Blacktown Boys' High School

2026

HSC Trial Examination

Mathematics Extension 2

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for this paper
- All diagrams are not drawn to scale
- In Questions 11 – 16, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 3 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7 – 12)

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Assessor: X. Chirgwin

Student Name: _____

Teacher Name: _____

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2026 Higher School Certificate Examination.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

Q1 What is the negation of the following statement?

"If you have good vision, then you don't need glasses."

A. You don't have good vision and you need glasses.

B. You have good vision and you need glasses.

C. If you don't have good vision, then you need glasses.

D. If you have good vision, then you need glasses.

Q2 Let the point P on an Argand diagram represent the complex number z. After being multiplied by another complex number ω, P is rotated $90°$ clockwise and $|z|$ is enlarged by a factor of 5. Which of the following is the value of ω?

A. $5i$

B. e^{-5i}

C. $-5e^{\frac{\pi}{2}i}$

D. $5e^{\frac{\pi}{2}i}$

Q3 Let $z = 3 + ki$, where $k \in \mathbb{R}$.
A value of k that makes $z^2 - 2iz + 6$ purely imaginary is

A. -5

B. -1

C. 1

D. 5

Q4 Which expression is equal to $\int \frac{1}{x^2+10x+27}\,dx$?

A. $\frac{1}{2}\ln\left|\frac{x+3}{x+9}\right|+c$

B. $\frac{1}{\sqrt{2}}\ln\left|\frac{x+9}{x+3}\right|+c$

C. $\frac{1}{2}\tan^{-1}\left(\frac{x+5}{2}\right)+c$

D. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+5}{\sqrt{2}}\right)+c$

Q5 A particle moving in a straight line obeys $v^2=7(11-6x-x^2)$ where x is its displacement from the origin in metres and v is its velocity in ms^{-1}. Where is the centre of the motion?

A. $x=3$

B. $x=1$

C. $x=-1$

D. $x=-3$

Q6 The relation $(z-4i)(\bar{z}+4i)=4$, when graphed on an argand diagram, would be

A. a circle with radius 2 with centre at $(0,4)$

B. a circle with radius 2 with centre at $(0,-4)$

C. a circle with radius 4 with centre at $(0,4)$

D. a circle with radius 4 with centre at $(0,-4)$

Q7 Each pair of lines given below intersects at $\begin{pmatrix}1\\2\\3\end{pmatrix}$.

Which pair of lines are perpendicular?

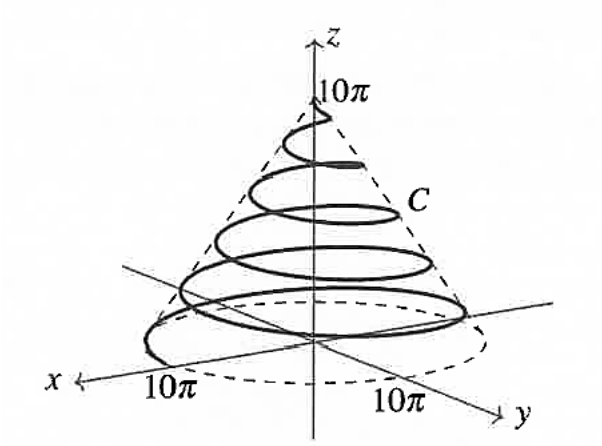
A. $\ell_1\colon \underset{\sim}{r} = \begin{pmatrix}2\\4\\6\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\ell_2\colon \underset{\sim}{r} = \begin{pmatrix}-2\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\3\end{pmatrix}$

B. $\ell_1\colon \underset{\sim}{r} = \begin{pmatrix}1\\2\\3\end{pmatrix} + \lambda\begin{pmatrix}-1\\-2\\1\end{pmatrix}$ and $\ell_2\colon \underset{\sim}{r} = \begin{pmatrix}3\\1\\3\end{pmatrix} + \lambda\begin{pmatrix}2\\-1\\0\end{pmatrix}$

C. $\ell_1\colon \underset{\sim}{r} = \begin{pmatrix}0\\-3\\2\end{pmatrix} + \lambda\begin{pmatrix}-1\\2\\-3\end{pmatrix}$ and $\ell_2\colon \underset{\sim}{r} = \begin{pmatrix}-1\\-2\\-3\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\3\end{pmatrix}$

D. $\ell_1\colon \underset{\sim}{r} = \begin{pmatrix}3\\-2\\8\end{pmatrix} + \lambda\begin{pmatrix}-2\\4\\-5\end{pmatrix}$ and $\ell_2\colon \underset{\sim}{r} = \begin{pmatrix}3\\4\\2\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\1\end{pmatrix}$

Q8 A curve C spirals around a cone as shown in the diagram.



A particle moves along C, starting at the point $(0, 0, 10\pi)$ and ending at the point $(10\pi, 0, 0)$.
Which of the following vector equations best describes the path of the particle for $0 \leq t \leq 10\pi$?

A. $\boldsymbol{r}_A = t\cos(t)\,\boldsymbol{i} + t\sin(t)\,\boldsymbol{j} + (10\pi - t)\boldsymbol{k}$

B. $\boldsymbol{r}_B = t\sin(t)\,\boldsymbol{i} + t\cos(t)\,\boldsymbol{j} + (10\pi - t)\boldsymbol{k}$

C. $\boldsymbol{r}_C = t\cos(t)\,\boldsymbol{i} + t\sin(t)\,\boldsymbol{j} + t\boldsymbol{k}$

D. $\boldsymbol{r}_D = t\sin(t)\,\boldsymbol{i} + t\cos(t)\,\boldsymbol{j} + t\boldsymbol{k}$

Q9 A complex number ω satisfies

$$\omega^2 + \frac{1}{1+\omega^2} = \omega$$

Which is the correct statement about ω^{2026}?

A. $\omega^{2026} = \omega$

B. $\omega^{2026} = -\omega$

C. $\omega^{2026} = \omega^2$

D. $\omega^{2026} = -\omega^2$

Q10 Which integral has the smallest value?

A. $\int_0^{\frac{\pi}{4}} \cos^2 x \, dx$

B. $\int_0^{\frac{\pi}{4}} \sin^2 x \, dx$

C. $\int_0^{\frac{\pi}{4}} \cos x \sin x \, dx$

D. $\int_0^{\frac{\pi}{4}} \tan x \sin x \, dx$

End of Section I

Section II

90 Marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question on the relevant pages of the writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 pages of the Writing Booklet.

(a) The complex numbers w and z is given by $w = 2e^{\frac{i\pi}{6}}$ and $z = 10e^{\frac{i\pi}{3}}$.

(i) Find the modulus and argument of wz. **2**

(ii) Find, in Cartesian form for $\frac{z}{w^5}$. **2**

(b) Consider the statement: 'If $a^2 - 6a$ is even, then a is even.'
Prove by contrapositive that the statement is true. **2**

(c) Determine the locus and shade the region of $z\bar{z} - 7(z + \bar{z}) \leq 0$. **3**

(d) Find $\int \frac{x^4}{9 + x^{10}}\, dx$ **2**

(e) (i) Show that $\sqrt{\frac{12 - x}{x}} = \frac{6 - x}{\sqrt{12x - x^2}} + \frac{6}{\sqrt{12x - x^2}}$. **1**

(ii) Hence evaluate $\int_0^3 \sqrt{\frac{12 - x}{x}}\, dx$. **3**

End of Questions 11

Question 12 (15 marks) Use the Question 12 pages of the Writing Booklet.

