VIRILE AGITUR

Name:________________________________

Teacher's Name: ________________________

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**2025**

**TRIAL HIGHER SCHOOL CERTIFICATE**

**YEAR 12 – Assessment Task 4**

**Monday 25th August**

# Mathematics Extension 2

**General Instructions**

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/ or calculations

**Total Marks: 100**

Section I – 10 marks (pages 1–3)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 4–12)

- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

| MCQ | 11 | 12 | 13 | 14 | 15 | 16 | Total |
|---|---|---|---|---|---|---|---|
| /10 | /15 | /15 | /15 | /15 | /15 | /15 | /100 |

## Section I

**10 marks**
**Attempt Questions 1–10**
**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Questions 1–10

1. Let $z = 1 + 3i$ and $w = 4 - 5i$ be two complex numbers.

   What is the projection of $z$ onto $w$?

   A. $-\frac{44}{41} + \frac{55}{41}i$

   B. $-\frac{44}{\sqrt{41}} + \frac{55}{\sqrt{41}}i$

   C. $-\frac{55}{41} + \frac{44}{41}i$

   D. $-\frac{44}{\sqrt{41}} - \frac{55}{\sqrt{41}}i$

2. A sphere of unit radius is tangent to the $xy$-plane, the $xz$-plane and the $yz$-plane. $A$ is the point on the sphere which is closest to the origin.

   What is the distance between $A$ and the origin?

   A. $1$

   B. $\sqrt{2}$

   C. $\sqrt{2} - 1$

   D. $\sqrt{3} - 1$

3. Which of the following is logically equivalent to the converse of the following statement?

   *"If there are no teachers in the room, then there are no students in the room."*

   A. If there is a teacher in the room, then there is a student in the room with them.

   B. If there is a student in the room, then there is a teacher in the room with them.

   C. If there are no teachers in the room, then there is a student in the room.

   D. If there no students in the room, then there is a teacher in the room.

4. Which of the following is a primitive of $\frac{5-x}{\sqrt{9-(x-5)^2}}$?

A. $\sqrt{9-(x-5)^2}+12\sin^{-1}\left(\frac{x-5}{3}\right)$

B. $\frac{1}{2}\sqrt{9-(x-5)^2}+6\sin^{-1}\left(\frac{x-5}{3}\right)$

C. $\sqrt{9-(x-5)^2}$

D. $\frac{1}{2}x\sqrt{9-(x-5)^2}+12\sin^{-1}\left(\frac{x-5}{3}\right)$

5. Two particles, $A$ and $B$, initiate simple harmonic motion from the origin at the same time, moving in the same direction. Both particles oscillate about the origin. Particle $A$ moves with period $T$, and Particle $B$ moves with period $2T$. The particles collide after a time interval of $\frac{1}{4}T$. What is the ratio $a_B : a_A$ of their amplitudes?

A. $1:\sqrt{2}$

B. $\sqrt{2}:1$

C. $\sqrt{3}:2$

D. $2:\sqrt{3}$

6. Which of the following is a true statement for *n* a natural number?

A. $\frac{n(n+1)}{2}$ is an odd number for $n \geq 1$.

B. $4n^2-1$ is divisible by 3 for $n \geq 1$.

C. $\sqrt{5n-4}$ is irrational for $n \geq 2$.

D. $n^3-n$ is divisible by 6 for $n \geq 2$.

7. The complex numbers $z_1$ and $z_2$ satisfy the following conditions:

$$z_1+z_2=10+24i$$

$$|z_1-z_2|=2$$

What is a possible value for $|z_1|$?

A. 9

B. 13

C. 24

D. 25

8. Consider the following statements:

I. $\int_0^{\pi}\left(\sin^4 x-\cos^4 x\right)dx=0$

II. $\int_0^1 \frac{dx}{1+x^3} > \int_0^1 \frac{dx}{1+x^5}$

Which of these statements is true?

A. Both I and II

B. I only

C. II only

D. Neither I nor II

9. A sphere $S$ passes through the points $(a,0,0)$, $(0,b,0)$, $(0,0,c)$ and the origin, where $a,b,c \neq 0$. What is the centre of the sphere?

