rectangular prism, $ABCDEFGH$, is placed in a three dimensional plane with its centre at the origin, O. M is the midpoint of EH.



The value of $\angle AMB$ is closest to which of the following?

A. 20^o

B. 25^o

C. 30^o

D. 35^o

End of Section I

Section II

45 marks
Attempt Questions 6 – 8

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

Question 6 (15 marks) Please begin a new Writing Booklet.

(a) Let $\underset{\sim}{p} = 4\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{q} = -\underset{\sim}{i} + 2\underset{\sim}{j} - 3\underset{\sim}{k}$. Find the following showing your working.

(i) $\underset{\sim}{p} + 2\underset{\sim}{q}$ **1**

(ii) $3\underset{\sim}{p} \cdot \underset{\sim}{q}$ **1**

(iii) The acute angle between $\underset{\sim}{p}$ and $\underset{\sim}{q}$ **1**

(iv) The area of $\triangle OPQ$ where $\overrightarrow{OP} = \underset{\sim}{p}$ and $\overrightarrow{OQ} = \underset{\sim}{q}$. Give you answer to 2 decimal places. **2**

(b) Consider vectors $\underset{\sim}{m} = \begin{bmatrix} 2 \\ 7 \\ -3 \end{bmatrix}$ and $\underset{\sim}{n} = \begin{bmatrix} -4 \\ n \\ 6 \end{bmatrix}$. Find the value of n when:

(i) $\underset{\sim}{m}$ and $\underset{\sim}{n}$ are parallel. **1**

(ii) $\underset{\sim}{m}$ and $\underset{\sim}{n}$ are perpendicular. **2**

Let $\overrightarrow{OM} = \underset{\sim}{m}$ and $\overrightarrow{ON} = \underset{\sim}{n}$.

(iii) Find the value of n given that $\triangle OMN$ is right angled with the right angle at M. **2**

(c) Consider the vectors $\overrightarrow{OA} = \underset{\sim}{i} - 3\underset{\sim}{j} + 2\underset{\sim}{k}$, $\overrightarrow{OB} = 4\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k}$ and $\overrightarrow{OC} = 5\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$

(i) Find the vector $\overrightarrow{AB}$. **1**

(ii) Hence, or otherwise, find the equation of the line through A and B. **2**

(iii) Find the smallest distance of C from the line found in part (ii). **2**

End of Question 6

Question 7 (15 marks) Please begin a new Writing Booklet.

(a) Find the Cartesian equation of the line perpendicular to both $4\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ and $4\underset{\sim}{j} + 3\underset{\sim}{k}$ and passing through $-3\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$ **2**

(b) Consider two spheres that are placed in the same three dimensional plane with vector equations

$$S_1 : (x-1)^2 + (y+2)^2 + (z+3)^2 = 36$$

$$S_2 : (x-1)^2 + (y-2)^2 + (z+3)^2 = 144$$

(i) State the Vector equations for S_1 and S_2 **2**

(ii) Find the equation of the circle on which the two circles intersect. **2**

(iii) S_2 is moved parallel to the x axis until both circles are tangential to each other at the point of intersection. **2**

Find a possible new equation of S_2.

(c) Consider the lines $L_1 : \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ 5 \\ 2 \end{bmatrix}$ and $L_2 : \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix} + \mu \begin{bmatrix} 4 \\ -5 \\ 1 \end{bmatrix}$.