(a) (i) Determine all real values of x and y such that $(x+iy)^2 = 5-12i$. **2**

(ii) Hence or otherwise, solve $z^2+4z-1+12i=0$. **2**

(b) The two lines, l_1 and l_2 are represented respectively by the following vector equations:

$$\boldsymbol{r_1} = \begin{pmatrix} p \\ -5 \\ 2 \end{pmatrix} + \lambda_1 \begin{pmatrix} 3 \\ 2 \\ -5 \end{pmatrix}, \quad \boldsymbol{r_2} = \begin{pmatrix} -5 \\ -9 \\ -6 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix},$$

where λ_1 and λ_2 are real.

The two lines intersect at the point $A(3r, -r, 6r)$.

(i) Find the values of r and λ_2. Hence write down the coordinates of A. **2**

(ii) Hence, find the value of p. **1**

(iii) Calculate the acute angle between l_1 and l_2, round to the nearest minute. **2**

(c) Assume that $|x|+|y| \geq |x+y|$. **2**

Prove that if $a, b \in \mathbb{R}$, then $|a-b| \geq \big||a|-|b|\big|$.

(d) $$\int \frac{x^2-4x+53}{(5-x)(x^2+4)}\,dx$$ **4**

End of Questions 12

Question 13 (15 marks) Use the Question 13 pages of the Writing Booklet.

(a) Find $\int x \tan^{-1} x \, dx$. **3**

(b) A particle moving in simple harmonic motion starts from rest at a distance 15 metres to the right of its centre of oscillation O. The period of the motion is 6 seconds.

(i) Find the exact speed of the particle when it is 10 metres from its starting point. **3**

(ii) Find the time taken by the particle to first reach the point 10 metres from its starting point, in seconds, correct to 2 decimal places. **1**

(c) A particle moves in a straight line with acceleration $\ddot{x} = (4x - 2)\ \text{ms}^{-2}$, where x is the displacement in metres. Initially the particle is at the origin with velocity $1\ \text{ms}^{-1}$.

(i) If the velocity at time t seconds is $v\ ms^{-1}$, show that **2**

$$v^2 = (1 - 2x)^2$$

(ii) Given that $v = 1 - 2x$ throughout the motion, find expressions for x and v in terms of t. **3**

(d) Find $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 x \sin^2 x \cot^2 x \, dx$ **3**

End of Questions 13

Question 14 (15 marks) Use the Question 14 pages of the Writing Booklet.

(a) Let $\boldsymbol{c} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ be a unit vector that is perpendicular to both $\boldsymbol{a} = 3\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{b} = -2\boldsymbol{i} + \boldsymbol{j} - 4\boldsymbol{k}$. **4**

Find all possible vectors $\boldsymbol{c}$.

(b) Use the substitution $t = \tan\frac{x}{2}$ to show that **4**

$$\int_0^{\frac{\pi}{2}} \frac{1}{7 + 5\sin x + 5\cos x}\, dx = \log_e\left(\frac{9}{8}\right)$$

(c) Suppose that $x > 0, y > 0, z > 0$.

(i) Prove that $xy^{-1} + yx^{-1} \geq 2$. **1**

(ii) Prove that $x^3 + y^3 \geq xyz(xz^{-1} + yz^{-1})$. **2**

(iii) Hence show that $x^3 + y^3 + z^3 \geq 3xyz$. **2**

(iv) For $a > 0, b > 0, c > 0, d > 0$, deduce that **2**

$$(a + b + c)(a + b + d)(a + c + d)(b + c + d) \geq 81abcd$$

End of Questions 14

Question 15 (15 marks) Use the Question 15 pages of the Writing Booklet.

(a) (i) Show that $2n + 3 > 2\sqrt{(n+1)(n+2)}$, for $n > 0$. **1**

(ii) Hence, use mathematical induction to prove that for $n \geq 1$, **3**

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + + \frac{1}{\sqrt{n}} \geq 2\left(\sqrt{n+1} - 1\right)$$

(b) (i) If $z = \cos\theta + i\sin\theta$, show that for positive integers n, **1**

$$z^n + \frac{1}{z^n} = 2\cos(n\theta)$$

(ii) Hence expand $\left(z + \frac{1}{z}\right)^4 + \left(z - \frac{1}{z}\right)^4$ to show that **2**

$$\cos^4\theta + \sin^4\theta = \frac{1}{4}(\cos 4\theta + 3)$$

(iii) Let $x = \cos\theta$, solve the equation $8x^4 + 8(1 - x^2)^2 = 7$ to show **2**

that the roots are $\pm\cos\frac{\pi}{12}, \pm\cos\frac{5\pi}{12}$.

(iv) Hence show that $\cos\frac{\pi}{12} + \cos\frac{5\pi}{12} = \frac{\sqrt{6}}{2}$. **4**

(v) Hence, determine the exact value of $\cos\frac{5\pi}{12}$. **2**

End of Questions 15

Question 16 (15 marks) Use the Question 16 pages of the Writing Booklet.

(a) Given that $I_n = \int_1^e (1 - \ln x)^n dx$, for some integer n, where $n \geq 0$.

(i) Show that $I_n = -1 + nI_{n-1}$, for all positive integer n. **3**

(ii) Hence evaluate $\int_1^e (1 - \ln x)^3 dx$. **2**

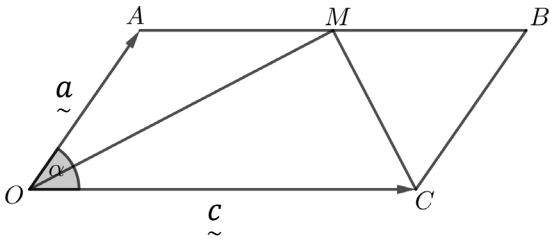
(iii) Show that $\frac{I_n}{n!} = e - \sum_{r=0}^{n} \frac{1}{r!}$, for positive integer n. **2**

(iv) Show that $0 \leq I_n \leq e - 1$. **1**

(b) The diagram below shows a parallelogram $OABC$ where $\angle AOC = \alpha$,

$\overrightarrow{OA} = \underset{\sim}{a}$, $\overrightarrow{OC} = \underset{\sim}{c}$, and $\left|\overrightarrow{OC}\right| = 2\left|\overrightarrow{OA}\right|$.

M is a point on AB such that $\overrightarrow{AM} = k\overrightarrow{AB}$, where $0 \leq k \leq 1$ and $\angle OMC = 90°$.