A. $\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)$

B. $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

C. $(a-b-c, b-a-c, c-a-b)$

D. $\left(\frac{a}{bc},\frac{b}{ac},\frac{c}{ab}\right)$

10. Let $P(z)=z^n+az^{n-1}+1$, be a polynomial with $z\in\mathbb{C}$ where $a$ is a real number satisfying $-2<a<2$. The complex number $\omega$ is a non-real root of $P(z)=0$ such that $|\omega|=1$. Which of the following cannot be true?

A. $\omega^{n-1}=\frac{-1}{a+\omega}$

B. $|\omega+a|=|\omega^n+a\omega^{n-1}+1|+1$

C. $\omega$ is an $(n-2)$-th root of unity.

D. $\omega$ is an $(n-1)$-th root of unity.

## Section II

**90 marks**
**Attempt Questions 11–16**
**Allow about 2 hours and 45 minutes for this section**

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

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**Question 11** (15 marks) Use a SEPARATE Writing Book

(a) Define the complex numbers $z = 3 + 4i$ and $w = 2 - 2i$. Find:

(i) $z + \overline{w}$ **1**

(ii) $\left|\frac{z}{w}\right|$ **2**

(b) Write down a vector equation for the line that passes through the point (3, −2, 1) and is parallel to the line with equation $\boldsymbol{r}(\lambda) = \begin{pmatrix} -1 \\ 0 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$. **1**

(c) Write the complex number $2\sqrt{3} + 2i$ in exponential form. **2**

(d) The acceleration of a particle is given by $\ddot{x} = \dfrac{3}{2+v}$, where $\ddot{x}$ represents the acceleration and $v$ represents the velocity. **3**

Given that the particle is initially stationary and located at $x = 1$, find the displacement $x$ of the particle when $v = 2$.

**Question 11 continues on page 5**

Question 11 (continued)

(e) The points $A(2, 1, 5)$, $B(-1, 9, 4)$ and $C(6, -3, -2)$ are three vertices of the parallelogram $ABCD$ in three-dimensional space with the vertices described in cyclic order. **2**

Find the coordinates of the point $D$.

(f) Evaluate $\int_0^{\pi/4} |x \sin x| \, dx$, leaving your answer in exact form. **4**

**End of Question 11**

**Question 12** (15 marks) Use a SEPARATE Writing Book

(a) Consider the two lines $l_1$ and $l_2$, defined below:

$$l_1: r_1(\lambda) = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$$

$$l_2: r_2(\mu) = \begin{pmatrix} -6 \\ 3 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 2 \\ 1 \end{pmatrix}$$

(i) Find the point of intersection of these two lines. **2**

(ii) Find the acute angle formed between the two lines at this point. **2**

Give your answer to the nearest degree.

(b) Consider the following partial fraction decomposition:

$$\frac{6x^2 - 9x + 17}{(2x^2 + 3)(x + 5)} = \frac{Ax + B}{2x^2 + 3} + \frac{C}{x + 5}$$

(i) Find the constants $A$, $B$ and $C$ in this partial fraction decomposition. **2**

(ii) Hence evaluate $\displaystyle\int_0^2 \frac{6x^2 - 9x + 17}{(2x^2 + 3)(x + 5)}\, dx$. **3**

(c) Consider the polynomials $P_1(z) = z^2 + 6z + 10$ and $P_2(z) = z^3 + az^2 + bz + 20$, **3**

where $a$ and $b$ are real numbers and $z \in \mathbb{C}$.

Given that the polynomials share a common root, find $a$.

(d) Let $a, b, c$ and $d$ be positive real numbers. **3**

You are given that $\frac{x+y}{2} \geq \sqrt{xy}$ for real $x, y \geq 0$. (DO NOT PROVE THIS.)

