SHORE

Exam Number:

Year 12

Mathematics Extension 2

HSC Trial Examination

2024

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black or blue pen
- Draw diagrams using pencil
- Board approved calculators may be used
- Answer Questions 1 - 10 on the Multiple Choice answer sheet provided
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations
- Start each of Questions 11 – 16 in a new writing booklet
- Write your examination number on the front cover of each booklet to be handed in
- A NESA reference sheet is provided.
- If you do not attempt a question, submit a blank booklet marked with your examination number and "N/A"

Total marks – 100

Section I Pages 2 - 5

10 marks

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II Pages 6 - 14

90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section.

Note: Any time you have remaining should be spent revising your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

BLANK PAGE

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

---

1. The velocity of a particle moving in simple harmonic motion along the x-axis is given by $V^2 = 60 + 8x - 4x^2$.

What is the centre of motion?

(A) $x = -5$

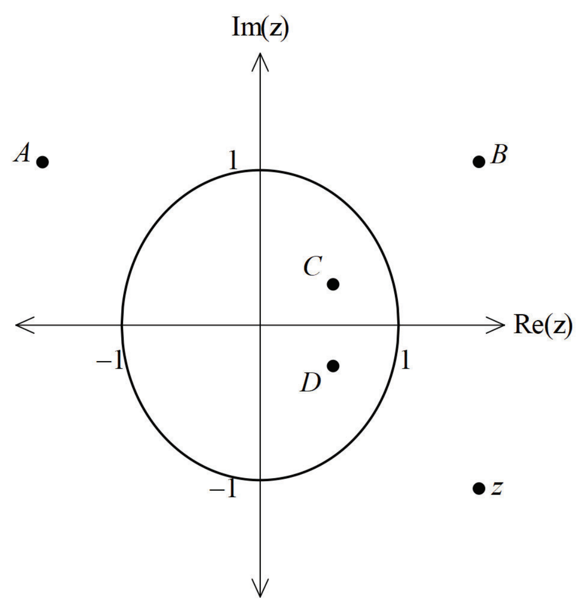
(B) $x = 3$

(C) $x = 1$

(D) $x = 60$

2. The diagram shows the complex number z in the fourth quadrant on the Argand diagram. The modulus of z is 2.

Which of the points marked A, B, C or D best shows the position of $\frac{1}{z}$?



(A) Point A

(B) Point B

(C) Point C

(D) Point D

3. Which expression is equal to $\int \tan^3 x \sec^2 x \, dx$?

(A) $\dfrac{\tan^4 x}{4} + c$

(B) $\tan^4 x + c$

(C) $\dfrac{\tan^5 x}{5} + \dfrac{\tan^3 x}{3} + c$

(D) $\tan^5 x + \tan^3 x + c$

4. What value of z satisfies $z^2 = 7 + 24i$?

(A) $4 + 3i$

(B) $-4 + 3i$

(C) $-3 + 4i$

(D) $3 + 4i$

5. Consider the complex, non-real cube roots of unity w and w^2.
What is the value of $\left(1 - w + w^2\right)\left(1 + w - w^2\right)$?

(A) 0

(B) 1

(C) 2

(D) 4

6. What is the size of the acute angle θ between the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} - \underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - 2\underset{\sim}{k}$?

(A) $\theta = \dfrac{\pi}{6}$

(B) $\theta = \dfrac{\pi}{5}$

(C) $\theta = \dfrac{\pi}{4}$

(D) $\theta = \dfrac{\pi}{3}$

7. A particle has initial velocity $2\underset{\sim}{i}\ \text{ms}^{-1}$. At time *t* seconds it has acceleration

$$(2t-3)\underset{\sim}{i} + \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right)\underset{\sim}{j}\ \text{ms}^{-2}.$$

When is the particle at rest?

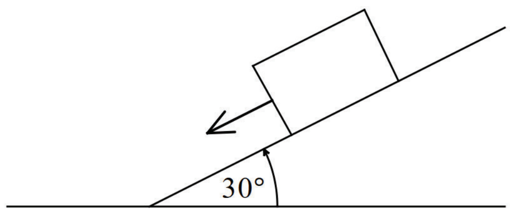
(A) Never.

(B) At time $t=1$ second only.

(C) At time $t=2$ seconds only.

(D) At time $t=1$ and $t=2$ seconds.

8. A 10 kg box on a plane inclined at an angle of $30°$ to the horizontal is undergoing uniform acceleration of 1.5 m/s^2 down the plane.
Take the acceleration g due to gravity to be 9.8 m/s^2.



What is the magnitude of the frictional force resisting the motion of the box?

