STUDENT'S NAME: ________________________________

TEACHER'S NAME: ________________________________

2025

HURLSTONE AGRICULTURAL HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE ASSESSMENT TASK 4

TRIAL EXAMINATION

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using a black or blue pen.
- NESA approved calculators may be used.
- A reference sheet is provided at the end of this question booklet.
- For questions in Section II, show all relevant mathematical reasoning and/or calculations.
- This examination paper is not to be removed from the examination centre.

Total marks: 100

Section I – 10 marks (pages 2 to 6)

- Attempt Questions 1 – 10. The multiple-choice answer sheet has been provided at the end of this question booklet.
- Allow about 15 minutes for this section.

Section II – 90 marks (pages 7 to 12)

- Attempt Questions 11 – 16, write your solutions in the answer booklets provided. Extra working pages are available if required.
- Allow about 2 hours and 45 minutes for this section.

Section 1

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. Let $z = a + ib$, where a and b are real non-zero numbers.
If $z + \frac{1}{z} \in \mathrm{R}$, which of the following must be **true**?

A. $\arg z = \frac{\pi}{4}$

B. $a = -b$

C. $a = b$

D. $|z| = 1$

2. Multiplying a non-zero complex number by $\frac{1+i}{1-i}$ results in a rotation about the origin on an Argand diagram.

What is the rotation?

A. Clockwise by $\frac{\pi}{4}$

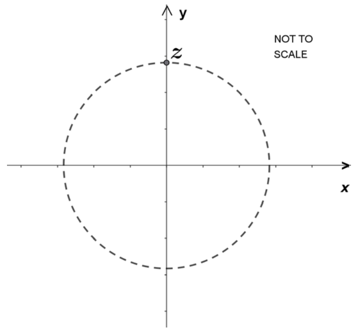
B. Clockwise by $\frac{\pi}{2}$

C. Anticlockwise by $\frac{\pi}{4}$

D. Anticlockwise by $\frac{\pi}{2}$

~ Section I continues ~

3. The diagram shows a point z on the circle $x^2 + y^2 = 2$ in an Argand Diagram. What value does z represent?



A. $2i$

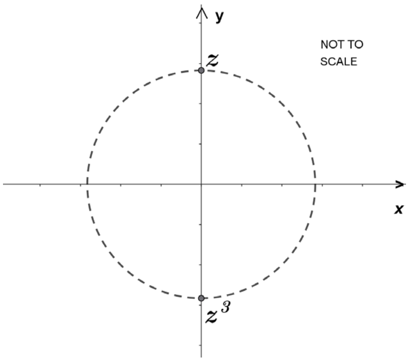
B. $i\sqrt{2}$

C. $2+2i$

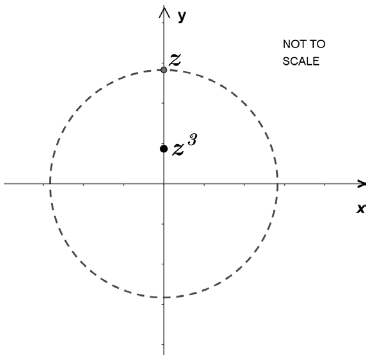
D. $-i\sqrt{2}$

4. Using the same diagram of z as in question 3, which of the following best indicates the position of z^3?

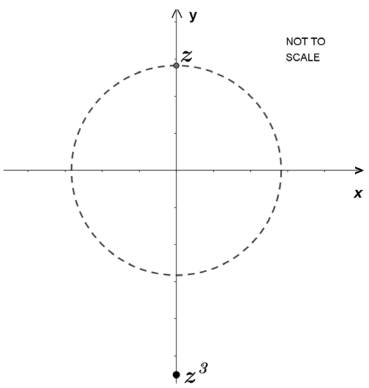
A.



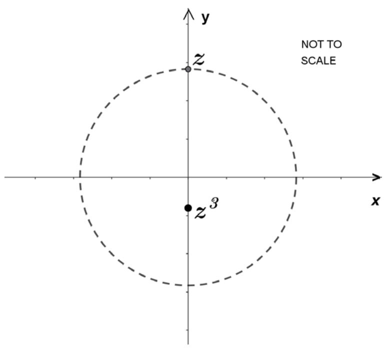
B.



C.



D.



~ Section I continues ~

5. It is given that a, b, c and d are consecutive integers.
Which of the following statements *may* be false?

A. $abcd$ is divisible by 3

B. $abcd$ is divisible by 8

C. $a+b+c+d$ is divisible by 2

D. $a+b+c+d$ is divisible by 4

6. Which of the following statements is true?

A. $\forall a,b \in \mathrm{R},\ \sin a = \sin b \Rightarrow a = b$

B. $\forall a,b \in \mathrm{R},\ |a+b| > |a-b|$

C. $\exists a,b \in \mathrm{R}$ such that $\log_a(a+b) = \log_a(ab)$

D. $\exists a,b \in \mathrm{C},\ |a+b| > |a|+|b|$

7. What is the value of p for which the vectors $\underset{\sim}{i} - 2p\underset{\sim}{j} + 3\underset{\sim}{k}$ and $p\underset{\sim}{i} - 4\underset{\sim}{j} + 3\underset{\sim}{k}$ are perpendicular?

A. -1

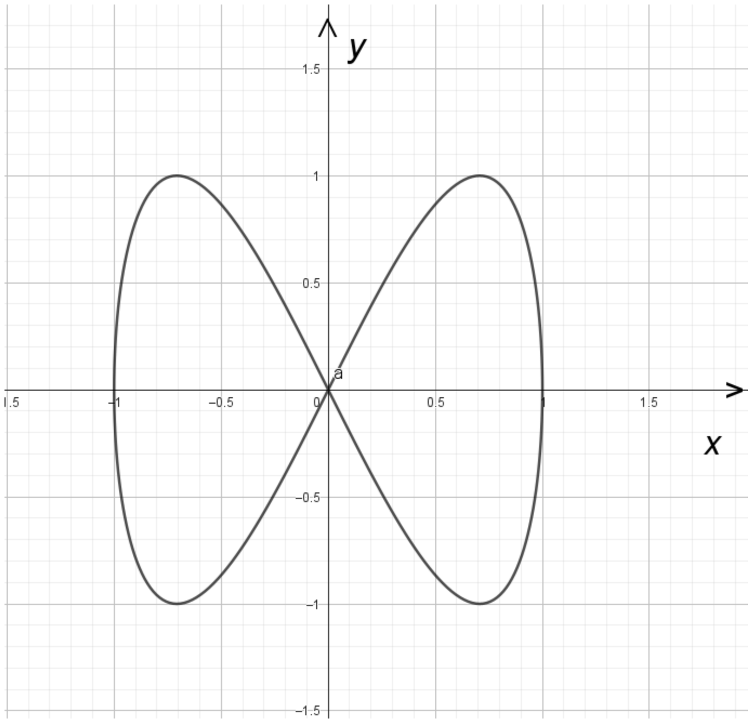
B. 2

C. -2

D. -3

~ Section I continues ~

8. Which of the following parametrisations will give the curve shown for $0 \le t \le 2\pi$?



A. $\underset{\sim}{r}(t) = (\sin 2t)\underset{\sim}{i} + (\sin t)\underset{\sim}{j}$

B. $\underset{\sim}{r}(t) = (\sin 2t)\underset{\sim}{i} + (\cos t)\underset{\sim}{j}$

C. $\underset{\sim}{r}(t) = (\sin t)\underset{\sim}{i} + (\cos 2t)\underset{\sim}{j}$

D. $\underset{\sim}{r}(t) = (\sin t)\underset{\sim}{i} + (\sin 2t)\underset{\sim}{j}$

9. If $\int_1^4 f(x)dx = k$ for some constant k, what is the value of $\int_1^4 f(5-x)dx$?

A. $-k$

B. $5-k$

C. $k+5$

D. k

10. It is given that $f(x)$ is a non-zero even function and $g(x)$ is a non-zero odd function,

Which expression is equal to $\int_{-a}^{a} f(x)+g(x)dx$?

