Mathematics Extension 2

HSC marking feedback 2025

General feedback

Students should:

- show relevant mathematical reasoning and/or calculations
- read the question carefully to ensure that they do not miss important components of the question
- have a clear understanding of key words in the question and recognise the intent of the question and its requirement, such as show, solve, evaluate, hence, calculate, derive
- use the Reference Sheet where appropriate
- ensure the solution is legible and follows a clear sequence
- engage with any stimulus material provided and refer to it in their response when required by the question
- check their solution answers the question
- round off numerical solutions only at the final step of the solution
- construct graphs neatly, with precision, and display all relevant information as required by the question
- interpret information presented in graphs across a range of contexts
- understand when to use relevant calculator functions
- carefully note any information in the questions which supplies units of measurement.

Section II

Question 11 (a)

In better responses, students were able to:

- sketch the conjugate of a complex number z in the complex plane by reflecting z in the real axis.

Areas for students to improve include:

- understanding the anticlockwise rotation of a complex number by $\frac{\pi}{2}$ about the origin when it is multiplied by i
- showing the correct position of a complex number with no change to the modulus after its multiplication with i.

Question 11 (b)

In better responses, students were able to:

- apply index laws to multiply complex numbers in Euler form
- identify the modulus and argument of a complex number in Euler form.

Areas for students to improve include:

- recognising the different forms of a complex number
- providing the modulus and argument of each complex number, rather than expressing the product of the complex numbers in modulus–argument form, as required by the question
- demonstrating accuracy by avoiding basic algebraic errors.

Question 11 (c) (i)

In better responses, students were able to:

- square the complex number
- provide the answer in the Cartesian form of the complex number.

Areas for students to improve include:

- reading the question carefully to understand what is being asked
- providing the answer in Cartesian form as required, clearly separating the real and imaginary parts.

Question 11 (c) (ii)

In better responses, students were able to:

- make the denominator real using the complex conjugate
- present the answer in the form specified by the question.

Areas for students to improve include:

- using $i^2 = -1$
- recognising and distinguishing between the various forms of complex numbers
- recognising that $\frac{1}{z}$ is not the same as $\bar{z}$.

Question 11 (d) (i)

In better responses, students were able to:

- show the calculations to determine the magnitude of a vector
- identify the direction of a vector
- calculate the unit vector in the direction of the given vector
- use the unit vector and the magnitude of the force to find the vector representing the force.

Areas for students to improve include:

- showing relevant and necessary calculations to fully justify their answer in 'show' questions
- checking simple arithmetic carefully.

Question 11 (d) (ii)

In better responses, students were able to:

- demonstrate the ability to add two vectors accurately to determine a resultant force.

Areas for students to improve include:

- showing relevant and necessary calculations to fully justify their answer in 'show' questions
- ensuring numerical values are carefully checked to prevent transcription errors.

Question 11 (d) (iii)

In better responses, students were able to:

- demonstrate the ability to calculate the scalar (dot) product of two vectors accurately.

Areas for students to improve include:

- understanding that the scalar (dot) product of two vectors is a scalar quantity, not a vector
- check addition and multiplication carefully, to enhance the accuracy of solutions.

Question 11 (e)

In better responses, students were able to:

- identify and explicitly state the correct assumption when performing a proof by contradiction
- show all relevant mathematical calculations, presenting the solution clearly and systematically
- perform calculations with inequalities carefully and accurately
- arrive at the concluding mathematical result $60 \leq 9$ that is clearly false
- state clearly when a contradiction has been reached and conclude that the original statement is therefore true.

Areas for students to improve include:

- interpreting assumptions involving inequalities through the equality sign
- recording logical mathematical reasoning to show that the given statement is false
- stating the contradiction that arises in the solution clearly
- stating the conclusion of the proof clearly
- providing clear and concise explanations of why a statement constitutes a contradiction
- checking simple arithmetic carefully when working with inequalities.

Question 11 (f)

In better responses, students were able to:

- complete the square in the denominator
- use the Reference Sheet to apply the standard integrals.

Areas for students to improve include:

- completing the square when the quadratics have a negative coefficient of x^2
- including an arbitrary constant C in the anti-derivative
- checking transcription of expressions to support calculation of the correct final integrand.

Question 12 (a)

In better responses, students were able to:

- demonstrate their knowledge of integration by parts
- determine which terms of the integrand to differentiate and integrate
- evaluate the relevant trigonometric ratios.

