St George Girls High School

Year 11 – Higher School Certificate Course

Assessment Task 1

2014

Mathematics Extension 2

Instructions to Candidates

- Working time – 70 minutes.
- Reading time – 5 minutes.
- Write using black or blue pen.
- Attempt all questions.
- Section – Multiple choice.
  Fill in the response oval on the Answer Sheet.
- Start each question in a new booklet, showing all working.
- Marks for each question are shown in right column
- All diagrams should be at least ½ a page in size.
- Complete each booklet cover clearly showing
  - your name
  - mathematics class and teacher

Total Marks: 50

Section I
Total marks (5)
Attempt Questions 1 – 5
Use the answer sheet provided

Section II
Total marks (45)
Attempt Questions 6 – 8
Start each question in a new booklet.

Section I

5 marks – 1 mark for each correct answer
Attempt Questions 1 – 5

Use the multiple choice answer sheet for Questions 1 – 5

1. The Principal argument of $-2-i$ is

(A) $\tan^{-1}\left(\frac{1}{2}\right)$.

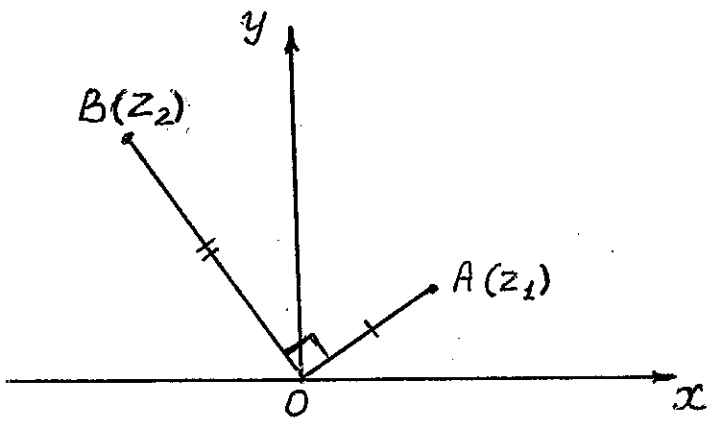
(B) $\pi-\tan^{-1}\left(\frac{1}{2}\right)$.

(C) $\pi+\tan^{-1}\left(\frac{1}{2}\right)$.

(D) $-\pi-\tan^{-1}\left(\frac{1}{2}\right)$.

(E) $-\pi+\tan^{-1}\left(\frac{1}{2}\right)$.

2.



The points A and B represent the complex numbers z_1 and z_2 respectively. It is given that $|z_1|<|z_2|$.

Which of the following is true?

(A) $|z_1+z_2|=|z_1|+|z_2|$.

(B) $\arg\left(\frac{z_1}{z_2}\right)=\frac{\pi}{2}$.

(C) $z_2=iz_1$.

(D) $\arg\left(\frac{z_2}{z_1}\right)=\frac{\pi}{2}$.

(E) $z_1=iz_2$.

Section I (cont'd)

3. If $z = \sqrt{3} - i$ which of the following is true?

(A) $|z| = 2$ and $\arg z = \frac{\pi}{6}$.

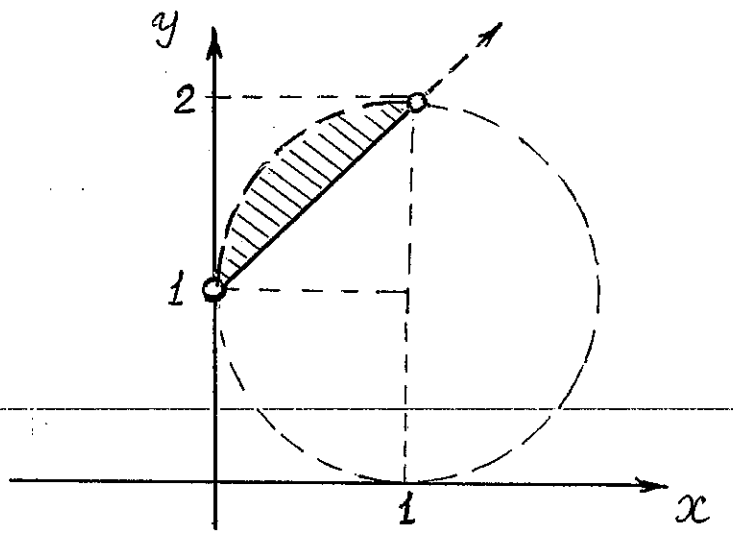
(B) $|z| = 4$ and $\arg z = -\frac{\pi}{6}$.