A. $2\int_{0}^{a} f(x)dx$

B. $2\int_{-a}^{a} g(x)dx$

C. $\int_{-a}^{a} g(x)dx$

D. $\int_{0}^{a} f(x)+g(x)dx$

~ END OF SECTION I ~

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section.

Answer the questions in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet. **MARKS**

a) Let $z = 5 - 12i$ and $w = 3 + 4i$.

(i) Find $z + \overline{w}$ in the form $a + bi$. **1**

(ii) Show that $\frac{z}{w} = -\frac{33}{25} - \frac{56}{25}i$ **1**

b) On an Argand diagram, sketch the region where the inequalities

$$|z| < 3 \text{ and } -\frac{2\pi}{3} \leq \arg(z+2) \leq \frac{\pi}{6} \text{ both hold.}$$

It is not necessary to find the intersection points. **3**

c) Let there be two arbitrary complex numbers *z* and *w*.

(i) Sketch the complex numbers *z*, *w* and *z* + *w* on an Argand diagram, representing each as a vector. **1**

(ii) Justify why $|z + w| \leq |z| + |w|$. **1**

(iii) If $|z| = 10$, prove that $\left|13z^2 - 2z + 5\right| \leq 1325$. **2**

d) If $z = 1 + i$ is a root of the equation $z^3 + pz^2 + qz + 6 = 0$ where *p* and *q* are real,

Find the values of *p* and *q*. **3**

e) The points *A* and *B* represent the complex numbers $z_1 = 2 - i$ and $z_2 = 8 + i$ respectively.

Find all possible complex numbers z_3, represented by *C* such that ΔABC is **isosceles** and **right-angled** at *C*. **3**

~ End of Question 11 ~

Question 12 (15 marks) Use the Question 12 Writing Booklet. **MARKS**

a) (i) Express $z=\sqrt{3}-i$ in exponential form. **1**

(ii) Express z^5 in the form $a+bi$. **2**

(iii) For $n>0$, what is the least value of n that will make z^n a positive real number? **1**

b) (i) Solve $z^3+1=0$ **2**

(ii) Let w be the value of z with the smallest positive argument.

Show that $w^2-w+1=0$ **1**

(iii) Hence, simplify $\left(1+w^2\right)^3\left(w-1\right)^2$ **3**

c) Let $z=\cos\theta+i\sin\theta$.

(i) Show that $z^n+z^{-n}=2\cos n\theta$. **1**

(ii) Expand $\left(z+z^{-1}\right)^4$. **1**

(iii) Use parts (i) and (ii) to show that

$$8\cos^4\theta=\cos 4\theta+4\cos 2\theta+3$$ **3**

~ End of Question 12 ~

Question 13 (15 marks) Use the Question 13 Writing Booklet. **MARKS**

a) Assuming $n > 0$, prove using contrapositive that if $4^n - 1$ is prime, then n is odd. **2**

b) If x and y are real and unequal, then $x^2 + y^2 > 2xy$. DO NOT PROVE THIS.

Deduce that for a, b and c, which are real and unequal, that:

(i) $a^2 + b^2 + c^2 > ab + bc + ac$. **1**

(ii) If $a + b + c = 6$, show that $ab + bc + ac < 12$. **2**

c) Consider the statement: $\forall a, b \in \mathbb{Z}^+,\ a^2 - 4b \neq 2$

(i) Write the *negation* of this statement in words. **1**

(ii) Use proof by contradiction to prove the statement is true. **3**

d) (i) By considering the integral $\int_k^{k+1} \frac{1}{x\sqrt{x}}\,dx$, or otherwise, show that for all positive integers $k \geq 1$, $\frac{1}{(k+1)\sqrt{k+1}} < \frac{2}{\sqrt{k}} - \frac{2}{\sqrt{k+1}}$. **2**

(ii) Hence, use the process of mathematical induction to prove that for all positive integers $n \geq 2$,

$$\frac{1}{1\sqrt{1}} + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \ldots + \frac{1}{n\sqrt{n}} < 3 - \frac{2}{\sqrt{n}}.$$ **4**

~ End of Question 13 ~

Question 14 (15 marks) Use the Question 14 Writing Booklet. **MARKS**

a) Find the vector equation of the straight line that passes through $(2,10,-3)$ and is parallel to the vector $2\underset{\sim}{i}-3\underset{\sim}{k}$. **1**

b) Determine whether the points $A(0,2,2)$, $B(4,10,18)$, and $C(6,14,26)$ are collinear. **3**

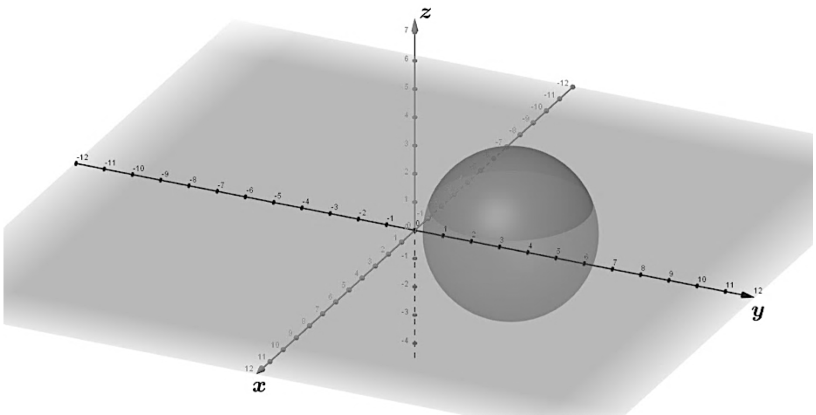
c) Find the distance of the point $P(7,6,-2)$

(i) from the x-axis. **1**

(ii) From the point $(0,-4,5)$. **1**

d) This diagram shows a sphere that satisfies the equation:

$$x^2+y^2+z^2+6x-4y+2z+6=0.$$



(i) Find the coordinates of its centre and its radius. **2**

(ii) Find the Cartesian equation of the circle in which the sphere intersects the x-y plane. **2**

e) Consider 2 vectors, $\underset{\sim}{a}=\begin{pmatrix}3\\1\\2\end{pmatrix}$ and $\underset{\sim}{b}=\begin{pmatrix}1\\-3\\4\end{pmatrix}$.