Areas for students to improve include:

- choosing the correct expressions of $u = x$ and $dv = \sin x$
- substituting the limits of the integral carefully
- evaluating the resulting trigonometric ratios.

Question 12 (b)

In better responses, students were able to:

- demonstrate their understanding of the steps involved in mathematical induction
- establish a clear initial case where both forms of $\frac{dy}{dx}$ give $(2x + 1)e^{2x}$
- interpret and use correct notation to represent higher-order derivatives.

Areas for students to improve include:

- showing the two separate forms of the initial case clearly
- recognising that $2^0 = 1$
- understanding that $\frac{d}{dx}\left(\frac{d^k y}{dx^k}\right)$ means to differentiate $P(k)$
- applying the product rule carefully to differentiate $\left(2^k x + k2^{k-1}\right)e^{2x}$.

Question 12 (c)

In better responses, students were able to:

- draw a neat, clear graph showing sufficient detail to support their response
- factorise $|z+5-i| > |z-3+3i|$ as $|z-(-5+i)| > |z-(3-3i)|$
- plot the points $(-5, 1)$ and $(3, -3)$ on a complex plane
- recognise that the region would be bounded by the perpendicular bisector of the two points
- use the formula for the modulus, $\sqrt{x^2+y^2}$, to establish that the equation of the perpendicular bisector was $y < 2x + 1$
- shade the region, including the point $3 - 3i$.

Areas for students to improve include:

- providing detail to support their response, for example, right angles, equal interval marking, labelling of coordinates of midpoints or x and y intercepts
- understanding that the points to be plotted are found after factorising
- knowing the geometric implications of $|z+5-i| > |z-3+3i|$, for example, recognising that $|z-z_1|$ represents the distance of the variable point z from the fixed point z_1.
- recognising that the line to be drawn should be dashed
- employing algebraic skills when using the modulus method
- determining the correct region to shade.

Question 12 (d)

In better responses, students were able to:

- use partial fractions to simplify the integrand
- integrate the resulting fractions correctly.

Areas for students to improve include:

- using correct setting up of partial fractions
- employing algebraic skills when determining the numerators of partial fractions
- recognising that the function should be in absolute value signs when an integration results in a logarithmic function.

Question 12 (e)

In better responses, students were able to:

- write a force equation correctly
- simplify this force equation to an acceleration equation
- use $\ddot{x} = v\frac{dv}{dx}$ or $\ddot{x} = \frac{d\left(\frac{1}{2}v^2\right)}{dx}$ to derive an expression for x in terms of v
- use definite integrals or determine the constants of integration
- simplify the resulting expression to obtain the answer supplied in the 'show' question.

Areas for students to improve include:

- establishing the correct force and/or acceleration equations from the supplied information
- realising that $\ddot{x} = v\frac{dv}{dx}$ or $\ddot{x} = \frac{d\left(\frac{1}{2}v^2\right)}{dx}$ are more relevant forms of acceleration for expressions involving x and of v
- integrating carefully to deal with negatives and constants
- showing sufficient working to demonstrate how to get the supplied answer in the 'show' question, in particular with the use of logarithmic laws
- considering the limits of the definite integral, if using an alternative solution approach, including making sure that the limits for the two variables are correctly matched.

Question 13 (a)

In better responses, students were able to:

- work fluently with an integral dx that included a constant m
- choose a suitable substitution $u = m - x$ to make the boundaries simpler in the second integral and put it in the form kA
- justify the existence of a constant k by evaluation
- keep track of which integral they were substituting into, depending upon the substitution method chosen, as this helped with recognising that they needed to obtain the kA as opposed to $\frac{1}{k}$.

Areas for students to improve include:

- focusing on the 'show that' element of the statement and using the laws of indices appropriately in this calculus context
- clarifying which substitution was being used, especially when the response trialled several options
- taking note of both the desired limits and the form of the new integral when making a decision about which substitution to use.

Question 13 (b)

In better responses, students were able to:

- use the equations derived from the scalar (dot) products to find two 'pair' relationships connecting x, y and z before moving onto the unit vector condition
- use the formula for the length of a unit vector to solve and find the equation of the $\underset{\sim}{c}$ vectors needed
- realise that there are two potential vectors $\underset{\sim}{c}$ and $-\underset{\sim}{c}$.