(i) Show that the two lines never intersect. **2**

(ii) Hence, show that L_1 and L_2 are skew. **1**

(iii) Show that L_1 and L_2 are not perpendicular. **1**

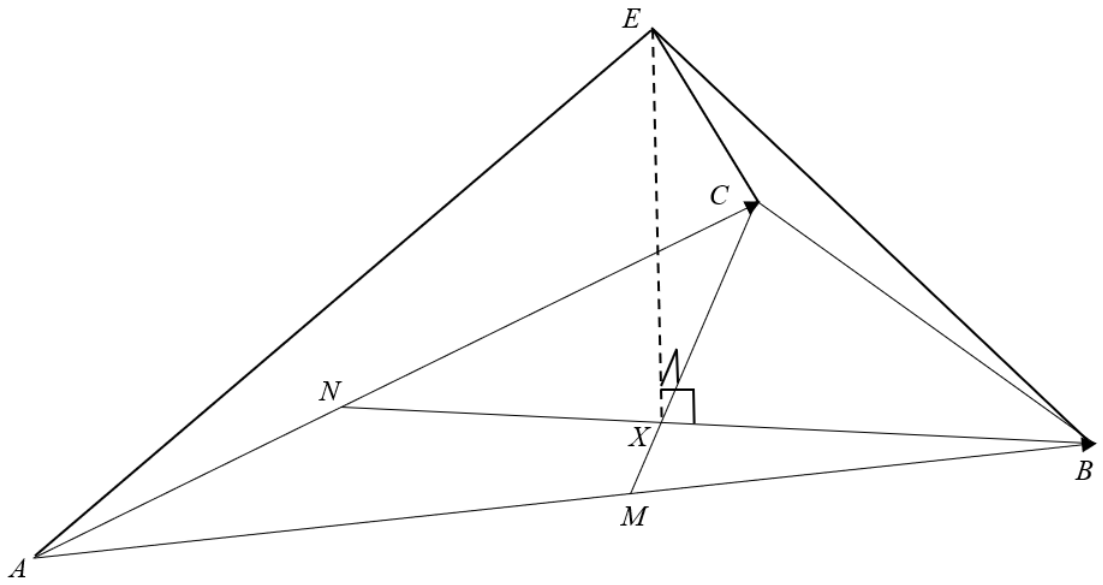
Let P lie on L_1 and Q on L_2.

(iv) Find $\overrightarrow{PQ}$ where $\left|\overrightarrow{PQ}\right|$ is a minimum. **3**

End of Question 7

Question 8 (15 marks) Please begin a new Writing Booklet.

(a) $\triangle ABC$ is the base of the pyramid $ABCE$. M is placed at the midpoint of $\overrightarrow{AB}$ and N is placed on $\overrightarrow{AC}$ so that $\dfrac{|\overrightarrow{AN}|}{|\overrightarrow{NC}|} = \dfrac{1}{3}$. E is vertically above X, the intersection of $\overrightarrow{NB}$ and $\overrightarrow{MC}$.



Let $\overrightarrow{AB} = \underset{\sim}{b}$, and $\overrightarrow{AC} = \underset{\sim}{c}$ and $\overrightarrow{AE} = \underset{\sim}{e}$

(i) Explain why $\overrightarrow{NX} = \lambda\left(\underset{\sim}{b} - \dfrac{1}{4}\underset{\sim}{c}\right)$ where $\lambda \in \mathbb{R}$. **2**

(ii) Hence, or otherwise, find an expression for $\overrightarrow{EX}$ in terms of $\underset{\sim}{c}$, $\underset{\sim}{b}$ and $\underset{\sim}{e}$. **3**

