HORNSBY GIRLS HIGH SCHOOL

Mathematics Extension 2

Year 12 Higher School Certificate

Online Practice Trial Examination Term 3 2021

General Instructions

- Reading Time – 10 minutes
- Working Time – 3 hours
- Write using black pen
- NESA-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for untidy and poorly arranged work
- Clearly label every question and part

Total marks – 100

Section I Pages 2 – 5

10 marks

Attempt Questions 1 – 10

Answer on the Multiple Choice Answer Sheet provided or a separate sheet of paper

Section II Pages 6 – 11

90 marks

Attempt Questions 11 – 16

Start each question on a new sheet of paper

Question	***1-10***	***11***	***12***	***13***	***14***	***15***	***16***	***Total***
Total	*/10*	*/15*	*/15*	*/15*	*/15*	*/15*	*/15*	*/100*

This assessment task constitutes 0% of the Higher School Certificate Course School Assessment

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet or a separate sheet of paper for Questions 1-10

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1. Find $\int \frac{2x}{\sqrt{1-x^4}}\,dx$

(A) $2\sin^{-1} x + c$

(B) $\cos^{-1} x^2 + c$

(C) $\sin^{-1} x^2 + c$

(D) $\frac{1}{2}\sin^{-1} x^2 + c$

2

For a certain complex number z where $\arg(z) = \frac{\pi}{5}$, $\arg(z^7)$ is:

(A) $\frac{-7\pi}{5}$

(B) $\frac{-3\pi}{5}$

(C) $\frac{2\pi}{5}$

(D) $\frac{3\pi}{5}$

3. Write down the contrapositive for the statement 'If I like walking, then I do not like cycling'

(A) If I dislike cycling, then I do not like walking.

(B) If I like cycling, then I like walking.

(C) If I dislike cycling, then I like walking.

(D) If I like cycling, then I do not like walking.

4. Find the radius and centre of the sphere: $x^2 + y^2 + z^2 + x - 3y + 2z + 2 = 0$

(A) Radius $= \frac{\sqrt{6}}{2}$ and centre $= \left(\frac{1}{2}, \frac{3}{2}, 1\right)$

(B) Radius $= \frac{\sqrt{6}}{3}$ and centre $= \left(\frac{-1}{4}, \frac{3}{2}, -3\right)$

(C) Radius $= \frac{\sqrt{6}}{2}$ and centre $= \left(\frac{-1}{2}, \frac{3}{2}, -1\right)$

(D) Radius $= \frac{3\sqrt{6}}{4}$ and centre $= \left(\frac{-1}{3}, \frac{3}{2}, -2\right)$

5.

Which of the following is an expression for $\int \frac{\sin x \cos x}{4 + \sin x}\, dx$?

(A) $-\sin x - 4\ln|4 + \sin x| + C$

(B) $-4\ln|4 + \sin x| + C$

(C) $\sin x - 4\ln|4 + \sin x| + C$

(D) $4\ln|4 + \sin x| + C$

6. $1 + e^{i(\alpha)} =$

(A) $2cos(\alpha/_2)e^{i(\alpha/_2)}$

(B) $isin(^{-\alpha}/_2)$

(C) $cos(\alpha/_2)$

(D) $3cos(\alpha)e^{i(\alpha/_2)}$

7.

Given that x and y are natural numbers, which of the following is a FALSE statement?

A. $\forall x \exists y \, (x - y = 0)$

B. $\forall x \exists y \, (3x - y = 0)$

C. $\forall x \exists y \, (x - 3y = 0)$

D. $\exists x \exists y \, (x + y = 8)$

8. Evaluate $\int_0^1 x(1-x)^{2021} dx$

(A) $\dfrac{1}{4090506}$

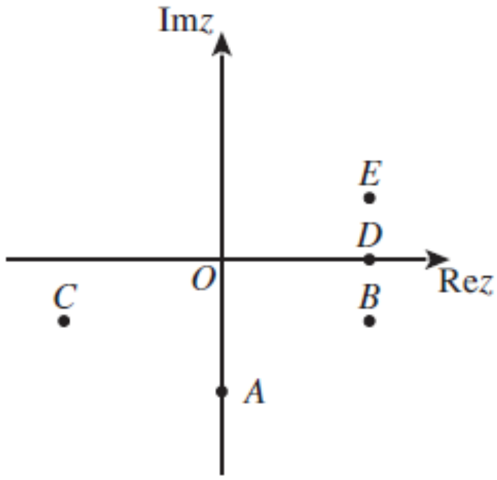
(B) 13

(C) 786

(D) 3 456

9.

