Name: ______________________________

Student number:

Teacher TNK VMA

IN FIDE SCIENTIAM

Newington College

Mathematics Extension 2
Trial Examination 2025

General Instructions

- Reading time: 10 minutes
- Working time: 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- Write your student number on the front of each exam writing booklet.

Total marks: 100

Section I - 10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II - 90 marks

- Attempt Questions 11–16
- Allow about 2 hour and 45 minutes for this section

Section 1

10 Marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for questions 1 – 10.

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1. What is the value of i^i?

(A) -1

(B) $e^{-\frac{\pi}{2}}$

(C) $e^{\frac{\pi}{2}}$

(D) 1

2. $OABC$ is a square with $\overrightarrow{OA} = a\underset{\sim}{i} - 2\underset{\sim}{j} - 2\underset{\sim}{k}$ and $\overrightarrow{OC} = 6\underset{\sim}{i} - 4\underset{\sim}{j} + a\underset{\sim}{k}$ for some constant a.

What is the value of a?

(A) -2

(B) -1

(C) 1

(D) 2

3. Which of the following is an expression for $\int \sin^{-1} x \, dx$?

(A) $x\sin^{-1}x - \frac{1}{2}\sqrt{1-x^2} + c$

(B) $x\sin^{-1}x + \frac{1}{2}\sqrt{1-x^2} + c$

(C) $x\sin^{-1}x - \sqrt{1-x^2} + c$

(D) $x\sin^{-1}x + \sqrt{1-x^2} + c$

4. Consider the following statement: $\forall x, y \in \mathbb{R}, xy \leq 9 \Rightarrow x \leq 3 \text{ or } y \leq 3$.

   Which of the following is the negation of this statement?

   (A) $\forall x, y \in \mathbb{R}, x \leq 3 \text{ or } y \leq 3 \Rightarrow xy \leq 9$

   (B) $\forall x, y \in \mathbb{R}, x > 3 \text{ and } y > 3 \Rightarrow xy > 9$

   (C) $\exists x, y \in \mathbb{R} : x > 3 \text{ or } y > 3 \text{ and } xy \leq 9$

   (D) $\exists x, y \in \mathbb{R} : xy \leq 9 \text{ and } x > 3 \text{ and } y > 3$

5. A particle is moving in simple harmonic motion along the x axis. At time t seconds it has displacement x metres from the origin O and velocity $v\ \mathrm{m\,s^{-1}}$ given by $v^2 = \frac{1}{9}\pi^2(4 - x^2)$.

   What is the distance travelled by the particle in the first minute of its motion?

   (A) 40 metres

   (B) 48 metres

   (C) 80 metres

   (D) 96 metres

6. Consider the following statement concerning positive integers $n > 1$:

   If n is odd, then n can be written as $x^2 - y^2$
   for positive integers x and y.

   Which of the following is correct?

   (A) The contrapositive statement is false and the converse statement is false.

   (B) The contrapositive statement is true and the converse statement is false.

   (C) The contrapositive statement is false and the converse statement is true.

   (D) The contrapositive statement is true and the converse statement is true.

7. The point P with position vector $\overrightarrow{OP} = 4\underset{\sim}{i} + 5\underset{\sim}{j} + 6\underset{\sim}{k}$ lies on a sphere with centre O. Which one of the following points also lies on the sphere?

(A) A with position vector $\overrightarrow{OA} = 2\underset{\sim}{i} + 5\underset{\sim}{j} + 7\underset{\sim}{k}$

(B) B with position vector $\overrightarrow{OB} = 5\underset{\sim}{i} + 5\underset{\sim}{j} + 5\underset{\sim}{k}$

(C) C with position vector $\overrightarrow{OC} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 8\underset{\sim}{k}$

(D) D with position vector $\overrightarrow{OD} = 2\underset{\sim}{i} + 6\underset{\sim}{j} + 6\underset{\sim}{k}$

8. Which one of the following is NOT equal to $\int_{a-b}^{a+b} f(x)\,dx$?

(A) $\int_{-b}^{b} f(x+a)\,dx$

(B) $\int_{-b}^{b} f(a-x)\,dx$

(C) $\int_{2b}^{0} f(a+b-x)\,dx$

(D) $\int_{-2b}^{0} f(a+b+x)\,dx$

9. Let $z = e^{i\theta}$ and $w = e^{i\phi}$.

Which one of the following is an expression for $\dfrac{(z+w)(z-w)}{zw}$?