(A) 34 N

(B) 64 N

(C) 70 N

(D) 100 N

9. The points *A, B* and *C* are collinear where $\overrightarrow{OA} = \underset{\sim}{i} - \underset{\sim}{j}$, $\overrightarrow{OB} = -3\underset{\sim}{j} - \underset{\sim}{k}$ and $\overrightarrow{OC} = 2\underset{\sim}{i} + a\underset{\sim}{j} + b\underset{\sim}{k}$ for some constants *a* and *b*. What are the values of *a* and *b*?

(A) $a=-1$ and $b=-1$

(B) $a=-1$ and $b=1$

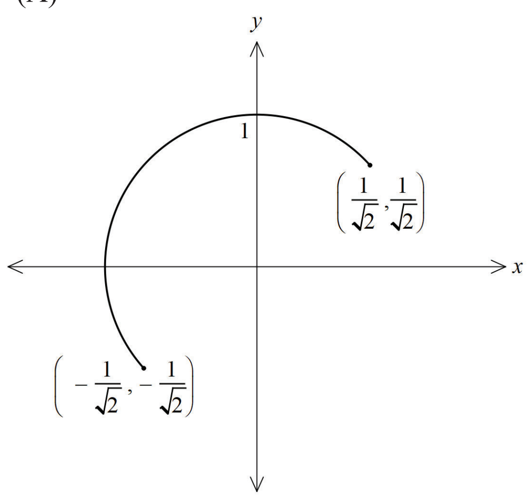
(C) $a=1$ and $b=-1$

(D) $a=1$ and $b=1$

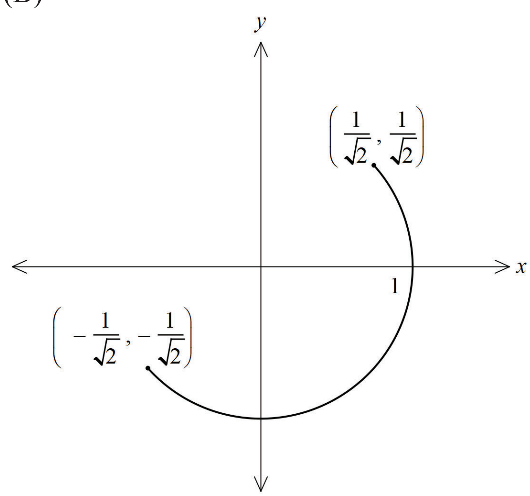
10. Which diagram best shows the curve described by the position vector

$$\underset{\sim}{r}(t)=\sin(t)\underset{\sim}{i}-\cos(t)\underset{\sim}{j} \text{ for } \frac{\pi}{4}\le t\le\frac{5\pi}{4}?$$

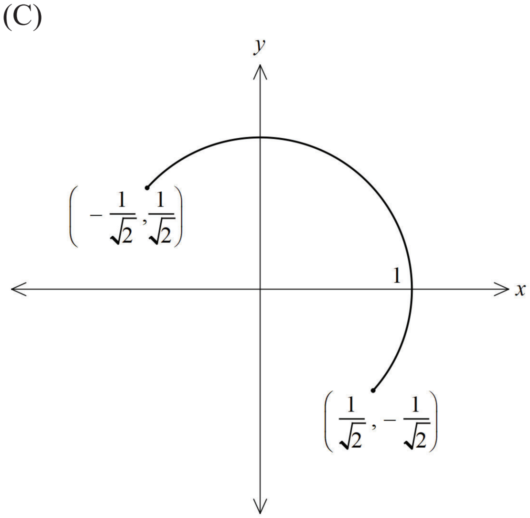
(A)



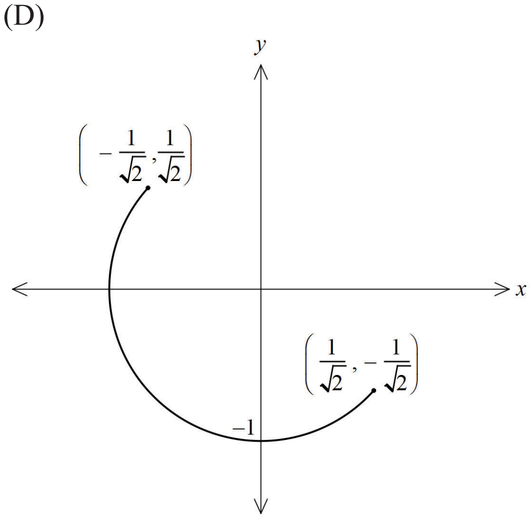
(B)



(C)



(D)



Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section.