Consider the Argand diagram, where $z = a + ib$.



Which of the following pairs of points in the complex plane could represent the square roots of z?

A. A and D

B. B and C

C. B and E

D. C and E

10. Solve the following quadratic equation: $ix^2 - 2(i+1)x + 10 = 0$.

(A) $x = -2 - 3i$ or $x = 2 + i$

(B) $x = -14 - 3i$ or $x = 3 + 4i$

(C) $x = -1 - 3i$ or $x = 3 + i$

(D) $x = -5 - 3i$ or $x = 5 + i$

Mathematics Extension 2

Section II

90 marks

Attempt Questions 11-16

Allow about 2 hours and 45 minutes for this section

Answer each question on your own writing paper. Start each of Questions 11 – 16 on a new sheet of paper so that they can be scanned/photographed and uploaded as separate questions.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks)

(a) Find $\int \frac{1}{e^x + e^{-x}} dx$. **2**

(b) On separate Argand diagrams sketch the following loci:

(i) $2 \geq |z| \geq 1$ **1**

(ii) $\frac{3\pi}{4} > \arg z > \frac{\pi}{4}$ **1**

(iii) $3 \geq \operatorname{Re} Z \geq 0$ and $3 \geq \operatorname{Im} Z \geq 1$ **2**

(c) Find $\int \frac{2x^2 - 2x + 1}{(x-2)(x^2+1)} dx$. **3**

(d) (i) Write down the moduli and argument of $-\sqrt{3} + i$ **2**

(ii) Write down the moduli and argument of $4 + 4i$ **2**

(iii) Hence express in modulus/argument form $\frac{-\sqrt{3} + i}{4 + 4i}$. **2**

End of Question 11

Question 12 (15 marks) (Start a new page)

(a) For the conditional statement 'If n is divisible by 16 then n is divisible by 4',

(i) write the converse statement. **1**

(ii) write the negation statement. **1**

(b) A particle moves along the x axis with its velocity v (cm/sec) at position x (cm), given by

$v = \sqrt{16x - x^2}$. Find the acceleration of the particle when $x = 5$ **2**

(c) Using vectors, determine whether the points $A(0, 4, 4)$, $B(8, 20, 36)$ and $C(12, 28, 52)$ are collinear. **3**

(d) A line passes through the points $A(-3, 2, 6)$ and $B(7, 4, -3)$

(i) Write a vector equation of the line. **1**

(ii) Write parametric equations for the line. **1**

(iii) Determine if $C(-13, 0, 15)$ lies on the line. **1**

(e) (i) Show that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$. **2**

(ii) Hence find $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x} dx$ **3**

End of Question 12

Question 13 (15 marks) (Start a new page)

(a) *A, B* and *C* are points defined by the position vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}$, $\underset{\sim}{b} = 3\underset{\sim}{i} + \underset{\sim}{j}$ and $\underset{\sim}{c} = 2\underset{\sim}{i} - 2\underset{\sim}{j} - \underset{\sim}{k}$. Find the magnitude of angle *ABC*, correct to one decimal place. **3**

(b) (i) Use the substitution $u = \frac{1}{x}$ to show that $\int_{\frac{1}{2}}^{1} \frac{\ln x}{1+x^2}.dx = \int_{2}^{1} \frac{\ln u}{1+u^2}.du$ **3**

(ii) Deduce the value of $\int_{\frac{1}{2}}^{2} \frac{\ln x}{1+x^2}.dx$ **1**

(c)

(i) Show that, for any integer *n*, $e^{in\theta} + e^{-in\theta} = 2\cos n\theta$. **1**

(ii) By expanding $\left(e^{i\theta} + e^{-i\theta}\right)^5$, show that $\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$. **3**

(iii) Hence, or otherwise, find $\int_0^{\frac{\pi}{2}} \cos^5\theta d\theta$. **2**

(iv) Using the result of part (ii), solve the equation $\cos 5\theta + 5\cos 3\theta + 9\cos\theta = 0$ for $0 \le \theta \le \pi$. **2**

End of Question 13

Question 14 (15 marks) (Start a new page)

(a) By considering the scalar product $\underset{\sim}{a} . \underset{\sim}{b}$ where $\underset{\sim}{a} = a_1\underset{\sim}{i} + a_2\underset{\sim}{j} + a_3\underset{\sim}{k}$ and $\underset{\sim}{b} = b_1\underset{\sim}{i} + b_2\underset{\sim}{j} + b_3\underset{\sim}{k}$

Prove that $(a_1b_1 + a_2b_2 + a_3b_3)^2 \leq (a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$ **3**

(b) Find $\int e^x \cos x dx$. **2**

(c) 1, ω and ω^2 are the three cube roots of unity.