(i) Find $\cos\theta$ as an exact value, where θ is the angle between the 2 vectors. **2**

(ii) Hence, find the exact area of the parallelogram that has vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ as diagonals. **3**

~ End of Question 14 ~

Question 15 (15 marks) Use the Question 15 Writing Booklet. **MARKS**

a) Using a relevant substitution, find $\int \frac{e^x}{e^{2x}+9}dx$. **2**

b) Evaluate $\int_0^2 x^2 \ln x dx$. **3**

c) (i) Differentiate $\sin^{-1}(x) - \sqrt{1-x^2}$. Give your answer in simplest form. **2**

(ii) Hence, show that $\int_0^{\alpha} \sqrt{\frac{1+x}{1-x}}dx = \sin^{-1}\alpha + 1 - \sqrt{1-\alpha^2}$ for $0 < \alpha < 1$. **2**

d) Let $I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx$.

(i) Show that $I_n = \frac{n-1}{n} I_{n-2}$, for $n \geq 2$. **3**

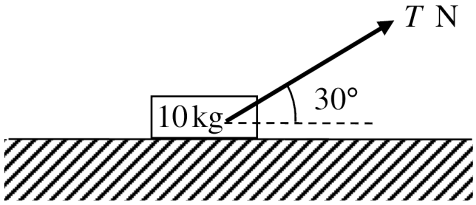
(ii) Hence, show that $I_{2n} = \frac{\pi(2n)!}{2^{2n+1}(n!)^2}$. **3**

~ End of Question 15 ~

Question 16 (15 marks) Use the Question 16 Writing Booklet. **MARKS**

a) At time t the displacement, x, of a particle satisfies $t = 4 - e^{-2x}$.
Find the acceleration of the particle as a function of x. **3**

b) A 10 kg box sits on frictionless ground and is subjected to a force of T N inclined 30° up from the horizontal.



If the box starts from rest and moves to the right, find its velocity after t seconds in terms of T. **2**

c) A particle is moving along the x-axis. The square of its velocity is given by

$$v^2 = -2x^4 + 14x^2 - 24$$

Where v m/s is the velocity and x is the displacement in metres from the origin. Initially the particle is stationary at $x = 2$.

(i) Show that the acceleration of the particle is $a = -4x\left(x^2 - \frac{7}{2}\right)$. **1**

(ii) Explain why the object starts moving in the negative direction. **1**

(iii) Where does the particle next come to rest? **2**

d) A particle is moving horizontally. Initially, the particle is at the origin moving with velocity 1 m/s.

The acceleration of the particle is given by

$$a = x - 1$$

where x is the displacement at time t.

(i) Show that the velocity of the particle is given by $v = 1 - x$. **3**

(ii) Find an expression for x as a function of t. **2**

(iii) Find the limiting position of the particle. **1**

~ End of Question 16 ~

~ End of Examination ~

2025 Yr12 HSC Assessment Task 4	
Multiple Choice	Solutions and Marking Guidelines

Solutions

1)

$$z+\frac{1}{z}=z+\frac{\overline{z}}{|z|}$$

$$=a+ib+\frac{a-ib}{\sqrt{a^2+b^2}} \quad (\text{to be } \in R,\ \text{coefficient of } i \text{ needs to be } 0)$$

$$b-\frac{b}{\sqrt{a^2+b^2}}=0$$

$$\sqrt{a^2+b^2}=1$$

$$|z|=1$$

Option D

2)

$$\frac{1+i}{1-i}\times\frac{1+i}{1+i}=\frac{2i}{1+1}=i$$

Which results in anticlockwise rotation by $\frac{\pi}{2}$.

Option D

3)

Modulus = $\sqrt{2}$; argument = $\frac{\pi}{4}$; $z=i\sqrt{2}$

Option B

4)

Modulus is cubed to give $2\sqrt{2}$, which is an increased modulus.
Argument is multiplied by 3.

Option C

5)

noting that A, B, C are obviously true 😊

Check D:

$$n+(n+1)+(n+2)+(n+3)$$

$$=4n+6$$

$$=4(n+1)+2$$

$\therefore$ *not* divisible by 4

Option D

6)

$\forall$ can be disproved with counter example.

So, A: $\sin 30° = \sin 150°$, but $30 \neq 150$... so not true

B: $\left|5+(-3)\right| > \left|5-(-3)\right|$

$|2| > |8|$ ... not true

C: $\exists$ ... find *one* example that works

$\ln(2+2) = \ln(2\times 2)$

$\ln 4 = \ln 4$ which *is* true

D: not true as A: $|a|+|b| \geq |a+b|$ by the triangle inequality $\forall$C

Option C

7)

Dot product needs to equal zero:

$p+8p+9=0 \rightarrow \quad p=-1$

Option A

8)

In one "lap" of the curve, the *x* parameter completes 1 period and the *y* parameter completes 2 periods. Hence *A* and *B* are eliminated.
Substituting *t=0* means the curve must pass through (0,0).

Option D

9)

Let $u = 5 - x$

$$\frac{du}{dx} = -x$$

When $x=1$ then $u=4$
When $x=4$ then $u=1$

$$-\int_4^1 f(u)\,du = \int_1^4 f(u)\,du = \int_1^4 f(x)\,dx = k$$

Option D

10)

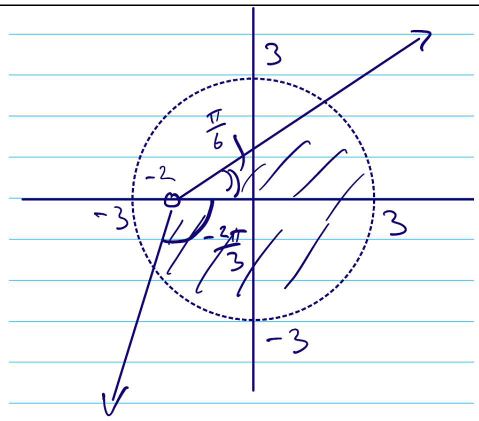
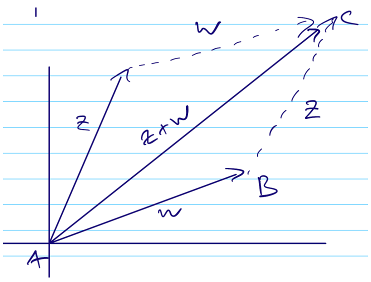
Since the odd function's integral will equal zero, it is just the value of the even function's integral.

Option A

Year 12	Mathematics Extension 2	Ass Task 4 2025 HSC
Question 8	Solutions and Marking Guidelines	TRIAL HSC

Outcomes Addressed in this Question

MEX 12-4 uses the relationship between algebraic and geometric representations of complex number techniques to prove results, model and solve problems.