Answer these questions in the Answer Book provided.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (17 marks) **Marks**

(a) Express $2\sqrt{2}\,e^{\frac{-3\pi}{4}i}$ in the form $x+iy$. **2**

(b) A particle starts at rest from the origin and moves in a straight line such that its displacement x metres at time t seconds is determined by the equation $\ddot{x}=-25x$. **2**

How long will it take for the particle to first return to the origin?

(c) Consider the two points $A(3,\,3,\,3)$ and $B(3,\,-3,\,3)$.

(i) Find $\overrightarrow{AB}$. **1**

(ii) Find $\left|\overrightarrow{AB}\right|$. **1**

(iii) Find $\angle AOB$. Give your answer correct to the nearest degree. **2**

(d) The function $f(x)$ is defined by

$$f(x)=\frac{6x+4}{(x-1)(x+1)(2x+3)}.$$

(i) Express $f(x)$ in terms of partial fractions. **2**

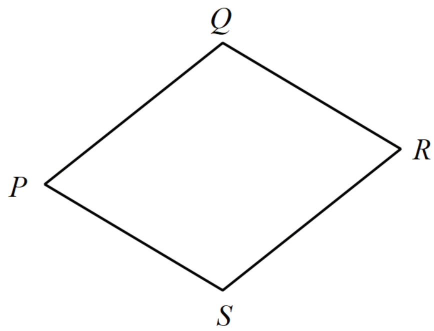
(ii) Hence, or otherwise, find $\displaystyle\int\frac{3x+2}{(x-1)(x+1)(2x+3)}\,dx$. **2**

(e) (i) Find the two square roots of $2i$, giving the answers in the form $x+iy$, where x and y are real numbers. **2**

(ii) Hence, or otherwise, solve $2z^2+2\sqrt{2}\,z+1-i=0$, leaving answers in the form $x+iy$. **3**

End of Question 11

Question 12 (15 marks) **Marks**



(a) $PQRS$ is a parallelogram. Given that $\overrightarrow{PQ} = 2\underset{\sim}{i} + 3\underset{\sim}{j} - 4\underset{\sim}{k}$ and $\overrightarrow{QR} = 5\underset{\sim}{i} - 2\underset{\sim}{k}$

(i) Show that $PQRS$ is a rhombus. **2**

(ii) Find the exact area of $PQRS$. **3**

(b) Evaluate $\displaystyle\int_0^{\frac{1}{\sqrt{2}}} \frac{x^3}{\sqrt{1-x^2}}\,dx$. **4**

(c) Prove by mathematical induction that $3n^2 - 3n \le 2^n - 1$ for $n \ge 7$. **3**

(d) Use the t-formula to evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{3 - \cos x}$. **3**

End of Question 12

Question 13 (14 marks) **Marks**

(a) A particle is initially 2 metres to the left of the origin, moving to the left at a speed of 3 m/s. The acceleration of the particle is given by $a = v + v^3$, where v is the velocity of the particle. **3**
Find an expression for x, the displacement of the particle, in terms of v.

(b) Consider the statement "If Sally studies, she will pass the test."

(i) Write down the converse of the statement. **1**

(ii) Write down the contrapositive of the statement. **1**

(c) (i) The complex number z is such that $\arg(z + 2i) = \tan^{-1}\frac{1}{2}$. **2**
It is given that $z = x + iy$, where x and y are real and $x > 0$.
Find an equation for y in terms of x.

(ii) The complex number z satisfies $\frac{-\pi}{2} \le \arg(z + 2i) \le \tan^{-1}\frac{1}{2}$. **2**
The region R is the locus of z. In your answer booklets, sketch the region R on the given Argand diagram.

(iii) z_1 is the point in R at which $|z|$ is minimum. Calculate the exact value of $|z_1|$. **1**

(iv) Express z_1 in the form $a + ib$ where a and b are real. **1**

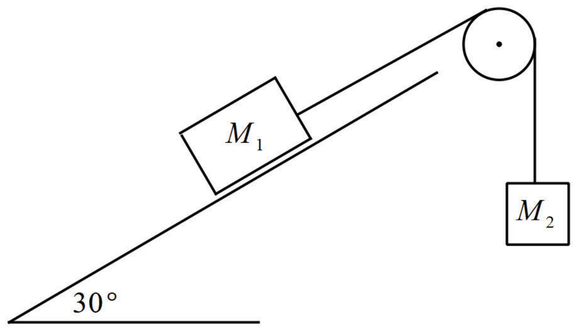
(d) The equations of two lines are $\underset{\sim}{r} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 3\underset{\sim}{k} + \lambda\left(\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}\right)$ and **3**
$\underset{\sim}{u} = \underset{\sim}{i} + 11\underset{\sim}{j} - 4\underset{\sim}{k} + \mu\left(-\underset{\sim}{i} + 3\underset{\sim}{j} - 2\underset{\sim}{k}\right)$.
Show that the lines intersect, stating the point of intersection.