(C) $\arg z^3 = \frac{\pi}{2}$.

(D) $\arg\left(\frac{1}{z}\right) = \frac{\pi}{6}$.

(E) $|\bar{z}| > |z|$.

4.



The shaded region on the Argand diagram is represented by:

(A) $|z - 1| = 1$ and $\arg(z - i) = \frac{\pi}{4}$.

(B) $|z - (1 + i)| = 1$ and $\arg(z - i) > \frac{\pi}{4}$.

(C) $|z - (1 + i)| < 1$ and $\arg(z - i) > \frac{\pi}{4}$.

(D) $|z - (1 + i)| \leq 1$ and $\arg(z - i) > \frac{\pi}{4}$.

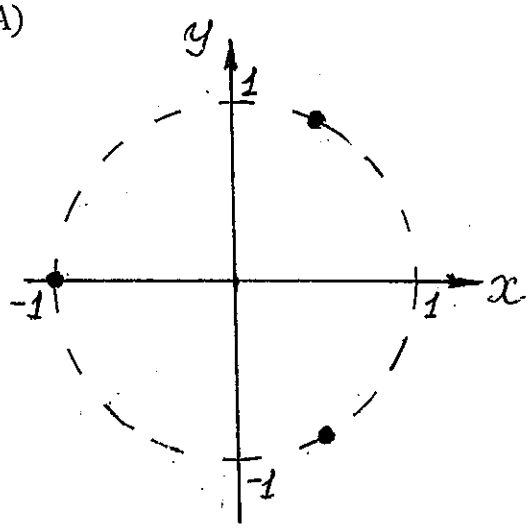
(E) $|z - (1 + i)| < 1$ and $\arg(z - i) \geq \frac{\pi}{4}$.

Section I (cont'd)

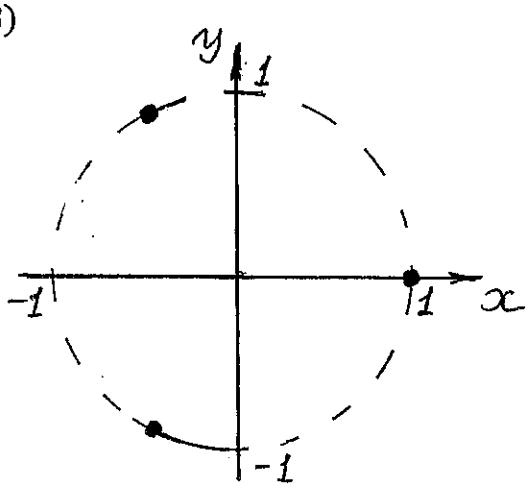
5. The solutions of $z^3 = i$ are drawn on an Argand diagram.

Which of the following best represents these solutions?

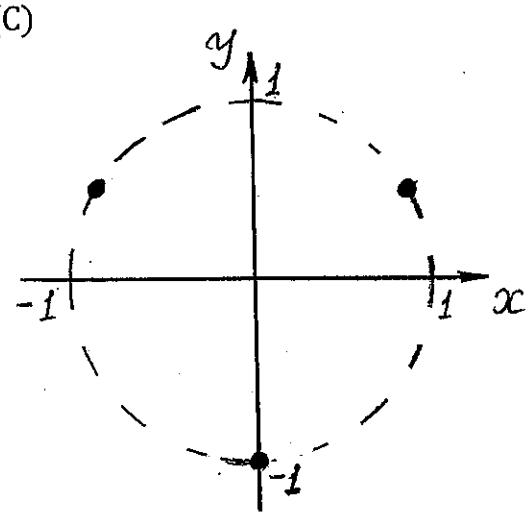
(A)



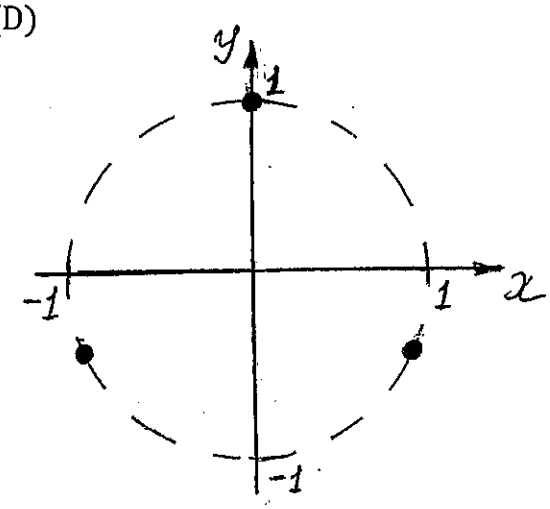
(B)



(C)



(D)



(E) none of the above.

