STUDENT'S NAME: _____

TEACHER'S NAME: _____

2026

HURLSTONE AGRICULTURAL HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE ASSESSMENT TASK 4

TRIAL EXAMINATION

Mathematics Extension 2

[redacted]

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using a black or blue pen.
- NESA approved calculators may be used.
- A reference sheet is provided at the end of this question booklet.
- For questions in Section II, show all relevant mathematical reasoning and/or calculations.
- This examination paper is not to be removed from the examination centre.

Total marks: 100

Section I – 10 marks (pages 2 to 5)

- Attempt Questions 1 – 10. The multiple-choice answer sheet has been provided at the end of this question booklet.
- Allow about 15 minutes for this section.

Section II – 90 marks (pages 6 to 13)

- Attempt Questions 11 – 16, write your solutions in the answer booklets provided. Extra working pages are available if required.
- Allow about 2 hours and 45 minutes for this section.

Disclaimer: Students are advised that this is a trial examination only and cannot in any way guarantee the content or the format of the 2026 HSC Mathematics Extension 2 Examination.

Section 1

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. $P(x) = x^4 + 2x^3 + 9x^2 + 8x + 20$ has a zero $x = 2i - 1$.

   What is the value of $P\left(\overline{2i-1}\right)$?

   A. $10 + 12i$

   B. $10 - 12i$

   C. $-10 - 12i$

   D. 0

2. The argument of $-iz$, where $z = -1 + i$, is which of the following?

   A. $\frac{\pi}{4}$

   B. $-\frac{3\pi}{4}$

   C. $\frac{3\pi}{4}$

   D. $\frac{5\pi}{4}$

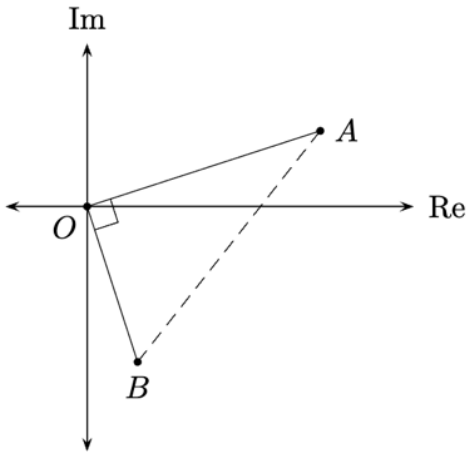
~ Section I continues ~

3. What is the value of i^i?

A. -1 B. $e^{-\frac{\pi}{2}}$

C. $e^{\frac{\pi}{2}}$ D. 1

4. The Argand diagram shows ΔOAB where A represents the complex number $3e^{i\frac{\pi}{10}}$, $\angle BOA = 90°$, and $\left|\overrightarrow{OB}\right| = \frac{2}{3}\left|\overrightarrow{OA}\right|$.



Which of the following represents the complex number B?

A. $2e^{-i\frac{2\pi}{5}}$ B. $2e^{-i\frac{9\pi}{10}}$

C. $2e^{i\frac{3\pi}{5}}$ D. $3e^{i\frac{3\pi}{5}}$

~ Section I continues ~

5. What is the negation of the quantified statement:

"For all real numbers x, there exists a real number y such that $x + y > 0$"?

A. There exists a real number x such that for all real numbers y, $x + y \leq 0$.

B. For all real numbers x, there exists a real number y such that $x + y \leq 0$.

C. There exists a real number x and there exists a real number y such that $x + y \leq 0$.

D. There exists a real number x such that for all real numbers y, $x + y < 0$.

6. Which of the following statements is logically equivalent to the implication:

"If n is an odd integer, then n^2 is an odd integer"?

A. If n is an even integer, then n^2 is an even integer.

B. If n^2 is an odd integer, then n is an odd integer.

C. If n^2 is an even integer, then n is an even integer.

D. If n is an odd integer, then n^2 is an even integer.

7. What is the scalar product of the two vectors $\underset{\sim}{u} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$ and $\underset{\sim}{v} = 4\underset{\sim}{i} - 6\underset{\sim}{j} - 3\underset{\sim}{k}$?

A. –11 B. 5

C. 23 D. 25

~ Section I continues ~

8. Consider the vectors $\overrightarrow{OA}=\begin{pmatrix}2\\-1\\3\end{pmatrix}$ and $\overrightarrow{OB}$ with $\left|\overrightarrow{OB}\right|=3$.

Given that $\overrightarrow{OA}\cdot\overrightarrow{OB}=6$, find in square units, the area of ΔOAB.