End of Question 13

Question 14 (16 marks) **Marks**

(a) Two masses, M_1 kilograms and M_2 kilograms, are attached by a light inextensible string. The string is placed over a smooth pulley, and the mass M_1 rests on a smooth frictionless plane inclined at 30 degrees to the horizontal as shown. **3**

The mass M_2 is suspended vertically by the string.



NOT TO SCALE

The mass M_1 accelerates down the inclined plane at 2 m/s^2, and $g = 10$ m/s^2.

Given $M_1 = kM_2$, find the value of k.

(b) Evaluate $\displaystyle\int_0^4 \frac{1}{\sqrt{1+\sqrt{x}}}\,dx$. **3**

Question 14 continues on page 9

(c) A missile is fired from point O with speed V m/s at an angle α above the horizontal where $\frac{\pi}{4} < \alpha < \frac{\pi}{2}$. The missile moves in a vertical plane under gravity where the acceleration due to gravity is g m/s^2. At time t seconds the position vector of the missile relative to O is $\underset{\sim}{r}(t) = (Vt\cos\alpha)\underset{\sim}{i} + \left(Vt\sin\alpha - \frac{1}{2}gt^2\right)\underset{\sim}{j}$.

(i) Show that the greatest height reached by the missile is $H = \frac{V^2 \sin^2 \alpha}{2g}$ metres and that when the missile is at its greatest height its angle of elevation from O is $\tan^{-1}\left(\frac{1}{2}\tan\alpha\right)$. **3**

(ii) When the missile reaches a height of h metres its angle of inclination to the horizontal falls to $\frac{\pi}{4}$. Show that $\frac{h}{H} = 2 - \operatorname{cosec}^2\alpha$. **3**

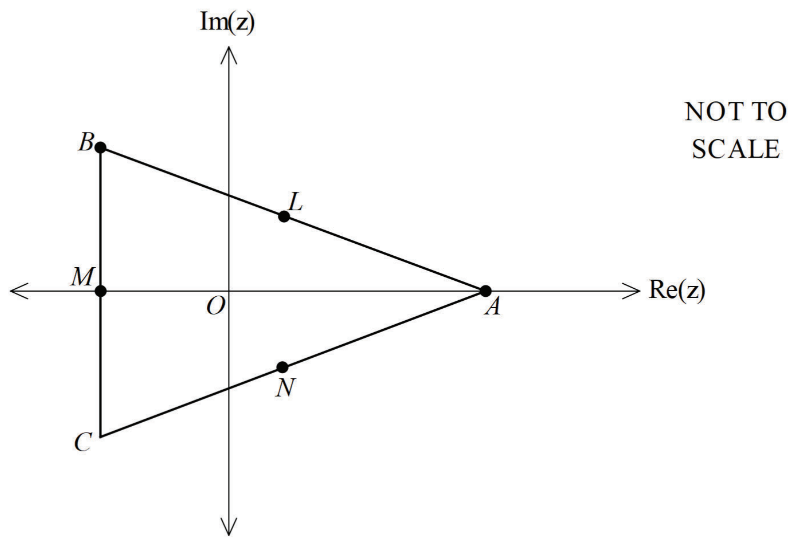
(d) (i) Prove that $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ **1**

(ii) Hence or otherwise, to find the value of $\int_0^{\pi} \frac{\sin x}{e^{\frac{\pi}{2}} + e^x}\,dx$. **3**

End of Question 14

Question 15 (15 marks) **Marks**

(a) The cube roots of unity are represented on the Argand diagram below by the points A, B and C. **3**



The points L, M and N are the midpoints of the line segments AB, BC and CA respectively.
Determine a degree 6 polynomial equation with integer coefficient whose roots are the complex numbers represented by the points A, B, C, L, M and N.