Not Drawn to Scale

(i) Use a vector method to show that **4**

$$\left|\underset{\sim}{a}\right|^2 (1 - 2k)(2\cos\alpha + 2k - 1) = 0$$

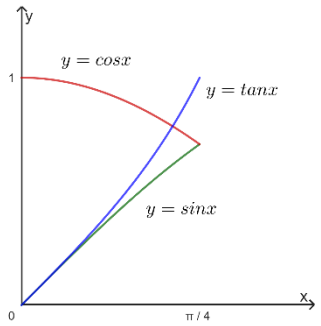
(ii) Determine the values of α such that there are two possible positions for M, satisfying all the given conditions. **3**

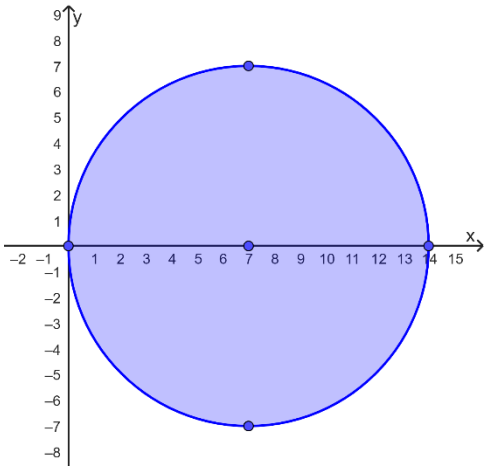
End of Paper

2026 Mathematics Extension 2 AT4 Trial Solutions

Section 1

Q1	**B** The negation of "If P then Q" is "P and not Q" You have good vision and you need glasses.	1 Mark
Q2	**C** $-5e^{\frac{\pi}{2}i} = -5i$, rotate $90°$ clockwise and enlargement factor of 5. Option A and D are the same, rotate $90°$ anticlockwise, which are incorrect. Option B modulus is 1 not 5, which is incorrect.	1 Mark
Q3	**D** $z^2 - 2iz + 6$ $= (3 + ki)^2 - 2i(3 + ki) + 6$ $= 9 + 6ki - k^2 - 6i + 2k + 6$ $= -k^2 + 2k + 3 + 6ki - 6i$ Purely imaginary, the real part is zero $-k^2 + 2k + 15 = 0$ $k^2 - 2k - 15 = 0$ $(k - 5)(k + 3) = 0$ $k = -3, k = 5$ $\therefore k = 5$ is a value that makes $z^2 - 2iz + 6$ purely imaginary.	1 Mark
Q4	**D** $\int \frac{1}{x^2 + 10x + 27}\,dx$ $= \int \frac{1}{x^2 + 10x + 25 + 2}\,dx$ $= \int \frac{1}{(x + 5)^2 + 2}\,dx$ $= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x + 5}{\sqrt{2}}\right) + c$	1 Mark
Q5	**D** $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$ $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2} \times 7(11 - 6x - x^2)\right)$ $\ddot{x} = \frac{7}{2}(-2x - 6)$ $\ddot{x} = -7(x + 3)$ $\therefore$ Centre of motion is $x = -3$	1 Mark
Q6	**A** $(z - 4i)(\bar{z} + 4i) = 4$ $\lvert z - 4i \rvert = 2$ Circle centre $(0, 4)$, radius 2	1 Mark

Q7	**B** Since the lines are perpendicular, the dot product of direction vectors equals zero. $\begin{pmatrix}-1\\-2\\1\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\0\end{pmatrix}=-2+2+0=0$	1 Mark
Q8	**A** When $t=0$, $\boldsymbol{r}_C=\boldsymbol{r}_D=0\boldsymbol{i}+0\boldsymbol{j}+0\boldsymbol{k}$. So not option C and D. When $t=10\pi$, $\boldsymbol{r}_A=10\pi\cos(10\pi)\,\boldsymbol{i}+10\pi\sin(10\pi)\,\boldsymbol{j}+(10\pi-10\pi)\boldsymbol{k}$ $\boldsymbol{r}_A=10\pi\boldsymbol{i}+0\boldsymbol{j}+0\boldsymbol{k}$. When $t=10\pi$, $\boldsymbol{r}_B=10\pi\sin(10\pi)\,\boldsymbol{i}+10\pi\cos(10\pi)\,\boldsymbol{j}+(10\pi-10\pi)\boldsymbol{k}$ $\boldsymbol{r}_B=0\boldsymbol{i}+10\pi\boldsymbol{j}+0\boldsymbol{k}$. So not option B.	1 Mark
Q9	**B** $\omega^2+\dfrac{1}{1+\omega^2}=\omega$ $\omega^2(1+\omega^2)+1=\omega(1+\omega^2)$ $\omega^2+\omega^4+1=\omega+\omega^3$ $\omega^4-\omega^3+\omega^2-\omega+1=0$ $\omega^5+1=0$ (where $\omega\neq-1$) $\omega^5=-1$ ω^{2026} $=(\omega^5)^{405}\times\omega$ $=(-1)^{405}\,\omega$ $=-\omega$	1 Mark
Q10	**B** For $0\leq x\leq\frac{\pi}{4}$, $\sin x<\cos x$ and $\sin x<\tan x$ $\sin^2 x<\sin x\cos x<\cos^2 x$ $\sin^2 x<\sin x\tan x$ ∴ The smallest value is $\int_0^{\frac{\pi}{4}}\sin^2 x\,dx$ 	1 Mark

Section 2		
Q11ai	$wz = 2e^{\frac{i\pi}{6}} \times 10e^{\frac{i\pi}{3}}$ $wz = 20e^{i\left(\frac{\pi}{6}+\frac{\pi}{3}\right)}$ $wz = 20e^{i\frac{i\pi}{2}}$ $\lvert wz\rvert = 20$ $Arg(wz) = \frac{\pi}{2}$	2 Marks Correct solution 1 Mark Correct modulus or argument or find wz
Q11aii	$\frac{z}{w^5} = \frac{10e^{\frac{i\pi}{3}}}{\left(2e^{\frac{i\pi}{6}}\right)^5}$ $\frac{z}{w^5} = \frac{10}{2^5}e^{i\left(\frac{\pi}{3}-\frac{5\pi}{6}\right)}$ $\frac{z}{w^5} = \frac{10}{32}e^{i\left(-\frac{3\pi}{6}\right)}$ $\frac{z}{w^5} = \frac{5}{16}e^{-\frac{i\pi}{2}}$ $\frac{z}{w^5} = \frac{5}{16}\left(\cos\left(-\frac{\pi}{2}\right) + i\sin\left(-\frac{\pi}{2}\right)\right)$ $\frac{z}{w^5} = -\frac{5}{16}i$	2 Marks Correct solution 1 Marl Finds $\frac{z}{w^5} = \frac{5}{16}e^{-\frac{i\pi}{2}}$
Q11b	The contrapositive of the statement is: 'If a is odd, then $a^2 - 6a$ is odd.' Let $a = 2k + 1, k \in \mathbb{Z}$ $a^2 - 6a$ $= (2k+1)^2 - 6(2k+1)$ $= 4k^2 + 4k + 1 - 12k - 6$ $= 4k^2 - 8k - 6 + 1$ $= 2(2k^2 - 4k - 3) + 1$ $= 2P + 1\ (P = 2k^2 - 4k - 3, P \in \mathbb{Z}$ since $k \in \mathbb{Z})$ $\therefore a^2 - 6a$ is odd. Since the contrapositive statement is true, then the original statement must be true.	2 Marks Correct solution 1 Mark State the contrapositive and makes progress in proof
Q11c	$z\bar{z} - 7(z + \bar{z}) \leq 0$ $(x^2 + y^2) - 7(2x) \leq 0$ $x^2 - 14y + y^2 \leq 0$ $(x-7)^2 + y^2 \leq 49$  	3 Marks Correct solution 2 Marks Correct locus and graph with most key features shown 1 Mark Correct locus