Question	Solutions	Marking Guidelines
(a)(i)	$z+\overline{w}=5-12i+3-4i$ $8-16i$	<u>1 mark</u>: correct solution
(a)(i)	$\dfrac{z}{w}=\dfrac{5-12i}{3+4i}$ $=\dfrac{5-12i}{3+4i}\times\dfrac{3-4i}{3-4i}$ * required $=\dfrac{15-20i-36i+48i^2}{9+16}$ $=\dfrac{15-56i-48}{9+16}$ * at least one of these lines was also required $=\dfrac{-33-56i}{25}=\dfrac{-33}{25}-\dfrac{56i}{25}$	<u>1 mark</u>: correct solution This is a *show* question
(b)	 	<u>3 marks</u>: *fully* correct solution <u>2 marks</u>: substantially correct solution <u>1 mark</u>: partially correct solution
(c)(i)	 	<u>1 mark</u>: correct diagram
(ii)	The sum of two sides of a triangle (ΔABC) is always longer than the third side. (The sum of two "sides" = the third "side" if *A, B, C* are collinear) $\therefore \quad \lvert AC\rvert \le \lvert AB\rvert + \lvert BC\rvert$ ie $\lvert z+w\rvert \le \lvert z\rvert + \lvert w\rvert$	<u>1 mark</u>: legitimate statement Just stating "*triangle inequality*" was not sufficient… what does that *mean*

<table>
<tr>
<td>(iii)</td>
<td>Question 8 continued

$|13z^2-2z+5| \le |13z^2|+|-2z|+|5|$
$=13|z^2|+2|z|+5$
$=13(100)+2(10)+5$
$=1325$</td>
<td><u>2 marks</u>: correct solution
Correct treatment of $|-2z|$ was required for full marks
ie $+|-2z| = +|-2||z| = +2|z|$
(not $-2|z|$)
<u>1 mark</u>: substantially correct solution
Some use of the result from (ii) was required.
Simply subbing in $z = 10$ to get 1325 was not worth any marks.</td>
</tr>
<tr>
<td>(d)</td>
<td>$z_1 = 1+i$ is a root
$z_2 = \overline{z_1} = 1-i$ is also a root ($P(x)$ has real coefficients)

$z_1 z_2 z_3 = -\frac{d}{a}$ (product of roots)
$(1+i)(1-i)z_3 = -\frac{6}{1}$
$(1+1)z_3 = -6$
$z_3 = -3$

$z_1+z_2+z_3 = -\frac{b}{a}$ (sum of roots)
$(1+i)+(1-i)-3 = -\frac{p}{1}$
$2-3 = -p$
$p = 1$

sub $z=-3$
into $z^3+z^2+qz+6=0$
$-27+9-3q+6=0$
$3q=-12$
$q=-4$ $\therefore p=1,\ q=-4$</td>
<td><u>3 marks</u>: correct solution
ie correctly finding p <u>and</u> q

<u>2 marks</u>: substantially correct solution
ie correctly finding p <u>or</u> q

<u>1 mark</u>: partially correct solution
Noting z & $\overline{z}$ are roots, <u>and</u> attempting to use product or sum of roots to find third root… or equivalent merit</td>
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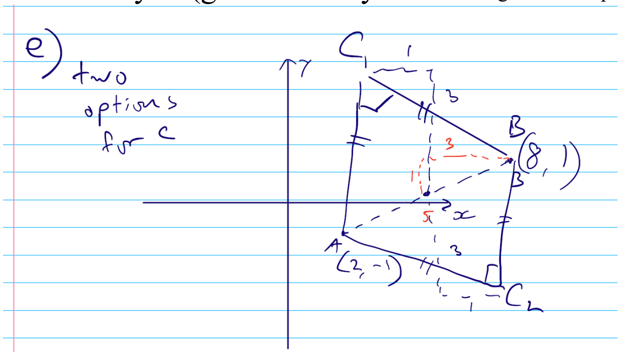
<td>(e)</td>
<td>There's a myriad of ways to do this question.
The best way… (geometrically… even using first attempt messy diagram 😊)


oh… AC_1BC_2 is a square
midpoint AB is $(5, 0)$
$C_1\ (5-1, 0+3) = (4, 3)$
$C_2\ (5+1, 0-3) = (6, -3)$</td>
<td><u>3 marks</u>: correct solution

<u>2 marks</u>: substantially correct solution
Eg vector attempt finds equations (1) and (2)
<u>1 mark</u>: partially correct solution
Eg vector attempt using dot product that doesn't find solutions</td>
</tr>
</table>

Question 8 (e) alternate methods	An algebraic version using complex numbers…	

$$C_1B = iC_1A$$
$$8+i-z_3 = i(2-i-z_3)$$
$$8+i-z_3 = 2i+1-iz_3$$
$$7-i = z_3 - iz_3$$
$$z_3 = \frac{7-i}{1-i}\times\frac{1+i}{1+i}$$
$$= \frac{7+7i-i+1}{1+1}$$
$$= \frac{8+6i}{2}$$
$$= 4+3i$$

$$C_2A = iC_2B$$
$$2-i-z_3 = i(8+i-z_3)$$
$$2-i-z_3 = 8i-1-iz_3$$
$$3-9i = z_3 - iz_3$$
$$z_3 = \frac{3-9i}{1-i}\times\frac{1+i}{1+i}$$
$$= \frac{3+3i-9i+9}{1+1}$$
$$= \frac{12-6i}{2}$$
$$= 6-3i$$

Vectors…

$$AC \perp BC$$
so $\overrightarrow{AC}\cdot\overrightarrow{BC} = 0$
$$\begin{pmatrix} x-2 \\ y+1 \end{pmatrix}\cdot\begin{pmatrix} x-8 \\ y-1 \end{pmatrix} = 0$$
$$(x-2)(x-8)+(y+1)(y-1) = 0$$
$$x^2-10x+y^2+15 = 0 \quad ... \ (1)$$

$$AC = BC$$
so $\sqrt{(x-2)^2+(y+1)^2} = \sqrt{(x-8)^2+(y-1)^2}$
$$x^2-4x+4+y^2+2y+1 = x^2-16x+64+y^2-2y+1$$
$$12x+4y-60 = 0$$
$$y = 15-3x \quad ... \ (2)$$

Sub (2) → (1)
$$x^2-10x+y^2+15 = 0$$
$$x^2-10x+(15-3x)^2+15 = 0$$
$$x^2-10x+225-90x+9x^2+15 = 0$$
$$10x^2-100x+240 = 0$$
$$x^2-10x+24 = 0$$
$$(x-4)(x-6) = 0$$
$$x = 4,\ 6$$

Which gives $(4,\ 3)$ and $(6,\ -3)$
ie $4+3i$ and $6-3i$

Question 12

Outcomes Addressed in this Question

MEX12-4 uses the relationship between algebraic and geometric representations of complex numbers and complex number techniques to prove results, model and solve problems

Question	Solutions	Marking Guidelines

(a)(i)

(i) $z = 2e^{\frac{-\pi i}{6}}$

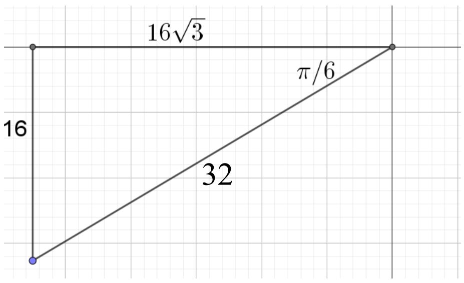
(a)(i) 1 mark: Correct answer.