End of Section I

Section II

45 marks
Attempt Questions 6 – 8

Answer each question in a separate writing booklet.

Question 6 – (15 Marks) – Start a New Booklet Marks

a) If $z = 3 - 2i$ and $w = -2 - i$ find each of the following in the form $a + bi$ where a, b are real numbers.

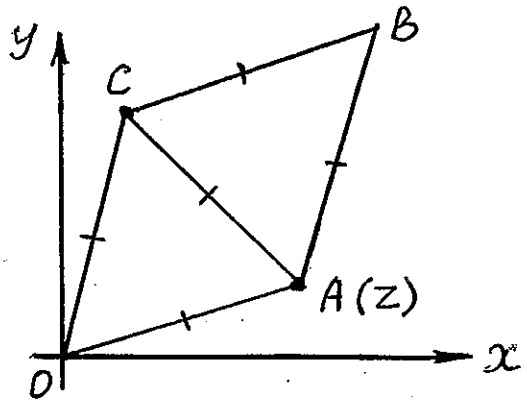
(i) zw. (ii) $\frac{z}{w}$. 1,1

(iii) $w\overline{w}$. (iv) z^2. 1,1

b) (i) Express $z = 1 - i$ in modulus argument form. 2

(ii) Hence find z^5 in the form $a + bi$ where a, b are real numbers. 3

c)



In the Argand diagram shown, the point A represents the complex number $z = 2 + i$.

If OAC is an equilateral triangle and $OABC$ is a rhombus, find the complex number represented by the point.

(i) C (ii) B 2,1

d) Solve for z: $z^2 = 8 + 6i$. 3

Question 7 – (15 Marks) – Start a New Booklet

Marks

a) If $z = \cos\theta + i\sin\theta$ use De Moivre's theorem to prove that 3

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta .$$

b) If $\arg\ (z-1) = -\frac{2\pi}{3}$ find:

(i) the minimum value of $|z|$. 2

(ii) $\arg\ (1-z)$. 2

c) (i) On an Argand diagram sketch the region satisfied by 2

$$|z-(1+2i)| \leq 1 \text{ and } \arg z \leq \frac{\pi}{4}.$$

(ii) If z is a complex number in this region find the minimum value of $\arg z$. 2
(Leave your answer in exact form).

d) The region defined by $\arg\left(\frac{z-2}{z}\right) = \frac{\pi}{3}$ is an arc of a circle on the Argand plane.

(i) Sketch this arc. 2

(ii) Find the radius of the circle. 2

Question 8 – (15 Marks) – Start a New Booklet

Marks

a) Draw separate Argand diagrams to represent each of the following:

(i) $|z-2i| = |z+1|$. 2

(ii) $\arg(z+1) = -\frac{\pi}{3}$. 2

(iii) $\arg(z-i) = \arg(z+1)$. 1

b) (i) Show that $z^5 - 1 = (z-1)(z^4 + z^3 + z^2 + z + 1)$. 1

(ii) Solve for z : $z^5 = 1$ clearly showing that the complex roots occur in conjugate pairs. 3

(iii) Use the solutions in (ii) to fully factor $z^5 - 1$ over the real numbers. 2

(iv) Compare your answers in (i) and (iii) to show that 1

$$2\cos\frac{\pi}{5} - 2\cos\frac{2\pi}{5} = 1.$$

(v) Hence show that $\cos\frac{\pi}{5} = \frac{1+\sqrt{5}}{4}$. 2

End of Paper

Student Name: _____

Class Teacher: _____

Year 11
HSC Course – Mathematics Extension 2 – Assessment Task 1 – 2014

Section I

Multiple-choice Answer Sheet – Questions 1 – 5

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:

correct ↙

A ⊗ B ⊗ C ○ D ○

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1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2} \right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$