Q11d	$\int \frac{x^4}{9+x^{10}}\,dx$ $=\frac{1}{5}\int \frac{5x^4}{3^2+(x^5)^2}\,dx$ $=\frac{1}{5}\times\frac{1}{3}\tan^{-1}\left(\frac{x^5}{3}\right)+c$ $=\frac{1}{15}\tan^{-1}\left(\frac{x^5}{3}\right)+c$	2 Marks Correct solution 1 Mark Finds $\tan^{-1}\left(\frac{x^5}{3}\right)$
Q11ei	$\sqrt{\frac{12-x}{x}}=\frac{\sqrt{12-x}}{\sqrt{x}}\times\frac{\sqrt{12-x}}{\sqrt{12-x}}$ $=\frac{12-x}{\sqrt{12x-x^2}}$ $=\frac{6-x}{\sqrt{12x-x^2}}+\frac{6}{\sqrt{12x-x^2}}$	1 Mark Correct solution
Q11eii	$\int_0^3 \sqrt{\frac{12-x}{x}}\,dx$ $=\int_0^3 \left(\frac{6-x}{\sqrt{12x-x^2}}+\frac{6}{\sqrt{12x-x^2}}\right)dx$ $=\int_0^3 \frac{6-x}{\sqrt{12x-x^2}}\,dx+\int_0^3 \frac{6}{\sqrt{36-(x-6)^2}}\,dx$ $=\left[\sqrt{12x-x^2}\right]_0^3+\left[6\sin^{-1}\left(\frac{x-6}{6}\right)\right]_0^3$ $=\left[\sqrt{12\times3-3^2}-\sqrt{12\times0-0^2}\right]+\left[6\sin^{-1}\left(\frac{3-6}{6}\right)-6\sin^{-1}\left(\frac{0-6}{6}\right)\right]$ $=3\sqrt{3}+6\left(-\frac{\pi}{6}-\left(-\frac{\pi}{2}\right)\right)$ $=3\sqrt{3}+2\pi$	3 Marks Correct solution 2 Marks Correct integration 1 Mark Correct integration of one component
Q12ai	$(x+iy)^2=5-12i$ $x^2-y^2+2xiy=5-12i$ $x^2-y^2=5\ldots\ldots(1)$ $2xy=-12$ $y=-\frac{6}{x}\ldots\ldots(2)$ Sub (2) into (1) $x^2-\left(-\frac{6}{x}\right)^2=5$ $x^2-\frac{36}{x^2}=5$	2 Marks Correct solution 1 Mark Correct x or y values

	$x^4 - 5x^2 - 36 = 0$ $(x^2 - 9)(x^2 + 4) = 0$ $x = \pm 3 \ (x \ \epsilon \ \mathbb{R})$ Sub back into (2) $y = -\frac{6}{\pm 3} = \mp 2$ $x = 3, y = -2 \qquad (3 - 2i)$ $x = -3, y = 2 \qquad (-3 + 2i)$	
Q12aii	$z = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times (-1 + 12i)}}{2 \times 1}$ $z = \frac{-4 \pm \sqrt{16 + 4 - 48i}}{2}$ $z = \frac{-4 \pm \sqrt{20 - 48i}}{2}$ $z = \frac{-4 \pm 2\sqrt{5 - 12i}}{2}$ $z = -2 \pm \sqrt{5 - 12i}$ $z = -2 \pm (3 - 2i)$ $z = 1 - 2i, -5 + 2i$	2 Marks Correct solution 1 Mark Finds $z = -2 \pm \sqrt{5 - 12i}$
Q12bi	$-5 + \lambda_2 = 3r \ldots\ldots (1)$ $-9 - 3\lambda_2 = -r \ldots\ldots (2)$ $-6 + 3\lambda_2 = 6r \ldots\ldots (3)$ $(2) + (3)$ $-15 = 5r$ $r = -3$ Sub into (3) $-6 + 3\lambda_2 = 6 \times (-3)$ $3\lambda_2 = -12$ $\lambda_2 = -4$ Verify by subbing into (1) $-5 - 4 = 3 \times -3$ is correct $\therefore A(-9, 3, -18)$	2 Marks Correct solution 1 Mark Finds the value of r
Q12bii	$-5 + 2\lambda_1 = 3$ $2\lambda_1 = 8$ $\lambda_1 = 4$ $p + 3 \times 4 = -9$ $p = -21$	1 Mark Correct solution
Q12biii	$\cos\theta = \frac{3 \times 1 + 2 \times (-3) + (-5) \times 3}{\sqrt{3^2 + 2^2 + (-5)^2} \times \sqrt{1^2 + (-3)^2 + 3^2}}$	2 Marks Correct solution 1 Mark

	$\cos\theta = \dfrac{-18}{\sqrt{38}\times\sqrt{19}}$ $\cos\theta = \dfrac{-18}{\sqrt{722}}$ $\theta \approx 132°3'31.05''$ Acute angle is $47°56'$ to the nearest minute.	Finds $\cos\theta = \dfrac{-18}{\sqrt{722}}$
Q12c	$\lvert x\rvert + \lvert y\rvert \geq \lvert x+y\rvert$ Let $x = a-b$ and $y = b$ $\lvert a-b\rvert + \lvert b\rvert \geq \lvert a-b+b\rvert$ $\lvert a-b\rvert + \lvert b\rvert \geq \lvert a\rvert$ $\lvert a-b\rvert \geq \lvert a\rvert - \lvert b\rvert$ Let $x = b-a$ and $y = a$ $\lvert b-a\rvert + \lvert a\rvert \geq \lvert b-a+a\rvert$ $\lvert b-a\rvert + \lvert a\rvert \geq \lvert b\rvert$ $\lvert b-a\rvert \geq \lvert b\rvert - \lvert a\rvert$ $\lvert a-b\rvert \geq \lvert b\rvert - \lvert a\rvert$ If $\lvert a\rvert - \lvert b\rvert \geq 0$, then $\big\lvert\lvert a\rvert - \lvert b\rvert\big\rvert = \lvert a\rvert - \lvert b\rvert \leq \lvert a-b\rvert$. If $\lvert a\rvert - \lvert b\rvert < 0$, then $\big\lvert\lvert a\rvert - \lvert b\rvert\big\rvert = -(\lvert a\rvert - \lvert b\rvert) = \lvert b\rvert - \lvert a\rvert \leq \lvert a-b\rvert$. $\therefore \lvert a-b\rvert \geq \big\lvert\lvert a\rvert - \lvert b\rvert\big\rvert$	2 Marks Correct solution 1 Mark Shows $\lvert a-b\rvert \geq \lvert a\rvert - \lvert b\rvert$
Q12d	$\int \dfrac{x^2-4x+53}{(5-x)(x^2+4)}dx$ $\dfrac{x^2-4x+53}{(5-x)(x^2+4)} = \dfrac{A}{5-x} + \dfrac{Bx+C}{x^2+4}$ $A(x^2+4) + (5-x)(Bx+C) = x^2-4x+53$ Let $x = 5$ $29A = 58$ $A = 2$ Let $x = 0$ $8 + 5C = 53$ $C = 9$ Let $x = 1$ $10 + 4(B+9) = 50$ $B = 1$ $\int \dfrac{x^2-4x+53}{(5-x)(x^2+4)}dx$ $= \int \left(\dfrac{2}{5-x} + \dfrac{x+9}{x^2+4}\right)dx$ $= \int \left(\dfrac{2}{5-x} + \dfrac{x}{x^2+4} + \dfrac{9}{x^2+4}\right)dx$ $= -2\ln\lvert 5-x\rvert + \dfrac{1}{2}\ln\lvert x^2+4\rvert + \dfrac{9}{2}\tan^{-1}\left(\dfrac{x}{2}\right) + c$	4 Marks Correct solution 3 Marks Makes significant progress 2 Marks Shows $\dfrac{x^2-4x+53}{(5-x)(x^2+4)}$ $= \dfrac{2}{5-x} + \dfrac{x+9}{x^2+4}$ 1 Mark Attempts to use partial fractions