(ii)

(ii) $z^5 = 2^5\left(e^{\frac{-\pi i}{6}}\right)^5$

$= 32e^{\frac{-5\pi i}{6}}$

$= -16\sqrt{3} - 16i$ (quadrant diagram below)



(ii) 2 marks: Correct solution. CFPA
1 mark: Correct use of DeMoivre's theorem;
or:
1 mark: Correct conversion from exponential to rectangular form.

(iii)

(iii) $\frac{-\pi}{6}n = -2k\pi$ When $k = 1, \frac{n\pi}{6} = 2\pi$, so

z^n will have an argument of -2π when $n = 12$.

(iii) 1 mark: Correct answer. CFPA

(b)(i)

(i) Solutions are the cube roots of -1
The real solution is $z = -1$ and the other solutions are obtained by dividing the unit circle into 3 sections with angles of $\frac{2\pi}{3}$ at the centre.

i.e. $z = -1, \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}, \cos\left(\frac{-\pi}{3}\right) + i\sin\left(\frac{-\pi}{3}\right)$

(Or equivalent answers in alternate form.)

(b)(i) 2 marks: All 3 roots correct.
1 mark: At least 1 root correct.

(ii)

(ii) $w = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

Method 1:

$z^3 + 1 = 0$

$(z+1)(z^2 - z + 1) = 0$

This has w as a solution, where $w \neq -1$. Therefore, w must satisfy $w^2 - w + 1 = 0$

When using this factorisation, you <u>must</u> say why the factor $(w+1)$ is discarded.

(ii) 1 mark: Correct solution (either method).

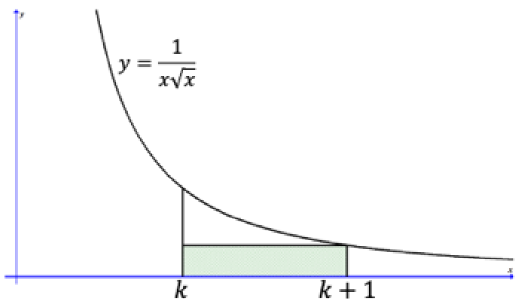
	Method 2: $w^2 - w + 1 = \left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) - \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) + 1$ $= \frac{-1}{2} + \frac{i\sqrt{3}}{2} - \frac{1}{2} - \frac{i\sqrt{3}}{2} + 1$ $= 0$	
(iii)	(iii) From part (ii): $1 + w^2 = w$ and $w - 1 = w^2$ Therefore: $\left(1+w^2\right)^3\left(w-1\right)^2 = \left(w\right)^3\left(w^2\right)^2$ $= w^7$ $= \left(w^3\right)^2\left(w\right)$ $= \left(-1\right)^2 w$ $= w$	**(iii) 3 marks:** Solution showing: simplification of initial factors; use of $w^3 = -1$; and correct final answer. **2 marks:** 2 of the above components correct. **1 mark:** 1 of the above components correct.
(c)(i)	(i) $\lvert z\rvert = 1 \quad \therefore \frac{1}{z} = \bar{z}$ $z^n + z^{-n} = \left(\cos\theta + i\sin\theta\right)^n + \left(\cos\theta - i\sin\theta\right)^n$ $= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$ $= 2\cos n\theta$	**(c)(i) 1 mark:** Correct solution.
(ii)	(ii) $\left(z+z^{-1}\right)^4 = z^4 + 4z^3z^{-1} + 6z^2z^{-2} + 4zz^{-3} + z^{-4}$ $= z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4}$ Note: The above solution is the "expansion" of the LHS. This was not where you should have expressed $\left(z+z^{-1}\right)^4$ in the terms shown in part (i). Read all 3 parts of the question before attempting it to try and avoid this scenario. The marker was very generous in awarding the "expansion" mark whether it was sighted in part (ii) or part (iii).	**(ii) 1 mark:** Correct solution.
(iii)	(iii) Let $n = 1$ in part (i) and equating with part (ii) gives: (Rearrange terms in (ii)) $\left(z+z^{-1}\right)^4 = \left(z^4 + z^{-4}\right) + 4\left(z^2 + z^{-2}\right) + 6$ $\left(2\cos\theta\right)^4 = 2\cos 4\theta + 4\left(2\cos 2\theta\right) + 6$ $16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$ $8\cos^4\theta = \cos 4\theta + 4\cos 2\theta + 3$	**(iii) 3 marks:** Simplify both sides of the equation correctly, leading to an expression for $16\cos^4\theta$. **2 marks:** One component of solution not shown. **1 mark:** One correct component.

Year 12	Mathematics Extension 2	Ass Task 4 2025 HSC
Question 13	Solutions and Marking Guidelines	TRIAL HSC

Outcomes Addressed in this Question

MEX 12-2 chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings

Question	Solutions	Marking Guidelines
(a)	Original statement: "If $4^n - 1$ is prime, then n is odd." Contrapositive: "If n is not odd *(ie even)*, then $4^n - 1$ is not prime. *(ie composite)* Suppose $n > 0$ is even ie $n = 2k$ where k is a positive integer $4^n - 1 = 4^{2k} - 1$ $a^2 + b^2 > 2ab$ — ① $b^2 + c^2 > 2bc$ — ② $a^2 + c^2 > 2ac$ — ③ ① + ② + ③ $2a^2 + 2b^2 + 2c^2 > 2ab + 2bc + 2ac$ *** $2(a^2 + b^2 + c^2) > 2(ab + bc + ac)$ $\therefore a^2 + b^2 + c^2 > ab + bc + ac$ (as required) ① tion)	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution Correctly *identifying* the contrapositive, or equivalent merit. Notes: justifying using examples is not the same as a proof. Also, if you are trying induction in a 2 mark question, something as been missed. *** this is an important statement, and omitting it should have prevented full marks being awarded. In this rare occasion its absence did not prevent full marks… but this is a rarity!
(b)(i)	$a^2 + b^2 > 2ab \quad ...(1)$ $b^2 + c^2 > 2bc \quad ...(2)$ $a^2 + c^2 > 2ac \quad ...(3)$ $2a^2 + 2b^2 + 2c^2 > 2ab + 2bc + 2ac \qquad (1) + (2) + (3)$ $a^2 + b^2 + c^2 > ab + bc + ac$	<u>1 mark</u>: *fully* correct solution
(ii)	$(a + b + c)^2 = 6^2$ $a^2 + b^2 + c^2 + 2(ab + ac + bc) = 36$ $ac + bc + ac + 2(ab + ac + bc) < 36 \qquad (\mathit{from}\ (i))$ $3(ab + ac + bc) < 36$ $ab + ac + bc < 12$	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution

(c)(i)	"There exists positive integers, a, b such that $a^2 - 4b = 2$.	<u>1 mark</u>: correct statement
(ii)	from (i)... $a^2 - 4b = 2$ $a^2 = 4b + 2$ $= 2(2b+2)$ $\therefore\ a^2$ is even. so $a = 2k$ for some integer k from $a^2 = 2(2b+1)$ $(2k)^2 = 2(2b+1)$ $4k^2 = 2(2b+1)$ $2k^2 = 2b+1$ LHS is even, and RHS is odd Which is a contradiction So our assumption (in (i)) is incorrect $\therefore \forall a,b \in Z^+,\ a^2 - 4b \neq 2$	<u>3 marks</u>: correct solution <u>2 marks</u>: substantially correct solution <u>1 mark</u>: partially correct solution Note: writing the initial statement incorrectly (in (i)) as a 'for all' statement generally meant only 1 mark was available for this part. Finding a counterexample to the $\forall$ statement is actually just giving one example where the initial given statement is true. An incorrect negation statement generally prevented a coherent contradiction proof being possible.
(d)(i)	 Area of shaded rectangle is $\frac{1}{(k+1)\sqrt{k+1}}$ $\text{Area of rectangle} < \int_k^{k+1} \frac{1}{x\sqrt{x}}\,dx$ $1 \times \frac{1}{(k+1)\sqrt{k+1}} < \left[-\frac{2}{\sqrt{x}}\right]_k^{k+1}$ $\frac{1}{(k+1)\sqrt{k+1}} < -\frac{2}{\sqrt{k+1}} - -\frac{2}{\sqrt{k}}$ $\frac{1}{(k+1)\sqrt{k+1}} < \frac{2}{\sqrt{k}} - \frac{2}{\sqrt{k+1}}$	<u>2 marks</u>: correct solution ***Correctly*** obtains inequality <u>1 mark</u>: substantially correct solution ***(Attempts)*** to obtain inequality Note: the integral in this question is a straightforward Maths *Advanced* integral. (integration by parts definitely ***NOT*** needed here!!!) While it was required, it was not enough for the first mark. Looking for the inequality relationship, *in conjunction with the integral* is where the first mark was awarded.

(d)(ii)	Let S_n be the statement $\dfrac{1}{1\sqrt{1}}+\dfrac{1}{2\sqrt{2}}+\dfrac{1}{3\sqrt{3}}+\ldots+\dfrac{1}{n\sqrt{n}}<3-\dfrac{2}{\sqrt{n}}$ **<u>Step 1</u>** – show true for $n = 2$ (base case) $\text{LHS}=\dfrac{1}{1\sqrt{1}}+\dfrac{1}{2\sqrt{2}} \approx 1.35355\ldots$ $\quad\quad$ $\text{RHS}=3-\dfrac{2}{\sqrt{2}} \approx 1.58579\ldots$ $\text{LHS}<\text{RHS} \quad \therefore S_2$ is true **<u>Step 2</u>** – assume S_k is true ie $\quad \dfrac{1}{1\sqrt{1}}+\dfrac{1}{2\sqrt{2}}+\dfrac{1}{3\sqrt{3}}+\ldots+\dfrac{1}{k\sqrt{k}}<3-\dfrac{2}{\sqrt{k}}$ **<u>Step 3</u>** – prove S_{k+1} is true ie, prove $\dfrac{1}{1\sqrt{1}}+\dfrac{1}{2\sqrt{2}}+\dfrac{1}{3\sqrt{3}}+\ldots+\dfrac{1}{k\sqrt{k}}+\dfrac{1}{(k+1)\sqrt{k+1}}<3-\dfrac{2}{\sqrt{k+1}}$ $\text{LHS}=\dfrac{1}{1\sqrt{1}}+\dfrac{1}{2\sqrt{2}}+\dfrac{1}{3\sqrt{3}}+\ldots+\dfrac{1}{k\sqrt{k}}+\dfrac{1}{(k+1)\sqrt{k+1}}$ $<3-\dfrac{2}{\sqrt{k}}+\dfrac{1}{(k+1)\sqrt{k+1}} \quad \left(\text{from } S_k\right)$ $<3-\dfrac{2}{\sqrt{k}}+\dfrac{2}{\sqrt{k}}-\dfrac{2}{\sqrt{k+1}} \quad \left(\text{from part(i)}\right)$ $=3-\dfrac{2}{\sqrt{k+1}}$ ie $\text{LHS}<\text{RHS}$ $\therefore$ true by the process of Mathematical Induction	<u>4 marks</u>: correct proof, including these details: Must ***<u>actually show</u>*** LHS>RHS in base case, correctly. At least show decimal equivalent, or simplify surds to a clear comparison. Must also reference, at least, when the assumption ***<u>or</u>*** result in part (i) was introduced. (Ideally both) Correct use of inequalities as results are introduced <u>3 marks</u>: substantially correct solution Makes use of part (i), or equivalent merit. Question sis HENCE!!! Ie part (i) needed to be used <u>2 marks</u>: partially correct solution Makes *some* progress towards an inductive style argument, or equivalent merit <u>1 mark</u>: partially correct solution Establishes the truth of S_2, or equivalent merit

Note: many candidates seem to think that $k+1\sqrt{k+1}=(k+1)\sqrt{k+1}$

Year 8 mathematics confirms this is not the case!

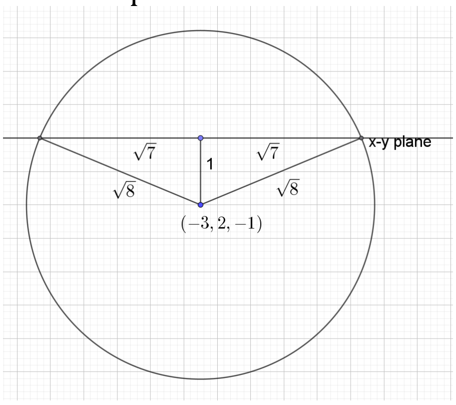
Solutions and Marking Guidelines

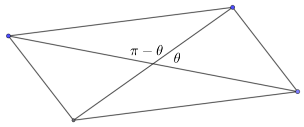
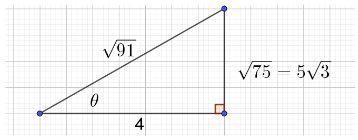
Question 14

Outcomes Addressed in this Question

MEX12-3 uses vectors to model and solve problems in two and three dimensions.