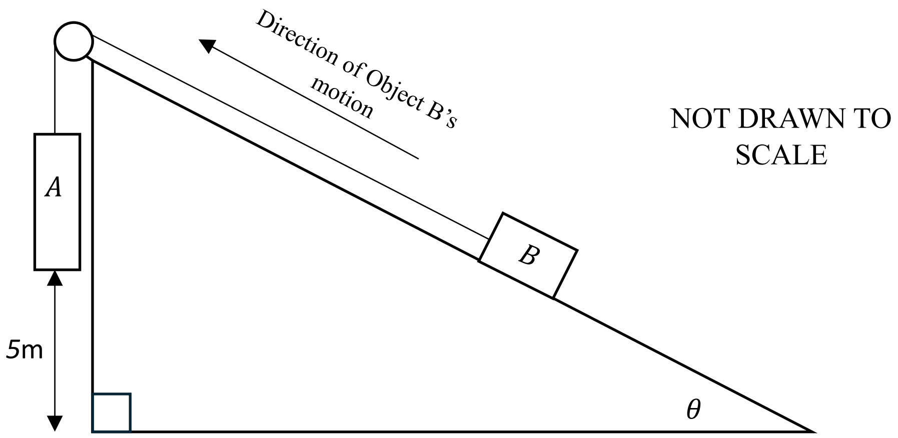
Show that $\dfrac{a^2 + b^2 + c^2 + d^2}{4} \geq \sqrt{abcd}$.

**End of Question 12**

**Question 13** (15 marks) Use a SEPARATE Writing Book

(a) Prove by contradiction that if $r^5 + 6r^3 + 2r > 3r^4 + 9r^2 + 6$, then $r$ is positive. **3**

(b) Objects *A* and *B* have masses 10 kg and 6 kg respectively and are attached via a light inextensible string running over a smooth pulley. The pulley is positioned at the top of an incline, angled at $\theta$ to the horizontal, upon which object *B* is situated. Object *A* hangs along the vertical side of the inclined block, 5 m above the floor.




The incline's surface exerts a frictional force of $0.4g$ with object *B* as it slides on the ramp. Apart from tension *T*, gravity and the normal force *N* on Object *B*, no other forces act on either object. Initially, both objects are stationary, with the acceleration of the system such that Object *A* descends and Object *B* slides up the slope. Take $g = 10\ m/s^2$

(i) By using force diagrams, show that the acceleration $a$ of the system satisfies: **3**

$$a = \frac{3}{8}g(1.6 - \sin\theta)$$

(ii) When Object *A* hits the floor, it is moving with a speed of $\sqrt{30}\ m/s$. Show that $\sin\theta = 0.8$. **2**

(iii) Once Object A hits the floor, it immediately stops moving. Object *B*'s momentum causes it to slide slightly further up the ramp to a maximal position *P*, before sliding back down until the string is taut. **2**

Show that during the final ascent of Object *B* to its maximal position *P*, its acceleration is given by $a = -\frac{26}{3}\ m/s^2$. Explain your reasoning carefully.

(iv) Hence, find the **total** distance travelled **up** the ramp by Object *B*. **2**

**Question 13 continues on page 8**

Question 13 (continued)

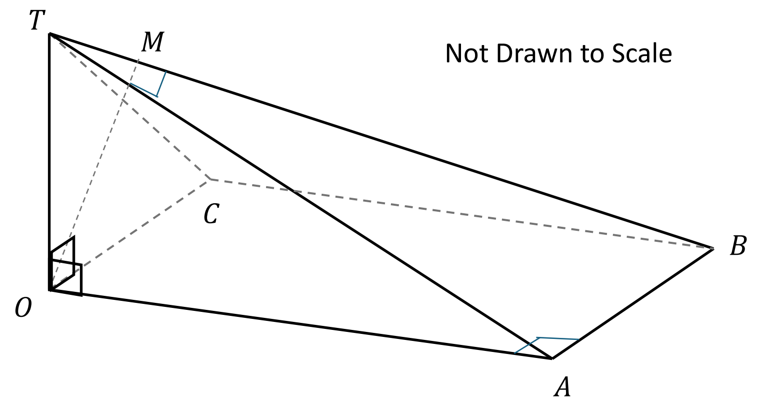
(c) Use mathematical induction to prove that $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} \geq \sqrt{n}$ **3**

for all positive integers $n \geq 1$.