Q13a	$I = \int x \tan^{-1} x \, dx$ $u = \tan^{-1} x \qquad v = \frac{1}{2}x^2$ $u' = \frac{1}{1+x^2} \qquad v' = x$ $I = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\int \frac{x^2}{1+x^2}\, dx$ $I = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2}\, dx$ $I = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2}\, dx$ $I = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\int 1\ dx + \frac{1}{2}\int \frac{1}{1+x^2}\, dx$ $I = \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}x\ + \frac{1}{2}\tan^{-1} x + c$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Correct application of integration by parts
Q13bi	Period $= 6 = \frac{2\pi}{n}$ $n = \frac{2\pi}{6} = \frac{\pi}{3}$ $x = 15\cos\left(\frac{\pi}{3}t\right)$ $v = -5\pi \sin\left(\frac{\pi}{3}t\right)$ $\cos^2\left(\frac{\pi}{3}t\right) + \sin^2\left(\frac{\pi}{3}t\right) = 1$ $\left(\frac{x}{15}\right)^2 + \left(\frac{v}{-5\pi}\right)^2 = 1$ $x = 5$, since it is 10 metres from the starting point $\left(\frac{5}{15}\right)^2 + \left(\frac{v}{-5\pi}\right)^2 = 1$ $\frac{v^2}{25\pi^2} = \frac{8}{9}$ $v^2 = \frac{200\pi^2}{9}$ $v = \pm\frac{10\sqrt{2}\pi}{3}$ $\therefore$ speed is $\frac{10\sqrt{2}\pi}{3}$ m/s.	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Find x in terms of t

Q13bii	$15\cos\left(\frac{\pi}{3}t\right) = 5$ $\cos\left(\frac{\pi}{3}t\right) = \frac{1}{3}$ $\frac{\pi}{3}t = \cos^{-1}\left(\frac{1}{3}\right)$ $t = \frac{3}{\pi}\cos^{-1}\left(\frac{1}{3}\right)$ $t = 1.175\ldots$ $t \approx 1.18$ seconds (2 d.p.)	1 Mark Correct solution
Q13ci	$\ddot{x} = \frac{d\left(\frac{1}{2}v^2\right)}{dx} = 4x - 2$ $\frac{1}{2}v^2 = 2x^2 - 2x + c$ $x = 0, v = 1, \rightarrow c = \frac{1}{2}$ $\frac{1}{2}v^2 = 2x^2 - 2x + \frac{1}{2}$ $v^2 = 4x^2 - 4x + 1$ $\therefore v^2 = (1 - 2x)^2$	2 Marks Correct solution 1 Mark Correct integration
Q13cii	$v = \frac{dx}{dt} = 1 - 2x$ $\int dt = \int \frac{1}{(1-2x)}dx$ $t = -\frac{1}{2}\ln\lvert 1 - 2x\rvert + c$ $1 - 2x = Ae^{-2t},\ \ A = \pm e^c$ $x = \frac{1 - Ae^{-2t}}{2}$ $t = 0, x = 0, \rightarrow A = 1$ $\therefore x = \frac{1 - e^{-2t}}{2}$ $v = 1 - 2 \times \frac{1 - e^{-2t}}{2}$ $\therefore v = e^{-2t}$	3 Marks Correct solution 2 Marks Makes significant progress 1 Mark Finds $x = \frac{1 - Ae^{-2t}}{2}$

Q13d	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 x \sin^2 x \cot^2 x \, dx$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos^2 x \sin^2 x \cos^2 x}{\sin^2 x} dx$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^4 x \, dx$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\frac{1}{2}(1+\cos 2x)\right)^2 dx$ $= \frac{1}{4}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (1 + 2\cos 2x + \cos^2 2x) \, dx$ $= \frac{1}{4}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(1 + 2\cos 2x + \frac{1}{2}(1+\cos 4x)\right) dx$ $= \frac{1}{4}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x\right) dx$ $= \frac{1}{4}\left[\frac{3}{2}x + \sin 2x + \frac{1}{8}\sin 4x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= \frac{1}{4}\left[\left(\frac{3}{2}\times\frac{\pi}{2} + \sin\left(2\times\frac{\pi}{2}\right) + \frac{1}{8}\sin\left(4\times\frac{\pi}{2}\right)\right) - \left(\frac{3}{2}\times\frac{\pi}{4} + \sin\left(2\times\frac{\pi}{4}\right) + \frac{1}{8}\sin\left(4\times\frac{\pi}{4}\right)\right)\right]$ $= \frac{1}{4}\left[\left(\frac{3\pi}{4}\right) - \left(\frac{3\pi}{8} + 1\right)\right]$ $= \frac{1}{4}\left[\frac{3\pi}{8} - 1\right]$ $= \frac{3\pi}{32} - \frac{1}{4}$	3 Marks Correct solution 2 Marks Correct integration 1 Mark Obtains $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\frac{1}{2}(1+\cos 2x)\right)^2 dx$
Q14a	Since $\boldsymbol{c}$ is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$. $\boldsymbol{a}\cdot\boldsymbol{c} = 0$: $\quad 3x - y + z = 0 \ldots\ldots (1)$ $\boldsymbol{b}\cdot\boldsymbol{c} = 0$: $\quad -2x + y - 4z = 0 \ldots\ldots (2)$ $(1) + (2)$ $x - 3z = 0$ $x = 3z$ Sub this into (1) $9z - y + z = 0$ $10z - y = 0$ $y = 10z$ Since $\boldsymbol{c}$ is a unit vector, $x^2 + y^2 + z^2 = 1 \ldots\ldots (3)$ $(3z)^2 + (10z)^2 + z^2 = 1$ $110z^2 = 1$	4 Marks Correct solution 3 Marks Makes significant progress and finds one $\boldsymbol{c}$ 2 Marks Obtains relationships between x and y in terms of z 1 Mark Establishes $3x - y + z = 0$