<table>
<tr><th>Part</th><th>Solutions</th><th>Marking Guidelines</th></tr>
<tr><td>(a)</td><td>$\underset{\sim}{r} = \begin{pmatrix} 2 \\ 10 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$</td><td>(a)1 mark: Correct answer</td></tr>
<tr><td>(b)</td><td>$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 10 \\ 18 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 16 \end{pmatrix}$
$\overrightarrow{BC} = \begin{pmatrix} 6 \\ 14 \\ 26 \end{pmatrix} - \begin{pmatrix} 4 \\ 10 \\ 18 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix}$
$\therefore \overrightarrow{AB} = 2\overrightarrow{BC}$, so the 2 vectors are parallel.
This is only a sample solution: you could include $\overrightarrow{AC}$ in your calculations.
<u>They also have a common point, B, so A, B and C are collinear.</u>
Stating why 2 vectors are parallel does not prove that they are collinear.</td><td>(b) 2 marks: Solution showing that a pair of vectors is parallel <u>and</u> has a named common point.
1 mark: 1 component performed correctly.</td></tr>
<tr><td>(c) (i)</td><td>(i) Note that following the components of the vector would imply travelling along the x-axis 7 units, and then moving 6 units in the y directions, followed by -2 in the z direction. Hence the distance from the x-axis comes from the y and z components only.
$d = \sqrt{6^2 + (-2)^2} = \sqrt{40}$ units.</td><td>(c) (i) 1 mark: Correct answer.</td></tr>
<tr><td>(ii)</td><td>(ii)
$d = \sqrt{(7-0)^2 + (6+4)^2 + (-2-5)^2}$
$= \sqrt{49 + 100 + 49}$
$= \sqrt{198}$ units</td><td>(ii) 1 mark: Correct answer.</td></tr>
<tr><td>(d)(i)</td><td>(i)
$x^2 + y^2 + z^2 + 6x - 4y + 2z + 6 = 0$
$x^2 + 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1 = 8$
$(x+3)^2 + (y-2)^2 + (z+1)^2 = 8$
$\therefore$ Centre $= (-3, 2, -1)$, Radius $= \sqrt{8}$ units</td><td>(d)(i) 2 marks: Correct separation into 3 perfect squares, followed by correct solutions for centre and radius.
1 mark: Incorrect centre and/or radius from correct squares; OR
Correct radius and centre from incorrect squares.</td></tr>
</table>

(ii)	The x-y plane has the equation $z=0$, so, substituting this into the equation in (i) gives: $(x+3)^2+(y-2)^2+(0+1)^2=8$ $(x+3)^2+(y-2)^2=7$ Alternate solution: Side-on view of sphere:  The centre is 1 unit below the x-y plane, so Pythagoras' Theorem enables us to see that the radius of the circle on the x-y plane is $\sqrt{7}$ units. Its centre is $(-3,2,0)$ Cartesian Equation: $(x+3)^2+(y-2)^2=7$	**(ii) 2 marks:** Correct understanding of the x-y plane, as well as correct Cartesian equation. **1 mark:** 1 component performed correctly.
(e)(i)	(i) $\cos\theta = \dfrac{\underset{\sim}{a}.\underset{\sim}{b}}{\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert} = \dfrac{3-3+8}{\sqrt{9+1+4}\sqrt{1+9+16}}$ $= \dfrac{8}{\sqrt{14}\sqrt{26}} = \dfrac{8}{2\sqrt{7}\sqrt{13}}$ $= \dfrac{4}{\sqrt{91}}$ Note: It is customary to not have to rationalise the denominator for a trig ratio, but simplifying the surds and hence the fraction is best practice. Many who finished this question with a more complicated answer than the one above often had trouble in part (ii) because of the burden of massive surds. See how the answer above leads to the right-angled triangle in part (ii) with adjacent side =4 and hypotenuse = $\sqrt{91}$.	**(e)(i) 2 marks:** Correct equation using dot product, and correct solution. **1 mark:** 1 component performed correctly.

(ii)	(ii)   If two vectors are the diagonals of a parallelogram, they bisect each other, and the acute angle between the diagonals is the angle between the vectors. If $\cos\theta = \dfrac{4}{\sqrt{91}}$, the right triangle shows that acute θ will have $\sin\theta = \dfrac{5\sqrt{3}}{\sqrt{91}}$ Now, $\sin(\pi-\theta)=\sin\theta$, using the quadrant relationship between supplementary angles. So the diagonals split the parallelogram into 4 triangles, each of which has an area of $$\frac{1}{2}\left(\frac{\sqrt{14}}{2}\right)\left(\frac{\sqrt{26}}{2}\right)\frac{5\sqrt{3}}{\sqrt{91}} = \frac{2\sqrt{91}\times 5\sqrt{3}}{8\sqrt{91}}$$ So the area of the parallelogram is: $A = \dfrac{4\times 2\sqrt{91}\times 5\sqrt{3}}{8\sqrt{91}} = 5\sqrt{3}$ units² Note: Many applications of the sin rule were attempted here, but you could only get full marks if you communicated why the 4 triangles have equal area despite not being congruent (I.e. explain the supplementary angle relationship.) Many answers received different mark totals out of 3, because they exhibited different levels of understanding of the question. For example, finding the area of a triangle rather than that of a parallelogram would not score as many marks, even if both calculations achieved the same final answer.	**(ii) 3 marks:** Solution including: Correct understanding of the diagonal properties of the parallelogram, **<u>including the obtuse and acute angles</u>**; Correct evaluation of $\sin\theta$; Correct exact answer. CFPA **2 marks:** 2 components completed correctly. **1 mark:** 1 component performed correctly.

2025 Yr12 HSC Assessment Task 4	
Question 15 Solutions and Marking Guidelines	
Outcomes Addressed in this Question	
MEX12-5 applies techniques of integration to structured and unstructured problems.	
Solutions	**Marking Guidelines**

a)

$$u = e^x$$
$$du = e^x dx$$

$$\int \frac{e^x}{e^{2x}+9}\,dx = \int \frac{du}{u^2+9}$$
$$= \frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right)+c$$
$$= \frac{1}{3}\tan^{-1}\left(\frac{e^x}{3}\right)+c$$

2 marks
Correct solution

1 mark
Sets up substitution

b)

$$u = \ln x \qquad dv = x^2$$
$$du = \frac{1}{x}dx \qquad v = \frac{x^3}{3}$$

$$\int_0^2 x^2 \ln x dx = \lim_{b\to 0}\left[\frac{x^3}{3}\ln x\right]_b^2 - \int_0^2 \frac{x^3}{3}\frac{1}{x}dx$$
$$= \frac{8\ln(2)}{3} - \lim_{b\to 0}\frac{\ln(b)b^2}{3} - \left[\frac{x^3}{9}\right]_0^2$$
$$= \frac{8\ln(2)}{3} - 0 - \frac{2^3}{9} - 0$$
$$= \frac{8\ln(2)}{3} - \frac{8}{9}$$

3 marks
Correct solution

2 marks
Correct integral

1 mark
Sets up substitution

c i)

$$\frac{d}{dx}\left(\sin^{-1}(x) - \sqrt{1-x^2}\right) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{2\sqrt{1-x^2}}(-2x)$$
$$= \frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}$$
$$= \frac{1+x}{\sqrt{1-x^2}}$$