**End of Question 13**

**Question 14** (14 marks) Use a SEPARATE Writing Book

(a) The rectangular pyramid $OABCT$ is constructed such that the ratio $|\overrightarrow{OT}|:|\overrightarrow{OA}|:|\overrightarrow{AB}|$ is 1:3:1. The angles, $\angle TOA$, $\angle TOC$ and $\angle OAB$ are all right angles, and the point $M$ is located on the edge $TB$ such that $OM \perp TB$.




(i) By considering a projection, show that $\overrightarrow{TM} = \frac{|\overrightarrow{TO}|^2}{|\overrightarrow{TB}|^2}\overrightarrow{TB}$ **3**

(ii) Hence find the ratio $|\overrightarrow{TM}|:|\overrightarrow{MB}|$. **3**

(b) Consider the equation $z^5 = 1$.

(i) Write down the solutions to this equation in exponential form. **1**

(ii) If $w$ represents the non-real solution to this equation with least positive argument, write down the exponential form of $w$. **1**

(iii) Represent all the roots in terms of $w$ and plot them on an Argand diagram indicating the circle they lie on. **3**

(iv) Show that $1 + w + w^2 + w^3 + w^4 = 0$. **1**

(v) Use the result in (iv) to show that $\frac{1}{2} + \cos\frac{2\pi}{5} + \cos\frac{4\pi}{5} = 0$. **2**

(vi) Hence show that $\cos\frac{\pi}{5}\cos\frac{3\pi}{5} = -\frac{1}{4}$. **1**

**End of Question 14**

**Question 15** (15 marks) Use a SEPARATE Writing Book

(a) A particle moves in simple harmonic motion according to the equation $\ddot{x} = -n^2(x-2)$. It begins at the origin, moving toward the positive $x$-axis at a speed of $u$ m/s, and it reaches the point $x = 5$ moving with speed $\frac{2}{3}u$.

(i) Find the amplitude $a$ of the particle's motion. **2**

(ii) Given that the maximum acceleration experienced by the particle is 6 m/s$^2$, find $u$. **2**

(b) The complex number $z$ lies in the first quadrant on the circle with centre at $C(1,0)$ and radius 1. Another complex number $w$ also lies on this circle, with $\text{Im}(w) = \text{Im}(z)$ and $Re(w) > \text{Im}(z)$. It can be shown that $Re(w) > 1 > Re(z)$. (DO NOT PROVE THIS.)
Let the points $A, B$ and $D$ represent the positions of $w, z$ and $\bar{z}$ respectively on the complex plane. Also, $Arg(\overrightarrow{CB}) = \theta$.

(i) Represent all this information neatly on a half-page Argand diagram. **1**

(ii) Describe the vector $\overrightarrow{CB}$ using complex numbers. **1**

(iii) Given $Arg(\overrightarrow{CB}) = \theta$, explain why $z = 1 + e^{i\theta}$ and write down the possible range of values for $\theta$. **2**

(iv) Given $z = 1 + e^{i\theta}$, write down similar expressions for $\bar{z}$ and $w$. **3**

(v) Deduce that $\frac{w}{\bar{z}} = i\tan\left(\frac{\theta}{2}\right)$. **2**

(vi) Using the above result, show that $\angle AOD = \frac{\pi}{2}$. **2**

**End of Question 15**

**Question 16** (15 marks) Use a SEPARATE Writing Book

(a) Consider the reduction formula: $I_n = \int_0^{1/2} \frac{x^n}{1-x}\,dx$ .

(i) Show that $I_n = -\frac{1}{n2^n} + I_{n-1}$ **2**

(ii) Hence show that: **2**

$$I_n = \ln 2 - \sum_{k=1}^{n} \frac{1}{k2^k}$$

(iii) Explain why: **1**

$$\int_0^{\frac{1}{2}} x^n dx \le I_n \le \int_0^{\frac{1}{2}} 2x^n dx$$

(iv) Deduce that: **2**

$$\ln 2 = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k2^k}$$

(b) A particle of mass $m$ is launched vertically upwards from the ground at a speed of $U$m/s. It experiences a downward gravitational force of $mg$, and air resistance of magnitude $kv$, where $v$ is the particle's velocity and $k$ is a positive constant.