(i) Show that $1+\omega+\omega^2 = 0$ where ω is a non-real root of unity **1**

(ii) Simplify each of the expressions $(1+3\omega+\omega^2)^2$ and $(1+\omega+3\omega^2)^2$ **2**

(iii) Find $(1+3\omega+\omega^2)^2 + (1+\omega+3\omega^2)^2$ **1**

(iv) Find the product of $(1+3\omega+\omega^2)^2$ and $(1+\omega+3\omega^2)^2$ **1**

(d) Evaluate $\int_0^{\frac{\pi}{6}} \cos x.\sin^3 x.dx$ **2**

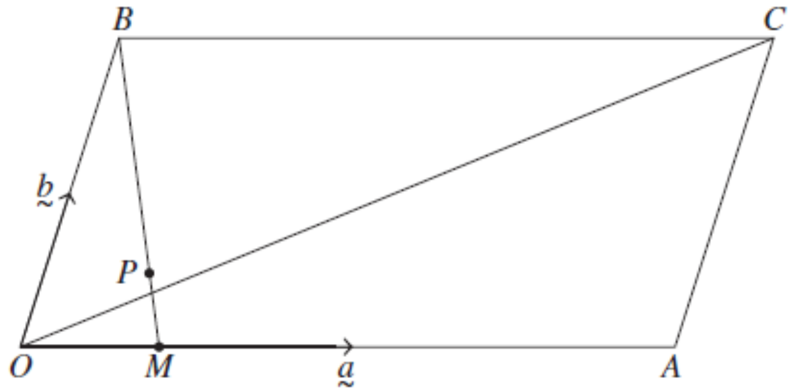
(e) Evaluate $\int_0^{\frac{\pi}{4}} \frac{1-\tan x}{1+\tan x}.dx$. **3**

End of Question 14

Question 15 (15 marks) (Start a new page)

(a)

Let $OACB$ be a parallelogram with $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$. M is a point on OA such that $|\overrightarrow{OM}| = \frac{1}{5}|\overrightarrow{OA}|$. P is a point on MB such that $|\overrightarrow{MP}| = \frac{1}{6}|\overrightarrow{MB}|$, as shown in the diagram.



(i) Show that P lies on OC. **3**

(ii) State the ratio of lengths $OP : PC$. **1**

(b) A cruise ship needs 24 metres of water to enter a harbour. At low tide the harbour is 16 metres deep and at high tide the depth is 28 metres. Low tide occurs at 10 a.m. and high tide occurs at 4 p.m.

Assume the tide rise and fall in simple harmonic motion, find the earliest time the ship can enter the harbour. **3**

(c) Prove by mathematical induction that for $n \geq 1$

$$1 \cdot \ln\frac{2}{1} + 2 \cdot \ln\frac{3}{2} + \ldots + n \cdot \ln\left(\frac{n+1}{n}\right) = \ln\left(\frac{(n+1)^n}{n!}\right).$$ **4**

(d) Evaluate $\int_0^{\pi/3} \frac{1}{9 - 8\sin^2 x} dx$. **4**

End of Question 15

Question 16 (16 marks) (Start a new page)

(a) If $a > 0$, $b > 0$, $c > 0$ and $d > 0$ show that $\dfrac{a+b+c+d}{4} \geq \sqrt[4]{abcd}$ **4**

(b) Let $t = \tan\dfrac{x}{2}$

(i) Show that $\dfrac{dx}{dt} = \dfrac{2}{1+t^2}$ 1

(ii) Without stating t-results, show that $\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} = \sin x$ 1

(iii) Hence, show that $\displaystyle\int_0^{\frac{\pi}{2}} \frac{1}{1+k\sin x}dx = \frac{2}{\sqrt{1-k^2}}\tan^{-1}\sqrt{\frac{1-k}{1+k}}$, where $0 < k < 1$. **4**

Let $I_n = \displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{2+\sin x}dx$, where $n = 0, 1, 2, \ldots$.

(iv) Show that $I_{n+1} + 2I_n = \displaystyle\int_0^{\frac{\pi}{2}} \sin^n x\,dx$. **1**

(v) Hence, or otherwise, find the value of I_2. Give your answer in the form $m\pi + 1$, where m is irrational. **4**

End of examination

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Year 12 Trial Examination Term 3 2021

STUDENT NUMBER:____

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A B C D

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A B C D

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

correct

A B C D

1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D
6.	A	B	C	D
7.	A	B	C	D
8.	A	B	C	D
9.	A	B	C	D
10.	A	B	C	D