2 marks
Correct solution

1 mark
Both terms differentiated correctly

ii) $$\frac{1+x}{\sqrt{1-x^2}}=\frac{1+x}{(1+x)^{\frac{1}{2}}(1-x)^{\frac{1}{2}}}$$ $$=\frac{(1+x)^{\frac{1}{2}}}{(1-x)^{\frac{1}{2}}}$$ $$=\sqrt{\frac{1+x}{1-x}}$$ OR $$\sqrt{\frac{1+x}{1-x}}\times\frac{\sqrt{1+x}}{\sqrt{1+x}}=\frac{1+x}{\sqrt{1-x^2}}$$ $$\int_0^\alpha \frac{1+x}{\sqrt{1-x^2}}dx=\int_0^\alpha \sqrt{\frac{1+x}{1-x}}dx$$ $$=\left[\sin^{-1}(x)-\sqrt{1-x^2}\right]_0^\alpha$$ $$=\sin^{-1}(\alpha)-\sqrt{1-\alpha^2}-\left(0-\sqrt{1}\right)$$ $$=\sin^{-1}(\alpha)-\sqrt{1-\alpha^2}+1$$ Since the domain for $\sin^{-1}(\alpha)$ is $-1\le\alpha\le1$ and $\sqrt{1-\alpha^2}$ is $-1\le\alpha\le1$, the result is valid for $0<\alpha<1$	**2 marks** Correct evaluation of integral **1 mark** Correctly shows how part (i) expression relates to expression to integrate in part (ii)
d i) $$I_n=\int_0^{\frac{\pi}{2}}\sin^{n-1}x\sin x dx$$ $$=\left[-\sin^{n-1}x\cos x\right]_0^{\frac{\pi}{2}}+(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}x\cos^2 x dx$$ $$=(n-1)\int_0^{\frac{\pi}{2}}\sin^{n-2}x\left(1-\sin^2 x\right)dx$$ $$=(n-1)\int_0^{\frac{\pi}{2}}\left(\sin^{n-2}x-\sin^n x\right)dx$$ $$=(n-1)I_{n-2}-(n-1)I_n$$ $$I_n(1+n-1)=(n-1)I_{n-2}$$ $$I_n=\frac{n-1}{n}I_{n-2}$$	**3 marks** Recognises I_{n-2} and I_n and simplifies correctly to obtain I_n **2 marks** Correctly sets up integration by parts **1 mark** Splits powers of sine appropriately

ii)

$$
\begin{aligned}
I_{2n} &= \int_0^{\frac{\pi}{2}} \sin^{2n} x\,dx \\
&= \frac{2n-1}{2n} I_{2n-2} \\
&= \frac{2n-1}{2n} \times \frac{2n-3}{2n-2} I_{2n-4} \\
&= \frac{2n-1}{2n} \times \frac{2n-3}{2n-2} \times \ldots \times \frac{1}{2} I_0 \\
&= \underbrace{\frac{2n-1}{2n}}_{\frac{2n}{2n}\times\frac{2n-1}{2n}} \times \underbrace{\frac{2n-3}{2n-2}}_{\frac{2n-2}{2n-2}\times\frac{2n-3}{2n-2}} \times \ldots \times \frac{1}{2} \times \frac{\pi}{2} \\
&= \frac{2n}{2n} \times \frac{2n-1}{2n} \times \frac{2n-2}{2n-2} \times \frac{2n-3}{2n-2} \times \ldots \times \frac{2}{2} \times \frac{1}{2} \times \frac{\pi}{2} \\
&= \frac{(2n)!}{4n^2 \times 4(n-1)^2 \times 4(n-2)^2 \times \ldots \times 4} \times \frac{\pi}{2} \\
&= \frac{(2n)!}{4^n (n!)^n} \times \frac{\pi}{2} \\
&= \frac{\pi(2n)!}{2^{2n+1}(n!)^2}
\end{aligned}
$$

3 marks
Both factorials in numerator and denominator are correct and simplifies the power of 2

2 marks
Correctly simplifies numerator OR denominator to get required factoral

1 mark
Uses reduction formula to write I_{2n} in expanded form.

2025 Yr12 HSC Assessment Task 4

Question 16 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MEX12-6 uses mechanics to model and solve practical problems.

Solutions	Marking Guidelines
a)	
$$\frac{dt}{dx}=2e^{-2x}$$ $$\frac{dx}{dt}=v=\frac{1}{2}e^{2x}$$ $$v^2=\frac{1}{4}e^{4x}$$	**3 marks** Finds correct a with prior parks correct **2 marks** Finds correct v
$$a=\frac{d}{dx}\left[\frac{1}{2}v^2\right]$$ $$=\frac{d}{dx}\left[\frac{1}{8}e^{4x}\right]$$ $$=\frac{1}{2}e^{4x}$$	**1 mark** Finds $\frac{dx}{dt}$
Note: Students need to review Leibniz notation and their understanding of motion.	
$$a=\frac{d^2x}{dt^2}\neq\frac{d^2t}{dx^2}\neq\frac{1}{\frac{d^2t}{dx^2}}$$	
b)	
$$10a=T\cos 30$$ $$a=\frac{T\cos 30}{10}$$ $$\frac{dv}{dt}=\frac{T\sqrt{3}}{20}$$ $$\int_0^v dv=\frac{T\sqrt{3}}{20}\int_0^t 1dt$$ $$v=\frac{T\sqrt{3}}{20}t$$	**2 marks** Correct solution **1 mark** Finds values of x
Note: students need to review evaluating fractions	
$$\frac{\cos(30)}{10}=\frac{\frac{\sqrt{3}}{2}}{10}=\frac{\sqrt{3}}{2}\div 10=\frac{\sqrt{3}}{2}\times\frac{1}{10}=\frac{\sqrt{3}}{20}$$	

c i) $a = \frac{d}{dx}\left[\frac{1}{2}v^2\right]$ $= \frac{d}{dx}\left[-x^4 + 7x^2 - 12\right]$ $= -4x^3 + 14x$ $= -4x\left(x^2 - \frac{7}{2}\right)$	**1 mark** Correct solution
ii) Since the particle is stationary ***and*** acceleration at the initial position $x = 2$ is negative, the particle will start moving to the negative direction. Note: just because acceleration is negative does not mean the particle is moving to the left.	**1 mark** Correct reasoning
iii) $0 = -2x^4 + 14x^2 - 24$ $0 = x^4 - 7x^2 + 12$ $0 = (x^2 - 4)(x^2 - 3)$ $x^2 = 4, 3$ $x = \pm 2, \pm\sqrt{3}$ The next time the particle comes to rest is at $x = \sqrt{3}$ since the particle is moving in the negative direction and $\sqrt{3} < 2$. Note: read the question carefully, the initial displacement is already given.	**2 marks** Correct solution with valid reasoning **1 mark** Finds values of x

d i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = x-2$ $\int_1^v d\left(\frac{1}{2}v^2\right) = \int_0^x (x-1)dx$ $\frac{1}{2}v^2 - \frac{1}{2} = \frac{1}{2}x^2 - x$ $v^2 = x^2 - 2x + 1$ $= (x-1)^2$ $v = \pm(x-1)$ Since $v = 1$ when $x = 0$, $v = 1 - x$	**3 marks** Correct solution with valid reasoning for selecting v **2 marks** solves for v^2 **1 mark** Sets up correct integral
ii) $\frac{dx}{dt} = 1 - x$ $\int_0^x \frac{dx}{1-x} = \int_0^t dt$ $\left[-\ln(1-x)\right]_0^x = t$ $t = -\ln(1-x)$ $e^{-t} = 1 - x$ $x = 1 - e^{-t}$	**2 marks** Finds x interms of t **1 mark** Separate variables and integrates correctly
iii) As $t \to \infty$, $e^{-t} \to 0$. So, $x \to 1$ (limiting position).	**1 mark** Explicitly reasons why $x \to 1$ to find limiting position.