(i) If $T$ is the time for the particle to reach its maximum height, show that: **2**

$$T = \frac{m}{k}\ln\left(1 + \frac{k}{mg}U\right)$$

(ii) If $H$ is the greatest height attained by the particle, show that: **2**

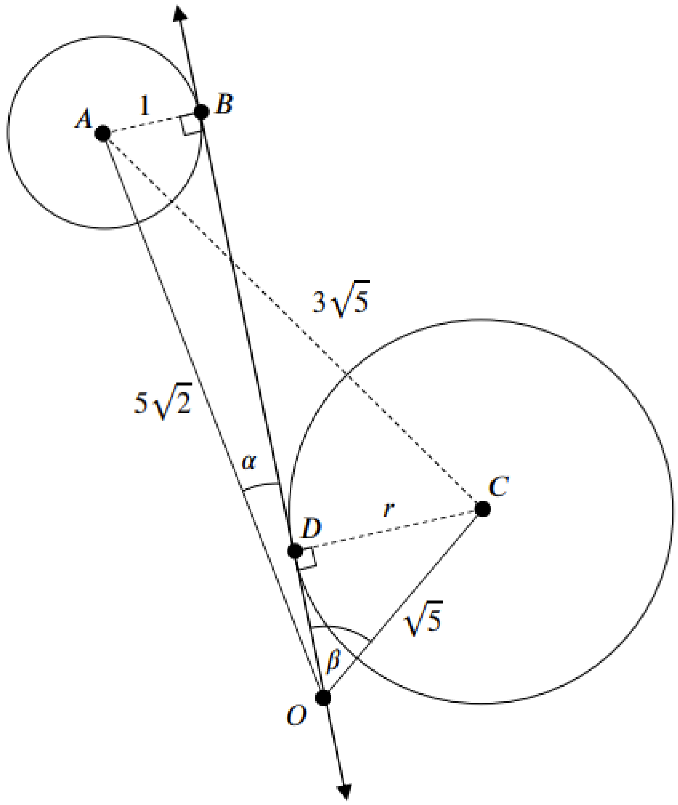
$$H = \frac{m}{k}U - \frac{m^2 g}{k^2}\ln\left(1 + \frac{k}{mg}U\right)$$

**Question 16 continues on page 12**

Question 16 (continued)

(c) A sphere $\mathcal{S}_1$ is defined in the $xyz$ co-ordinate space, with unit radius and centre at $A(0,5,5)$. Another sphere $\mathcal{S}_2$ is centred at $C(2,0,1)$ with radius $r$. A line $l$ passing through the origin is a common tangent to $\mathcal{S}_1$ and $\mathcal{S}_2$ touching $\mathcal{S}_1$ at $B$ and $\mathcal{S}_2$ at $D$.

The plane containing the line $l$ and the points $A, B, C, D$ and $O$ is shown below.




You are given that:

- $AB = 1$ as $\mathcal{S}_1$ has unit radius
- $OA = 5\sqrt{2}$ representing the distance of the point $A(0,5,5)$ from the origin $O$
- $OB = 7$ from Pythagoras' theorem in $\Delta AOB$
- $OC = \sqrt{5}$ representing the distance of the point $C(2,0,1)$ from the origin $O$
- $AC = 3\sqrt{5}$ representing the distance of the point $A(0,5,5)$ from $C(2,0,1)$

(i) Show that $\cos(\alpha+\beta) = \dfrac{1}{\sqrt{10}}$ where $\alpha = \angle AOB$ and $\beta = \angle COD$. **1**

(ii) Hence find the value of $r$ for which the line $l$ is a common tangent to $\mathcal{S}_1$ and $\mathcal{S}_2$. **3**

**End of paper**