	$z = \pm\frac{1}{\sqrt{110}}$ $c = \pm\left(\frac{3}{\sqrt{110}}\boldsymbol{i} + \frac{10}{\sqrt{110}}\boldsymbol{j} + \frac{1}{\sqrt{110}}\boldsymbol{k}\right)$	and $-2x + y - 4z = 0$
Q14b	$I = \int_0^{\frac{\pi}{2}} \frac{1}{7 + 5\sin x + 5\cos x}\,dx$ $t = \tan\frac{x}{2}$ $\frac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2} = \frac{1}{2}(1 + t^2) \rightarrow dx = \frac{2}{1+t^2}\,dt$ $x = \frac{\pi}{2}, t = 1$ $x = 0, t = 0$ $I = \int_0^1 \frac{1}{7 + 5\left(\frac{2t}{1+t^2}\right) + 5\left(\frac{1-t^2}{1+t^2}\right)} \times \frac{2}{1+t^2}\,dt$ $I = \int_0^1 \frac{2}{7 + 7t^2 + 10t + 5 - 5t^2}\,dt$ $I = \int_0^1 \frac{2}{2t^2 + 10t + 12}\,dt$ $I = \int_0^1 \frac{1}{t^2 + 5t + 6}\,dt$ $I = \int_0^1 \frac{1}{(t+2)(t+3)}\,dt$ $\frac{A}{t+2} + \frac{B}{t+3} = \frac{1}{(t+2)(t+3)}$ $A(t+3) + B(t+2) = 1$ Let $t = -2, A = 1$ Let $t = -3, B = -1$ $I = \int_0^1 \left(\frac{1}{t+2} + \frac{-1}{t+3}\right)dt$ $I = [\ln(t+2) - \ln(t+3)]_0^1$ $I = [\ln(1+2) - \ln(1+3)] - [\ln(0+2) - \ln(0+3)]$ $I = [\ln 3 - \ln 4] - [\ln 2 - \ln 3]$ $I = [2\ln 3 - \ln(4 \times 2)]$ $I = [\ln 3^2 - \ln 8]$ $I = \ln\left(\frac{9}{8}\right)$	4 Marks Correct solution 3 Marks Correct integration 2 Marks Correct partial fractions 1 Mark Correct substitution and simplifies to $\int_0^1 \frac{1}{t^2 + 5t + 6}\,dt$

Q14ci	$(x-y)^2 \geq 0$ $x^2+y^2-2xy \geq 0$ $x^2+y^2 \geq 2xy$ Divide both side by xy $xy^{-1}+yx^{-1} \geq 2$	1 Mark Correct solution
Q14cii	$x^3+y^3=(x+y)(x^2-xy+y^2)$ $x^3+y^3 \geq (x+y)(2xy-xy)$ $x^3+y^3 \geq xy(x+y)$ $x^3+y^3 \geq xyz(xz^{-1}+yz^{-1})$	2 Marks Correct solution 1 Mark Makes significant progress
Q14ciii	$x^3+y^3 \geq xyz(xz^{-1}+yz^{-1}) \dots (1)$ $y^3+z^3 \geq xyz(yx^{-1}+zx^{-1}) \dots (2)$ $x^3+z^3 \geq xyz(xy^{-1}+zy^{-1}) \dots (3)$ $(1)+(2)+(3)$ $2(x^3+y^3+z^3)=xyz(xz^{-1}+yz^{-1}+yx^{-1}+zx^{-1}+xy^{-1}+zy^{-1})$ $2(x^3+y^3+z^3)=xyz(xz^{-1}+zx^{-1}+yz^{-1}+zy^{-1}+yx^{-1}++xy^{-1})$ $2(x^3+y^3+z^3) \geq xyz(2+2+2)$ $x^3+y^3+z^3 \geq 3xyz$	2 Marks Correct solution 1 Mark Makes significant progress
Q14civ	Let $x=\sqrt[3]{a}, y=\sqrt[3]{b}, z=\sqrt[3]{c}$ $x^3+y^3+z^3 \geq 3xyz \rightarrow a+b+c \geq 3\sqrt[3]{abc} \dots (1)$ Similarly $a+b+d \geq 3\sqrt[3]{abd} \dots (2)$ $a+c+d \geq 3\sqrt[3]{acd} \dots (3)$ $b+c+d \geq 3\sqrt[3]{bcd} \dots (4)$ $(1) \times (2) \times (3) \times (4)$ $(a+b+c)(a+b+d)(a+c+d)(b+c+d) \geq 81\sqrt[3]{a^3b^3c^3d^3}$ $(a+b+c)(a+b+d)(a+c+d)(b+c+d) \geq 81abcd$	2 Marks Correct solution 1 Mark Makes significant progress
Q15ai	$(2n+3)^2=4n^2+12n+9$ $\left(2\sqrt{(n+1)(n+2)}\right)^2$ $=4(n+1)(n+2)$ $=4(n^2+3n+2)$ $=4n^2+12n+8$ $4n^2+12n+9>4n^2+12n+8$, for $n>0$ $(2n+3)^2>\left(2\sqrt{(n+1)(n+2)}\right)^2$ $\therefore 2n+3>2\sqrt{(n+1)(n+2)}$	1 Mark Correct solution
Q15aii	1. Prove true for $n=1$ $LHS=1$ $RHS=2\left(\sqrt{1+1}-1\right)=2\left(\sqrt{2}-1\right)=0.828\dots$ $LHS>RHS$ $\therefore$ Statement is true for $n=1$	3 Marks Correct solution 2 Marks 1 Mark

	2. Assume true for $n = k,\ k \in \mathbb{Z}^+$ $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + + \frac{1}{\sqrt{k}} \geq 2\left(\sqrt{k+1} - 1\right)$ 3. Prove statement is true for $n = k + 1$ i.e. $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} \geq 2\left(\sqrt{k+2} - 1\right)$ $LHS = 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}}$ $LHS \geq 2\left(\sqrt{k+1} - 1\right) + \frac{1}{\sqrt{k+1}}$ $LHS \geq 2\left(\sqrt{k+1}\right) + \frac{1}{\sqrt{k+1}} - 2$ $LHS \geq \frac{2(k+1)+1}{\sqrt{k+1}} - 2$ $LHS \geq \frac{2k+3}{\sqrt{k+1}} - 2$ From part i) $2k + 3 > 2\sqrt{(k+1)(k+2)}$ $LHS \geq \frac{2\sqrt{(k+1)(k+2)}}{\sqrt{k+1}} - 2$ $LHS \geq 2\sqrt{(k+2)} - 2$ $LHS \geq 2\left(\sqrt{k+1} - 1\right)$ $LHS \geq RHS$ ∴ Statement is true by mathematical induction for all positive integer n.	Proves initial case
Q15bi	$z = \cos\theta + i\sin\theta$ $z^n = \cos(n\theta) + i\sin(n\theta)$ (De Moivre's theorem) $z^{-n} = \cos(-n\theta) + i\sin(-n\theta)$ (De Moivre's theorem) $z^n + \frac{1}{z^n}$ $= \cos(n\theta) + i\sin(n\theta) + \cos(-n\theta) + i\sin(-n\theta)$ $= \cos(n\theta) + i\sin(n\theta) + \cos(n\theta) - i\sin(n\theta)$ $= 2\cos(n\theta)$	1 Mark Correct solution