(A) $2i\sin(\theta-\phi)$

(B) $i\sin 2(\theta-\phi)$

(C) $\cos 2(\theta-\phi)$

(D) $2\cos(\theta-\phi)$

10. A body of mass m kg is projected from point O with initial speed $V\,\mathrm{m}\,s^{-1}$ at an angle α above the horizontal. It moves in a vertical plane under gravity in a medium in which the resistance to motion has magnitude mkv Newtons where $v\,ms^{-1}$ is its speed and $k > 0$ is a constant.

What is an expression for the horizontal distance x metres travelled by the body in the first t seconds of its motion?

(A) $x = Vt\cos\alpha$

(B) $x = V\cos\alpha(1 - e^{-kt})$

(C) $x = kV\cos\alpha(1 - e^{-kt})$

(D) $x = \dfrac{1}{k}V\cos\alpha(1 - e^{-kt})$

Section II

90 Marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section.

Answer the questions in writing booklets provided. Use a separate writing booklet for each question. In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Use a separate writing booklet.**

(a) The complex number z is given by $z = \dfrac{a+3i}{2+ai}$ where a is real. Find all values of a such that z is real. **2**

(b) Use the substitution $t = \tan\dfrac{x}{2}$ to find $\displaystyle\int \frac{1+\tan\frac{x}{2}}{1+\sin x}\,dx$. **2**

(c) Find all complex numbers u and v such that $u - v = 2i$ and $uv = 6$. **3**

(d) $1 + 3i$ is a root of the equation $z^3 + bz^2 + d = 0$ where b and d are real.

(i) Find the other two roots of the equation in the form $x + iy$ where x and y are real. **2**

(ii) Hence, find the values of b and d. **2**

(e) Consider the vectors $\underset{\sim}{u} = 2\sqrt{2}\underset{\sim}{i} + 2\sqrt{2}\underset{\sim}{j} + 8\underset{\sim}{k}$ and $\underset{\sim}{v} = 6\underset{\sim}{i} + 6\underset{\sim}{j} + 2\sqrt{2}\underset{\sim}{k}$.

(i) Find the angle between the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$. **2**

(ii) Find the projection of the vector $\underset{\sim}{u}$ onto the vector $\underset{\sim}{v}$. **2**

Question 12 (15 marks) **Use a separate writing booklet.**

(a) Prove that $8^n + 1$ is a composite number for all positive integers $n \geq 1$. **2**

(b) Use the substitution $u = 1 + xe^{-x}$ to find $\int \frac{1-x}{e^x + x}\,dx$. **3**

(c) A particle is moving in simple harmonic motion along the x axis. At time t seconds it has displacement x metres from the origin O, given by $x = b + a\sin nt$ for some constants b, a and n, where $a > 0$ and $0 < n < \pi$. Initially the particle is 2 metres to the right of O. After 1 second, it is 5 metres to the right of O and after 2 seconds it is 1 metre to the left of O. **3**

Find, in simplest exact form, the values of b, a and n.

(d) Use integration by parts to find $\int \cos(\ln x)\,dx$. **3**

(e) (i) In an Argand diagram, draw the circle containing all the points representing complex numbers z such that $\left|z - \left(2 + 2\sqrt{3}i\right)\right| = 2$. Show coordinates of the centre and any intercepts on the coordinate axes. **2**

(ii) If the point P on this circle represents the complex number z_1 which has the smallest principal argument, find z_1 in the form $re^{i\theta}$ where $r > 0$ and $-\pi < \theta \leq \pi$. **2**

Question 13 (15 marks) **Use a separate writing booklet.**

(a) Prove that $\log_a(ab) + \log_b(ab) \geq 4$ for all real numbers $a > 1$ and $b > 1$. **3**