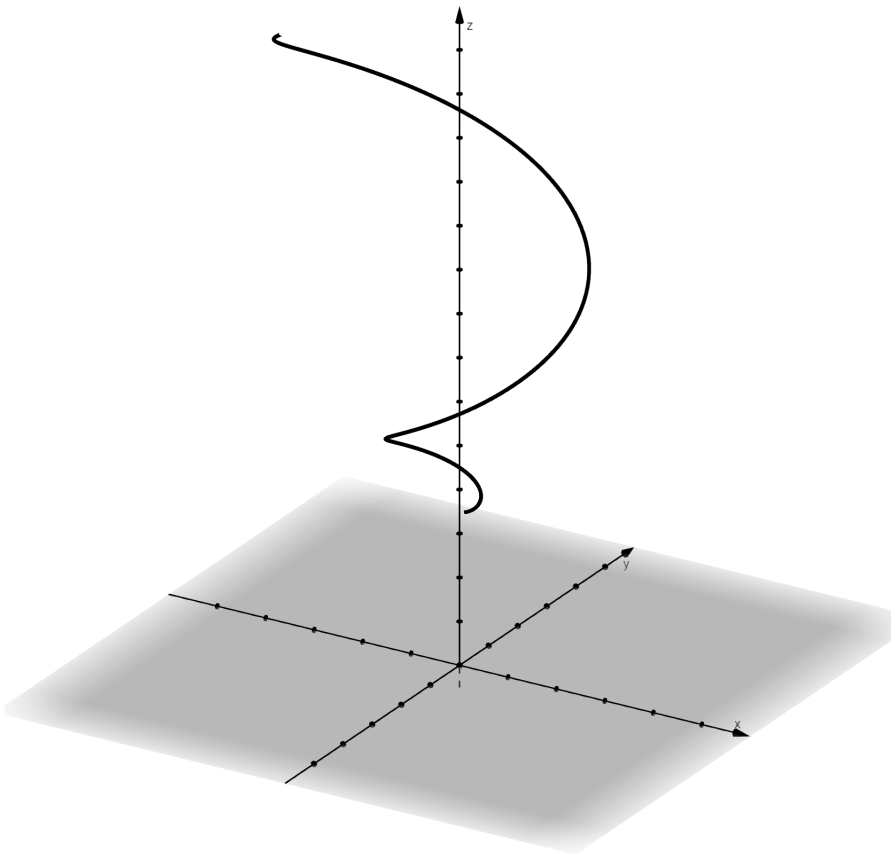
Question 8 continues on page 10

(b) Brónagh, an avid bird watcher, spots an Arctic Warbler (a small, rare, flying bird) and tracks it path.

Taking Brónagh's eyes as the origin of a three dimensional plane and the ground as parallel the *xy* plane, the Arctic Warbler's path is defined by the following vector equation.

$$\underset{\sim}{r}(t) = \begin{bmatrix} (5t+5)\cos\left(\frac{\pi t}{10}\right) \\ (5t+5)\sin\left(\frac{\pi t}{10}\right) \\ \frac{(t+1)^2}{50}+7 \end{bmatrix}$$

A graph of the path of the Arctic Warbler is given below.



Question 8 continues on page 11

Question 8 (continued)

(i) Find the initial position of the Arctic Warbler. **1**

(ii) From Brónagh's point of view, describe the path the Arctic Warbler takes including the initial direction in which it flies. **2**

As the Arctic Warbler moves, Brónagh rotates through the xy plane at the same rate, so that she can stare directly at the bird. To do so, she must change her 'Observation angle', which is the angle of elevation of the Arctic Warbler from the xy plane.

Let the Observation angle be θ.

(iii) Show that $\tan\theta = \dfrac{t^2+2t+351}{250(t+1)}$ **2**

(iv) Hence find the position of the Arctic Warbler when the Observation angle is a minimum. Give your answers to 2 decimal places where appropriate. **3**

The second time the Observation angle is $\tan^{-1}\sqrt{\dfrac{6}{5}}$, the Arctic Warbler spots Brónagh and changes direction so that it is flying directly away from her with a speed of 45 ms^{-1}.

(v) Find the vector representing the Warbler's velocity after it changes direction. **2**

End of Exam

NSW Education Standards Authority

2020 **HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}\left(a + b\right)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$\left(x - h\right)^2 + \left(y - k\right)^2 = r^2$$

Financial Mathematics

$$A = P\left(1 + r\right)^n$$

Sequences and series

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a\left(1 - r^n\right)}{1 - r} = \frac{a\left(r^n - 1\right)}{r - 1},\ r \neq 1$$

$$S = \frac{a}{1 - r},\ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

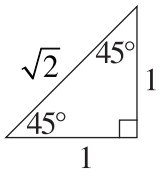
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

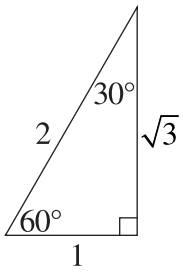
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A},\ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

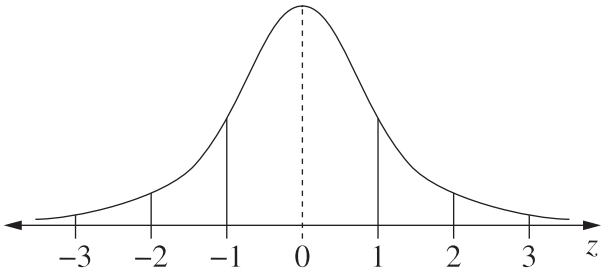
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\text{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

Continuous random variables

$$P(X \leq r) = \int_a^r f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow\ P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x},\ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\text{Var}(X) = np(1 - p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)\left[f(x)\right]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+\left[f(x)\right]^2}$

Integral Calculus

$$\int f'(x)\left[f(x)\right]^n dx = \frac{1}{n+1}\left[f(x)\right]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}\,dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln\left|f(x)\right| + c$$

$$\int f'(x)a^{f(x)}\,dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - \left[f(x)\right]^2}}\,dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + \left[f(x)\right]^2}\,dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f\left(x_1\right) + \cdots + f\left(x_{n-1}\right)\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^{n} = r^{n}(\cos n\theta + i\sin n\theta)$$
$$= r^{n}e^{in\theta}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$

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