Areas for students to improve include:

- taking care that all conditions stated in the question are considered before starting a response
- taking care to solve equations involving negative values.

Question 13 (c) (i)

In better responses, students were able to:

- use a learned starting point to prove a familiar relationship, in this case the perfect square of the difference of two terms being non-negative.

Areas for students to improve include:

- taking more care when working algebraically from a key starting point
- knowing a standard proof.

Question 13 (c) (ii)

In better responses, students were able to:

- apply the AM-GM inequality proven in part (i) and work concisely algebraically to reveal the proof statement required.

Areas for students to improve include:

- applications of the AM-GM inequality, realising part (i) provides a scaffold to complete part (ii). The 'hence' option is almost exclusively the more straightforward option from 'hence or otherwise'.
- understanding from the question that working with factors of $2n+1,\ 2n+2$ and $2n+3$ is the main focus
- realising that if $A > B$ and $C > D$, it does not follow that $\frac{A}{C} > \frac{B}{D}$.

Question 13 (d)

In better responses, students were able to:

- differentiate and substitute for u, including integral bounds, providing a clear delineation between the integrals du and dx
- use the complementary angles results and simplify the integral to reveal like integrals.
- collect like integrals and apply the t-formulae substitutions to evaluate the result, with correct bounds.

Areas for students to improve include:

- recognising that a factor of –1 will swap the bounds
- reviewing their solutions to ensure that in the multi-step process, the response does not stop before halving an integral or including a common factor of $\pi/2$.

Question 14 (a) (i)

In better responses, students were able to:

- use $cot^2\ \theta = cosec^2\ \theta - 1$
- replace integral with I_{n-1} .

Areas for students to improve include:

- recognising that this question did not involve integration by parts.

Question 14 (a) (ii)

In better responses, students were able to:

- use the recurrence relation.

Areas for students to improve include:

- understanding how recurrence relations work.

Question 14 (b) (i)

In better responses, students were able to:

- use $v\frac{dv}{dx}\ or\ \frac{d}{dx}(\frac{1}{2}v^2)$ to support working out to 'show'
- find the integral $v^2 or \frac{1}{2}v^2$
- apply knowledge that the initial velocity is negative.

Areas for students to improve include:

- using algebraic techniques
- choosing the correct format for integration.

Question 14 (b) (ii)

In better responses, students were able to:

- successfully integrate resulting in the inverse tangent function
- use $x = -3$ when evaluating the integral.

Areas for students to improve include:

- answering in radians, not degrees.

Question 14 (c) (i)

In better responses, students were able to:

- use sum of a geometric progression
- show all correct working.

Areas for students to improve include:

- identifying that the sum of a geometric progression could be used in this question.

Question 14 (c) (ii)

In better responses, students were able to:

- connect the relationship between the roots of unity and the roots of the quadratic
- understand that $\bar{\alpha}$ is the other root.

Areas for students to improve include:

- showing all working
- understanding the relationship between the roots of unity and the roots of the quadratic.

Question 14 (c) (iii)

In better responses, students were able to:

- follow their working through to a final answer.

Areas for students to improve include:

- showing all working and relating to earlier parts of the question.

Question 14 (d)

In better responses, students were able to:

- demonstrate understanding that $\frac{1}{b} = \frac{1}{a} + k$
- show $b + c$ or $a + d$ in terms of a and k
- use $ab > cd$ or $\frac{ab}{cd} > 1$ to introduce inequality
- obtain $a - b > c - d$ from correct working.

Areas for students to improve include:

- reducing the number of algebraic errors
- practising establishing an inequality.

Question 15 (a)

In better responses, students were able to:

- recognise that the area of a parallelogram is given by $base \times height$, then calculate the height using vector methods.

Areas for students to improve include:

- applying understanding of properties of a parallelogram, for example, diagonals are not perpendicular
- showing appropriate working for a 4-mark question
- checking simple arithmetic carefully when working across scalar (dot) products and vectors.

Question 15 (b)

In better responses, students were able to:

- correctly identify the period $T = \frac{1}{2}$ and $n = 4\pi$
- efficiently use appropriate formulae for velocity and acceleration
- find the correct amplitude which is independent of the period for the given conditions $x = \frac{1}{4}$ and $\dot{x} =$ half maximum speed
- find the maximum acceleration with the correct values of $n = 4\pi$ and $A = \frac{1}{\sqrt{12}}$.