Q15bii	$\left(z+\frac{1}{z}\right)^4+\left(z-\frac{1}{z}\right)^4$ $=\left(z^4+4z^3\times\frac{1}{z}+6z^2\times\frac{1}{z^2}+4z\times\frac{1}{z^3}+\frac{1}{z^4}\right)$ $+\left(z^4-4z^3\times\frac{1}{z}+6z^2\times\frac{1}{z^2}-4z\times\frac{1}{z^3}+\frac{1}{z^4}\right)$ $=2\left(z^4+\frac{1}{z^4}+6\right)\ldots\ldots(1)$ $z+\frac{1}{z}=2\cos\theta,\qquad z-\frac{1}{z}=2i\sin\theta$ $\left(z+\frac{1}{z}\right)^4+\left(z-\frac{1}{z}\right)^4$ $=(2\cos\theta)^4+(2i\sin\theta)^4$ $=16\cos^4\theta+16\sin^4\theta\ldots\ldots(2)$ Equate (1) and (2) $16\cos^4\theta+16\sin^4\theta=2\left(z^4+\frac{1}{z^4}+6\right)$ $\cos^4\theta+\sin^4\theta=\frac{1}{8}(2\cos(4\theta)+6)$ $\cos^4\theta+\sin^4\theta=\frac{1}{4}(\cos(4\theta)+3)$	2 Marks Correct solution 1 Mark Correct expansion to obtain $2\left(z^4+\frac{1}{z^4}+6\right)$
Q15biii	$8x^4+8(1-x^2)^2=7,\qquad x=\cos\theta$ $8\cos^4\theta+8(1-\cos^2\theta)^2=7$ $8\cos^4\theta+8\sin^4\theta=7$ $8\left(\frac{1}{4}(\cos(4\theta)+3)\right)=7$ $2(\cos(4\theta)+3)=7$ $\cos(4\theta)=\frac{1}{2}$ $4\theta=\frac{\pi}{3},\frac{5\pi}{3},2\pi+\frac{\pi}{3},2\pi+\frac{5\pi}{3},\ldots$ $\theta=\frac{\pi}{12},\frac{5\pi}{12},\frac{7\pi}{12},\frac{11\pi}{12},\ldots$ $x=\cos\frac{\pi}{12},\cos\frac{5\pi}{12},\cos\frac{7\pi}{12},\cos\frac{11\pi}{12}$ $x=\cos\frac{\pi}{12},\cos\frac{5\pi}{12},-\cos\frac{5\pi}{12},-\cos\frac{\pi}{12}$ $x=\pm\cos\frac{\pi}{12},\pm\cos\frac{5\pi}{12}$	2 Marks Correct solution 1 Mark Determines θ

Q15biv	$8x^4 + 8(1-x^2)^2 = 7$ $8x^4 + 8(1 - 2x^2 + x^4) = 7$ $8x^4 + 8 - 16x^2 + 8x^4 = 7$ $16x^4 - 16x^2 + 1 = 0$ $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$ $\cos\frac{\pi}{12} \times \cos\frac{5\pi}{12} + \cos\frac{\pi}{12} \times \left(-\cos\frac{5\pi}{12}\right) + \cos\frac{\pi}{12} \times \left(-\cos\frac{\pi}{12}\right)$ $+ \cos\frac{5\pi}{12} \times \left(-\cos\frac{5\pi}{12}\right) + \cos\frac{5\pi}{12} \times \left(-\cos\frac{\pi}{12}\right)$ $+ \left(-\cos\frac{5\pi}{12}\right) \times \left(-\cos\frac{\pi}{12}\right) = \frac{-16}{16}$ $-\cos^2\frac{\pi}{12} - \cos^2\frac{5\pi}{12} = -1$ $\cos^2\frac{\pi}{12} + \cos^2\frac{5\pi}{12} = 1 \ldots\ldots (1)$ $\alpha\beta\gamma\delta = \frac{e}{a}$ $\cos\frac{\pi}{12} \times \cos\frac{5\pi}{12} \times \left(-\cos\frac{5\pi}{12}\right) \times \left(-\cos\frac{\pi}{12}\right) = \frac{1}{16}$ $\cos^2\frac{\pi}{12}\cos^2\frac{5\pi}{12} = \frac{1}{16}$ $\cos\frac{\pi}{12} > 0$ and $\cos\frac{5\pi}{12}$, since $\frac{\pi}{12}$ and $\frac{5\pi}{12}$ are both in the first quadrant. $\cos\frac{\pi}{12}\cos\frac{5\pi}{12} = \frac{1}{4} \ldots\ldots (2)$ $(1) + (2) \times 2$ $\cos^2\frac{\pi}{12} + \cos^2\frac{5\pi}{12} + 2\cos\frac{\pi}{12}\cos\frac{5\pi}{12} = 1 + \frac{2}{4}$ $\left(\cos\frac{\pi}{12} + \cos\frac{5\pi}{12}\right)^2 = \frac{3}{2}$ $\cos\frac{\pi}{12} + \cos\frac{5\pi}{12} = \sqrt{\frac{3}{2}}$ $\therefore \cos\frac{\pi}{12} + \cos\frac{5\pi}{12} = \frac{\sqrt{6}}{2}$	4 Marks Correct solution 3 Marks Makes significant progress 2 Marks Finds $\cos^2\frac{\pi}{12} + \cos^2\frac{5\pi}{12} = 1$ and $\cos\frac{\pi}{12}\cos\frac{5\pi}{12} = \frac{1}{4}$ 1 Mark Finds $\cos^2\frac{\pi}{12} + \cos^2\frac{5\pi}{12} = 1$

Q14bv	Sum of roots: $\cos\frac{\pi}{12}+\cos\frac{5\pi}{12}=\frac{\sqrt{6}}{2}$ Product of roots: $\cos\frac{\pi}{12}\cos\frac{5\pi}{12}=\frac{1}{4}$ $\cos\frac{\pi}{12}, \cos\frac{5\pi}{12}$ are roots of the quadratic equation: $x^2-\frac{\sqrt{6}}{2}x+\frac{1}{4}=0$ $x=\frac{\frac{\sqrt{6}}{2}\pm\sqrt{\left(-\frac{\sqrt{6}}{2}\right)^2-4\times1\times\frac{1}{4}}}{2\times1}$ $x=\frac{\frac{\sqrt{6}}{2}\pm\sqrt{\frac{3}{2}-1}}{2\times1}$ $x=\frac{\frac{\sqrt{6}}{2}\pm\frac{\sqrt{2}}{2}}{2\times1}$ $x=\frac{\sqrt{6}\pm\sqrt{2}}{4}$ Since $\cos\frac{5\pi}{12}>\cos\frac{\pi}{12}$ $\therefore\cos\frac{5\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}$	2 Marks Correct solution 1 Mark Finds quadratic equation with roots $\cos\frac{\pi}{12}, \cos\frac{5\pi}{12}$
Q15ai	$I_n=\int_1^e(1-\ln x)^n dx$ $u=(1-\ln x)^n \qquad v=x$ $u'=-\frac{n}{x}(1-\ln x)^{n-1} \qquad v'=1$ $I_n=[x(1-\ln x)^n]_1^e-\int_1^e-\frac{n}{x}(1-\ln x)^{n-1}\times x dx$ $I_n=[e(1-\ln e)^n-1(1-\ln 1)^n]+n\int_1^e(1-\ln x)^{n-1}dx$ $I_n=[e(1-1)^n-1(1-0)^n]+nI_{n-1}$ $I_n=-1+nI_{n-1}$	3 Marks Correct solution 2 Marks Correct integration 1 Mark Use integration by parts
Q15aii	$I_3=\int_1^e(1-\ln x)^3 dx$ $I_3=-1+3I_2$ $I_2=-1+2I_1$ $I_1=-1+I_0$	2 Marks Correct solution 1 Mark Finds I_0