(b) With respect to a fixed origin O, two lines have equations

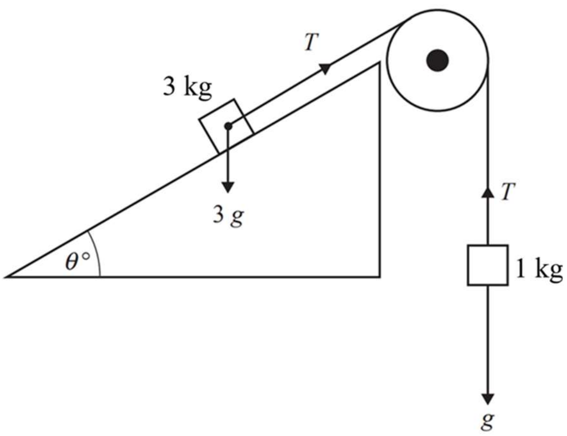
L_1: $\underset{\sim}{r} = (3\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}) + \lambda((2-a)\underset{\sim}{i} + 2\underset{\sim}{j} + \underset{\sim}{k})$ and

L_2: $\underset{\sim}{r} = (4\underset{\sim}{i} + 4\underset{\sim}{j} + 2\underset{\sim}{k}) + \mu(a\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k})$ for some constant a, where λ and μ are scalar parameters. The lines L_1 and L_2 are perpendicular to each other.

(i) Find the two possible values of a. **2**

(ii) Show that for one of these values of a, the lines L_1 and L_2 are skew and for the other value of a, the lines L_1 and L_2 intersect each other. **3**

(c) The 3 kg mass is placed on a smooth plane inclined at an angle of θ to the horizontal. The tension in the string is T newtons.



(i) When $\theta = 30°$, the acceleration of the 1 kg mass upwards is b ms^{-2}. Find the value of b in terms of g. **2**

(ii) For what angle $\theta°$ will the 3 kg mass be at rest on the plane? Give your answer to the nearest degree. **1**

(d) A particle moves along the x axis. At time t seconds it has displacement x metres from the origin O, velocity $v\ \mathrm{m\,s^{-1}}$ and acceleration $a\ \mathrm{m\,s^{-2}}$ given by $a = -\dfrac{\ln 2}{4^x}$.
Initially the particle is at O and is moving to the right of O with speed $1\ \mathrm{m\,s^{-1}}$.

(i) Use integration to show that $v = \dfrac{1}{2^x}$. **2**

(ii) Hence use integration to show that $x = \log_2(\ln(e2^t))$. **2**

Question 14 (15 marks) **Use a separate writing booklet.**

(a) (i) Use the substitution $u = \frac{\pi}{2} - x$ to show that $\int_0^{\pi} \frac{\sin x}{1+\sin x} dx = \pi - 2$. **3**

(ii) Hence, use the substitution $u = \pi - x$ to evaluate, in simplest exact form **2**

$\int_0^{\pi} \frac{x \sin x}{1+\sin x} dx$.

(b) A body of mass m kg moves along the x axis with initial speed V ms^{-1} subject to a resistance of magnitude $\frac{1}{2}m(1+v^4)$ Newtons when its speed is v ms^{-1}.

(i) Show that the particle travels $\tan^{-1}(V^2)$ by the time it stops moving. **2**

(ii) When the initial speed of the particle is $\frac{9}{4}U\ ms^{-1}$, the particle travels twice the distance compared to when the initial speed is $U\ ms^{-1}$. **3**

Find the value of U in simplest exact form.

(c) $I_n = \int_0^{\pi} x^n \sin x\, dx$ for $n = 0, 1, 2, \ldots$.

(i) Show that $I_n = \pi^n - n(n-1)I_{n-2}$ for $n = 2, 3, 4, \ldots$. **3**

(ii) Hence, evaluate $\int_0^{\pi} x^4 \sin x\, dx$ in simplest exact form. **2**

Question 15 (15 marks) **Use a separate writing booklet.**

(a) Vector $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are inclined at an acute angle of θ to each other.