Areas for students to improve include:

- recognising the connection between period $T = \frac{1}{2}$ and $n = 4\pi$
- using known simple harmonic motion formula hT.

Question 15 (c) (i)

In better responses, students were able to:

- use efficient methods to introduce the half angle into the identity
- apply double angle formulae with sound algebra
- apply correct conjugate methods to realise the denominator, in this case, needing the conjugate of $1 - \cos\theta - i\sin\theta$.

Areas for students to improve include:

- showing all necessary steps for 'show' questions.

Question 15 (c) (ii)

In better responses, students were able to:

- apply De Moivre's Theorem correctly to the equation $(\cos\theta + i\sin\theta)^6 = 1$ to show the 6 given roots.

Areas for students to improve include:

- applying knowledge that $\cos 6\theta + i\sin 6\theta = \cos\pi + \ i\sin\pi$ using De Moivre's Theorem
- realising that $-1 = cis\,(\pi + 2k\pi)$.

Question 15 (c) (iii)

In better responses, students were able to:

- solve the given equation $\left(\frac{z-1}{z+1}\right)^6 = -1$ correctly by seeking to make z the subject
- observe the value in the substitution $w = \frac{z-1}{z+1}$ correctly, leading to $z = \frac{1+w}{1-w}$
- make the connection between parts (i) and (ii) to obtain the correct solutions.

Areas for students to improve include:

- realising that solving an equation in z means making z the subject of the equation
- using substitutions to simplify equations.

Question 16 (a)

In better responses, students were able to:

- recognise that the negation of all solutions lying outside the circle $|z| = \frac{1}{2}$ is at least one solution on or inside the circle
- take the modulus of the sum of the n terms on the left-hand side of the equation and show that this is equal to 1
- show that the modulus of the sum of the n terms is less than or equal to the sum of the n moduli
- use modulus properties to rewrite the moduli in the form $|z|^k|\cos[k\theta]|$
- show that since $\cos\alpha \leq 1$ for all α, the sum of the n moduli is less than or equal to a geometric series with n terms, with the first term and common ratio both equal to $|z|$
- suppose by contradiction that $|z| \leq \frac{1}{2}$
- show that the sum of the geometric series is less than 1
- show that there is a contradiction as the modulus of the left-hand side cannot be equal to 1 and less than 1
- interpret the contradiction to mean that the assumption is incorrect and so all solutions must lie outside the circle $|z| = \frac{1}{2}$.

Areas for students to improve include:

- finding the correct negation of regions on the complex plane
- recognising that the triangle inequality only applies to the sum of non-negative real numbers, so moduli and absolute values are required for any expression which is not a non-negative real number
- recognising that the left-hand and right-hand sides of the triangle inequality can be equal, so use either $\leq$ or $\geq$ as appropriate, not $<$ or $>$
- recognising the need to use $\cos\alpha \leq 1$ rather than $\cos\alpha < 1$
- recognising the need to use $|z| \leq \frac{1}{2}$ rather than $|z| = \frac{1}{2}$ or $|z| < \frac{1}{2}$
- understanding that $\cos[n\theta] = \frac{1}{2}\mathrm{Re}(z^n)$ only when z is defined as $|z| = 1$ with $\mathrm{Arg}(z) = \theta$.

Question 16 (b)

In better responses, students were able to:

- use integration with respect to t to find expressions for $\dot{x}$ and x as functions of t
- use integration with respect to t to find expressions for $\dot{y}$ and y as functions of t
- rearrange the equation for x to find t as a function of x
- substitute the equations for x and t into the equation for y to find y as a function of x.

Areas for students to improve include:

- understanding that to create a Cartesian equation they need to find x and y as functions of t then combine them to create an equation for y as a function of x, unlike for most resisted motion where we use $\ddot{x} = \dot{x}\frac{d\dot{x}}{dx}$ and $\ddot{y} = \dot{y}\frac{d\dot{y}}{dy}$
- understanding that when integrating for both x and y it is more efficient to integrate each of them separately, rather than trying to present them in vector notation, for example,
$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \displaystyle\int \ddot{x}\,dx \\ \displaystyle\int \ddot{y}\,dy \end{pmatrix}$$
- understanding that $\ddot{x}$ means $\frac{d}{dt}(\dot{x})$ and $\dot{x}$ means $\frac{d}{dt}(x)$, and so they need to integrate with respect to t
- understanding that replacing $\ddot{y}$ with $\dot{y}\frac{d\dot{y}}{dy}$ will not support the solution since $\dot{y}$ is not a factor of $-k\dot{y} - 10$
- using definite integrals to achieve the answer more efficiently and accurately
- using dot notation throughout all steps in the integration and avoiding the use of v or introducing new variables for horizontal or vertical velocity, particularly using v for both horizontal and vertical velocity
- understanding that when integrating vertical motion, we use $\dot{y}$ and y rather than $\dot{x}$ and x

- understanding how to use the information in the question including the diagram to find the correct initial horizontal and vertical velocities
- using absolute values when finding the anti-derivative of a function in the form $\frac{f'(x)}{f(x)}$, unless information in the question shows that $f(x) > 0$ for all x
- understanding that when we rearrange an exponential equation and introduce logarithms that we do not introduce absolute values that were not already part of the equation.

Question 16 (c) (i)

In better responses, students were able to:

- note that at the closest point between B and ℓ the vector is perpendicular to ℓ
- solve $\left(\underset{\sim}{b} - \underset{\sim}{a} - \lambda_0 \underset{\sim}{d}\right) \cdot \underset{\sim}{d} = 0$ to find λ_0 as a function of $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{d}$
- use the axis of symmetry or calculus to find the value of λ_0 that minimises $[f(\lambda)]^2$ and $f(\lambda)$.

Areas for students to improve include:

- drawing a diagram showing all points and vectors to guide their solution
- understanding that the product of a scalar and a vector, for example a vector projection, is a vector
- understanding that an expression that involves the sum or difference of a vector and a scalar is undefined
- knowing how to perform calculations involving a mix of products and dot products.

Question 16 (c) (ii)

In better responses, students were able to:

- find an expression for $\overrightarrow{PB}$ and use the result in part (i) to show that the dot product of $\overrightarrow{PB}$ and $\underset{\sim}{d}$ is zero since $\underset{\sim}{d} \cdot \underset{\sim}{d} = \left|\underset{\sim}{d}\right|^2 = 1$, so $\overrightarrow{PB} \perp \ell$
- explain that since P has position vector $\underset{\sim}{a} + \lambda_0 \underset{\sim}{d}$, that $\overrightarrow{PB}$ is the shortest vector joining B and ℓ, so $\overrightarrow{PB} \perp \ell$.

Areas for students to improve include:

- understanding the need to show that $\overrightarrow{PB}$ is perpendicular to the direction vector of ℓ, $\underset{\sim}{d}$ or $\lambda_0 \underset{\sim}{d}$, rather than being perpendicular to $\overrightarrow{OP}$, $\underset{\sim}{a} + \lambda_0 \underset{\sim}{d}$
- expanding the products and dot products clearly, and using the fact that $\left|\underset{\sim}{d}\right| = 1$, to prove that the dot product is zero
- understanding that if they find $\overrightarrow{PB} = \underset{\sim}{0}$ then find $\overrightarrow{PB} \cdot \underset{\sim}{d} = 0 \cdot \underset{\sim}{d} = 0$ that this does not show that the vectors are perpendicular.

Question 16 (c) (iii)

In better responses, students were able to:

- draw a clear diagram showing all points and vectors
- find the square of the distance from the origin to ℓ by letting $\underset{\sim}{b} = \underset{\sim}{0}$ in the result from part (i)
- find the above result by noting that ΔOAP is right-angled, that $\left|\overrightarrow{OB}\right| = 1$ and $\underset{\sim}{b} \cdot \underset{\sim}{d} = 0$
- find the distance from the origin by letting $\underset{\sim}{b} = \underset{\sim}{0}$ in $\left|\overrightarrow{OP}\right| = \left|\underset{\sim}{a} - \underset{\sim}{b} + \lambda_0 \underset{\sim}{d}\right|$ and so the minimum distance is $\left|\underset{\sim}{a} - \left(\underset{\sim}{a} \cdot \underset{\sim}{d}\right) \underset{\sim}{d}\right| - 1$ units, and stating that the minimum distance is zero if the line intersects the unit sphere.

Areas for students to improve include:

- working on vector questions using results previously proven, combined with dot products, products and geometry.