	$I_0 = \int_1^e (1 - \ln x)^0 dx$ $I_0 = [x]_1^e$ $I_0 = e - 1$ $I_1 = -1 + e - 1 = e - 2$ $I_2 = -1 + 2(e - 2) = 2e - 5$ $I_3 = -1 + 3(2e - 5) = 6e - 16$	
Q16aiii	Let $A_n = \frac{I_n}{n!}$ Use the recurrence: $I_n = -1 + nI_{n-1}$ $\frac{I_n}{n!} = \frac{-1}{n!} + \frac{nI_{n-1}}{n!}$ $\frac{I_n}{n!} = \frac{-1}{n!} + \frac{I_{n-1}}{(n-1)!}$ $A_n = A_{n-1} - \frac{1}{n!}$ $A_0 = \frac{I_0}{0!} = e - 1 = e - \frac{1}{0!}$ $A_1 = A_0 - \frac{1}{1!} = e - \frac{1}{0!} - \frac{1}{1!}$ $A_2 = A_1 - \frac{1}{2!} = e - \frac{1}{0!} - \frac{1}{1!} - \frac{1}{2!}$ $A_{n-1} = e - \sum_{r=0}^{n-1} \frac{1}{r!}$ $A_n = \left(e - \sum_{r=0}^{n-1} \frac{1}{r!}\right) - \frac{1}{n!}$ $A_n = e - \sum_{r=0}^{n} \frac{1}{r!}$ $\therefore \frac{I_n}{n!} = e - \sum_{r=0}^{n} \frac{1}{r!}$	2 Marks Correct solution 1 Mark Makes significant progress
Q16avi	$1 \leq x \leq e$ $\ln 1 \leq \ln x \leq \ln e$ $0 \leq \ln x \leq 1$ $0 \geq -\ln x \geq -1$ $1 \geq 1 - \ln x \geq 0$ $0 \leq (1 - \ln x)^n \leq 1$ $0 \leq \int_1^e (1 - \ln x)^n dx \leq \int_1^e 1\, dx$	1 Mark Correct solution

	$\int_1^e 1\,dx$ $= [x]_1^e$ $= e - 1$ $\therefore 0 \leq I_n \leq e - 1$	
Q16bi	Since $\angle OMC = 90°$, then $OM \perp MC$ $\overrightarrow{OM} \cdot \overrightarrow{MC} = 0$ $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \underset{\sim}{a} + k\underset{\sim}{c}$ $\overrightarrow{MC} = \overrightarrow{MB} + \overrightarrow{BC} = (1-k)\underset{\sim}{c} - \underset{\sim}{a}$ $\left(\underset{\sim}{a} + k\underset{\sim}{c}\right) \cdot \left((1-k)\underset{\sim}{c} - \underset{\sim}{a}\right) = 0$ $(1-k)\underset{\sim}{a} \cdot \underset{\sim}{c} - \underset{\sim}{a} \cdot \underset{\sim}{a} + k(1-k)\underset{\sim}{c} \cdot \underset{\sim}{c} - k\underset{\sim}{a} \cdot \underset{\sim}{c} = 0$ $(1-2k)\underset{\sim}{a} \cdot \underset{\sim}{c} - \left\lvert\underset{\sim}{a}\right\rvert^2 + k(1-k)\left\lvert\underset{\sim}{c}\right\rvert^2 = 0$ Since $\lvert\overrightarrow{OC}\rvert = 2\lvert\overrightarrow{OA}\rvert$, then $\left\lvert\underset{\sim}{c}\right\rvert = 2\left\lvert\underset{\sim}{a}\right\rvert$ $(1-2k)\left\lvert\underset{\sim}{a}\right\rvert\left\lvert\underset{\sim}{c}\right\rvert\cos\alpha - \left\lvert\underset{\sim}{a}\right\rvert^2 + k(1-k)\left(2\left\lvert\underset{\sim}{a}\right\rvert\right)^2 = 0$ $2(1-2k)\left\lvert\underset{\sim}{a}\right\rvert^2\cos\alpha - \left\lvert\underset{\sim}{a}\right\rvert^2 + 4k(1-k)\left\lvert\underset{\sim}{a}\right\rvert^2 = 0$ $\left\lvert\underset{\sim}{a}\right\rvert^2\left(2(1-2k)\cos\alpha - 1 + 4k(1-k)\right) = 0$ $\left\lvert\underset{\sim}{a}\right\rvert^2(2(1-2k)\cos\alpha - 1 + 4k - 4k^2) = 0$ $\left\lvert\underset{\sim}{a}\right\rvert^2(2(1-2k)\cos\alpha - (1-2k)^2) = 0$ $\left\lvert\underset{\sim}{a}\right\rvert^2\left((1-2k)\left(2\cos\alpha - (1-2k)\right)\right) = 0$ $\left\lvert\underset{\sim}{a}\right\rvert^2\left((1-2k)(2\cos\alpha + 2k - 1)\right) = 0$	4 Marks Correct solution 3 Marks Makes significant progress 2 Marks Obtains $(1-2k)\underset{\sim}{a} \cdot \underset{\sim}{c} - \left\lvert\underset{\sim}{a}\right\rvert^2$ $+ k(1-k)\left\lvert\underset{\sim}{c}\right\rvert^2 = 0$ 1 Mark Finds $\overrightarrow{OM}$ and $\overrightarrow{MC}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{c}$
Q16bii	$\left\lvert\underset{\sim}{a}\right\rvert^2\left((1-2k)(2\cos\alpha + 2k - 1)\right) = 0$ $(1-2k) = 0 \quad or \quad (2\cos\alpha + 2k - 1) = 0$ $k = \frac{1}{2} \quad or \quad k = \frac{1 - 2\cos\alpha}{2}$ If $k = \frac{1}{2}$, then M is the midpoint, then this is only one position. So $k \neq \frac{1}{2}$.	3 Marks Correct solution 2 Marks Determines $\frac{\pi}{3} \leq \alpha \leq \frac{2\pi}{3}$ 1 Mark Finds both k cases or shows why $k \neq \frac{1}{2}$

	Given that $0 \leq k \leq 1$, $0 \leq \frac{1-2\cos\alpha}{2} \leq 1$ $0 \leq 1-2\cos\alpha \leq 2$ $-1 \leq -2\cos\alpha \leq 1$ $\frac{1}{2} \geq \cos\alpha \geq -\frac{1}{2}$ $-\frac{1}{2} \leq \cos\alpha \leq \frac{1}{2}$ $\frac{\pi}{3} \leq \alpha \leq \frac{2\pi}{3}$ Since $k \neq \frac{1}{2}$, then $\frac{1-2\cos\alpha}{2} \neq \frac{1}{2}$ $1-2\cos\alpha \neq 1$ $\cos\alpha \neq 0$ $\alpha \neq \frac{\pi}{2}$ $\therefore \frac{\pi}{3} \leq \alpha < \frac{\pi}{2} \cup \frac{\pi}{2} < \alpha \leq \frac{2\pi}{3}$	