(b) (i) Differentiate $\left(16+t^2\right)^{\frac{3}{2}}$ with respect to t. **1**

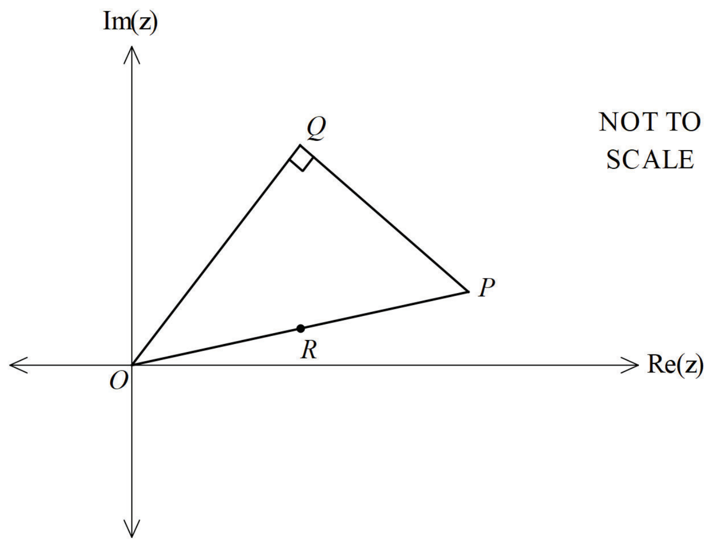
(ii) Let $I_n = \int_0^3 t^n \sqrt{16+t^2}\, dt$ for integers $n \geq 1$. Show that, for $n \geq 3$, **3**

$$(n+2)I_n = 125 \times 3^{n-1} - 16(n-1)I_{n-2}.$$

(c) A particle of mass m is projected vertically upward under gravity with speed u in a medium in which the resistance is mk times the speed, where k is a positive constant. **4**
If the particle reaches its greatest height H in time T, show that $u = gT + kH$.

Question 15 continues on page 11

(d) In the Argand diagram below, OPQ is a triangle which is right-angled at Q. The point R is the midpoint of OP.



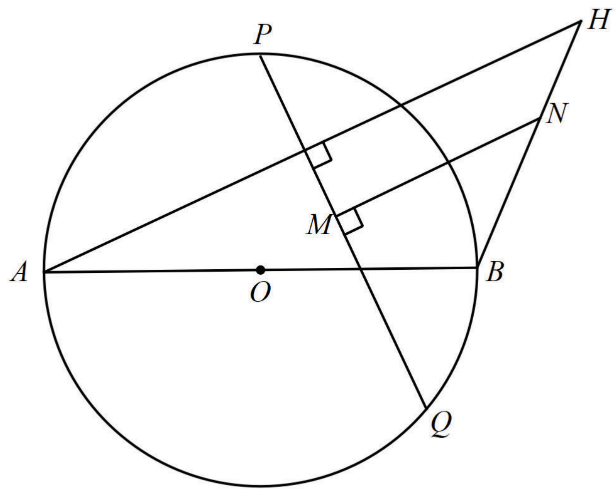
(i) If $\overrightarrow{OP} = z$ and $\overrightarrow{OQ} = w$ show that $\overrightarrow{OR} = \frac{1}{2}(1 - ki)w$ where k is a constant, $k > 0$. **2**

(ii) Express $\overrightarrow{RQ}$ in terms of w and hence show $\left|\overrightarrow{OR}\right| = \left|\overrightarrow{RQ}\right|$. **2**

End of Question 15

Question 16 (13 marks) **Marks**

(a)



NOT TO SCALE

In the diagram, AB is a diameter of a circle with centre O and PQ is a chord of the circle that is not perpendicular to AB. The perpendicular from A to PQ is produced to the point H outside the circle. M is the midpoint of PQ and N is the point on BH such that $MN \perp PQ$. Let $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OP} = \underset{\sim}{p}$, $\overrightarrow{OQ} = \underset{\sim}{q}$ and $\overrightarrow{AH} = \underset{\sim}{h}$.

(i) By writing $\overrightarrow{OM}$ in terms of $\underset{\sim}{p}$ and $\underset{\sim}{q}$, show that the points O, M and N are collinear. **2**

(ii) If $\overrightarrow{BN} = \lambda \overrightarrow{BH}$ for some scaler λ, express $\overrightarrow{ON}$ in terms of $\underset{\sim}{a}$, $\underset{\sim}{h}$ and λ. **2**

(iii) Hence, show that N is the midpoint of BH. **3**

(b) (i) By considering the graph of $y = \dfrac{1}{x\sqrt{x}}$, or otherwise, show that for all positive integers $k \geq 1$, $\dfrac{1}{(k+1)\sqrt{k+1}} < \dfrac{2}{\sqrt{k}} - \dfrac{2}{\sqrt{k+1}}$. **3**

(ii) Hence, use Mathematical Induction to show that for all positive integers $n \geq 2$, **3**

$$\frac{1}{1\sqrt{1}} + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \ldots + \frac{1}{n\sqrt{n}} < 3 - \frac{2}{\sqrt{n}}.$$

END OF PAPER