(i) Show that $|\underset{\sim}{u}+\underset{\sim}{v}|^2 = |\underset{\sim}{u}|^2 + |\underset{\sim}{v}|^2 + 2|\underset{\sim}{u}||\underset{\sim}{v}|\cos\theta$, and hence that $|\underset{\sim}{u}+\underset{\sim}{v}| \le |\underset{\sim}{u}| + |\underset{\sim}{v}|$. **2**

(ii) Hence show that for all real numbers a and b,

$$\sqrt{a^2+b^2} \le \frac{\sqrt{4a^2+b^2}+\sqrt{a^2+4b^2}}{3}.$$ **2**

(b) A particle of mass m kg falls vertically from rest under gravity in a medium for which the resistance to motion has magnitude $\frac{mv^2}{g}$ Newtons, where $v\,\text{ms}^{-1}$ is its speed and $g\ \text{ms}^{-2}$ is the acceleration due to gravity. After time t seconds the particle has fallen x metres, has velocity $v\,\text{ms}^{-1}$ and acceleration $a\,\text{ms}^{-2}$ given by

$$a = \frac{g^2 - v^2}{g}.$$

(i) Use integration to show that $t = \frac{1}{2}\ln\left(\frac{g+v}{g-v}\right)$. **3**

(ii) Hence, show that $v = g\left(\frac{e^{2t}-1}{e^{2t}+1}\right)$ and find the terminal velocity of the particle. **2**

(iii) Hence find the percentage of its terminal velocity reached by the particle after $\ln 7$ seconds. **1**

(c) Consider the sequence of propositions P_n, $n = 1, 2, 3, \ldots$ defined by

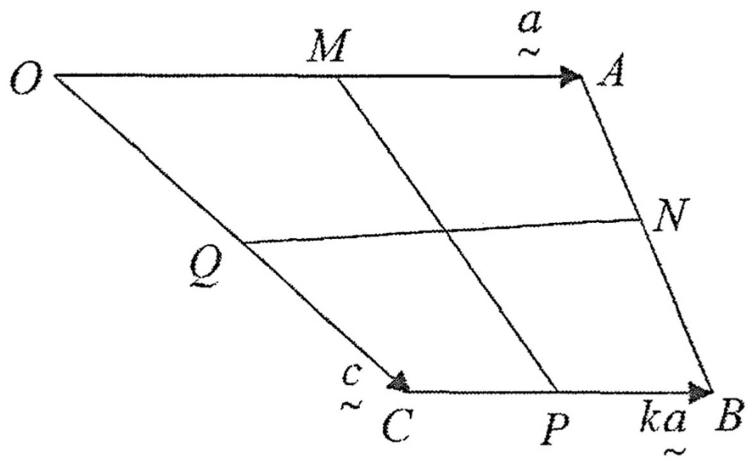
P_n: 14^n can be expressed as the sum of the squares of three different positive integers.

(i) Show that P_1 and P_2 are true. **2**

(ii) Use Mathematical Induction to prove that P_n is true for all positive integers $n \ge 1$. **3**

Question 16 (15 marks) **Use a separate writing booklet.**

(a)



In the diagram, $OABC$ is a trapezium in which $\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OC} = \underset{\sim}{c}$ and $\overrightarrow{CB} = k\overrightarrow{OA}$ for some constant $0 < k < 1$. The points M, N, P and Q are the midpoints of OA, AB, BC and CO respectively.

(i) Use vector methods to show that MP and QN bisect each other. **3**

(ii) If $MP \perp QN$, use vector methods to show that $\left|\overrightarrow{OC}\right|^2 = \left|\overrightarrow{AB}\right|^2$. **3**

(b) Use proof by contradiction to prove that if $a^2 + b^2 = c^2$ for positive integers a, b and c, then a and b cannot both be odd. **4**

(c) (i) Show that

$$1 + \sum_{k=1}^{n} (-1)^k \, {}^nC_k \cos^k \theta (\cos k\theta + i \sin k\theta) = (-i)^n \sin^n \theta (\cos n\theta + i \sin n\theta).$$ **3**

(ii) Hence, if

$S = 1 - {}^nC_1 \cos\theta \cos\theta + {}^nC_2 \cos^2\theta \cos 2\theta - \ldots + (-1)^n \, {}^nC_n \cos^n \theta \cos n\theta,$ **2**

find the value of S in terms of n and θ in each of the cases $n = 4m + 1$ and $n = 4m + 2$ for some integers $m = 0, 1, 2, \ldots$.

End of paper