A. $3\sqrt{10}$

B. $\dfrac{3\sqrt{14}}{2}$

C. $\dfrac{3\sqrt{10}}{4}$

D. $\dfrac{3\sqrt{10}}{2}$

9. What values of a and b give $\dfrac{2x-11}{(x+2)(x-1)}\equiv\dfrac{a}{x+2}+\dfrac{b}{x-1}$?

A. $a=-5,\ b=-3$

B. $a=-5,\ b=3$

C. $a=5,\ b=-3$

D. $a=5,\ b=3$

10. If $\int_a^x f(t)\,dt=g(x)$, which of the following is a primitive of $f(x)g(x)$?

A. $\dfrac{1}{2}\left(f(x)\right)^2$

B. $\dfrac{1}{2}\left(f'(x)\right)^2$

C. $\dfrac{1}{2}\left(g(x)\right)^2$

D. $\dfrac{1}{2}\left(g'(x)\right)^2$

~ END OF SECTION I ~

Section II

90 marks

Attempt Questions 11 – 16

Allow about 2 hours and 45 minutes for this section.

Answer the questions in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet. **MARKS**

a) (i) Write $\dfrac{1+i\sqrt{3}}{1+i}$ in the form $x+iy$, where x and y are real. **2**

(ii) By expressing both $1+i\sqrt{3}$ and $1+i$ in modulus-argument form, write $\dfrac{1+i\sqrt{3}}{1+i}$ in modulus-argument form **3**

(iii) Hence, find $\cos\dfrac{\pi}{12}$ in surd form. **1**

b) Sketch the region in the complex plane, where the inequalities **4**

$|z-2+i| \le 2$ and $\text{Im}(z) \ge 0$ both hold.
Include all points of intersection of the regions in your sketch.

c) Let $z = \cos\theta + i\sin\theta$. By considering z^2, prove $\cos 2\theta = \cos^2\theta - \sin^2\theta$. **2**

d) R is a positive real number and z_1 and z_2 are complex numbers. **3**

Show that the points on the Argand diagram which represent the numbers

z_1, z_2 and $\dfrac{z_1 - iRz_2}{1-iR}$ respectively, form the vertices of a right-angled triangle.

~ End of Question 11 ~

Question 12 (15 marks) Use the Question 12 Writing Booklet. **MARKS**

a) Express $2\sqrt{2}\,e^{-\frac{3\pi}{4}i}$ in the form $x+iy$ **2**

b) Find all the 5th roots of -1 in modulus-argument form and sketch them on an Argand diagram. **3**

c) (i) Write $-1-i\sqrt{3}$ in the form $r\left(\cos\theta+i\sin\theta\right)$. **2**

(ii) Hence, or otherwise, find $\left(-1-i\sqrt{3}\right)^{17}$ in the form $a+ib$, where a and b are integers. **3**

d) The number z is a fifth root of unity where $z\neq 1$, that is $z^5=1$.

(i) Show that $\left(z+z^{-1}\right)^2+\left(z+z^{-1}\right)-1=0$. **2**

(ii) If $z=e^{i\theta}$, show $\cos\theta=\dfrac{z+z^{-1}}{2}$. **1**

(iii) Hence show $\cos\dfrac{2\pi}{5}=\dfrac{\sqrt{5}-1}{4}$. **2**

~ End of Question 12 ~

Question 13 (15 marks) Use the Question 13 Writing Booklet. **MARKS**

a) Use proof by contradiction to prove that $\log_6 12$ is irrational. **3**

b) (i) Show that $2ab \le a^2 + b^2$ for all real numbers a and b. **1**

(ii) Hence, deduce that $3(ab+bc+ca) \le (a+b+c)^2$ for all real numbers a, b and c. **3**

c) Let $z_2 = 1+i$ and, for $n > 2$, let $z_n = z_{n-1}\left(1+\dfrac{i}{|z_{n-1}|}\right)$. **3**

Use mathematical induction to prove that $|z_n| = \sqrt{n}$ for all integers $n \ge 2$.

d) The polynomial $P(x)$ is given by $P(x) = x^5 - 5cx + 1$, where c is a real number.

(i) By considering the turning points, prove that if $c < 0$, $P(x)$ has just one real root, which is negative. **2**

(ii) Prove that $P(x)$ has three distinct real roots if $c > \left(\dfrac{1}{4}\right)^{\frac{4}{5}}$. **3**

~ End of Question 13 ~

Question 14 (15 marks) Use the Question 14 Writing Booklet. **MARKS**

a) It is given that the point R is $(2,1,-1)$, $\overrightarrow{RS}=\begin{pmatrix}-4\\-1\\2\end{pmatrix}$ and $\overrightarrow{RT}=3\overrightarrow{RS}$. **2**

Find the coordinates of T.

b) For two vectors $\underset{\sim}{m}$ and $\underset{\sim}{p}$, it is known that $|\underset{\sim}{m}|=5$, $|\underset{\sim}{p}|=3$ and $\underset{\sim}{m}\cdot\underset{\sim}{p}=9$. **2**

Find $|\underset{\sim}{m}-\underset{\sim}{p}|$.

c) Consider the two lines $l_1:\begin{bmatrix}7\\-1\\2\end{bmatrix}+\lambda\begin{bmatrix}1\\3\\-5\end{bmatrix}$ and $l_2:\begin{bmatrix}-1\\2\\-6\end{bmatrix}+\mu\begin{bmatrix}2\\-3\\6\end{bmatrix}$

(i) Find the point of intersection of l_1 and l_2. **3**

(ii) Find the values of μ and k given $B(9,k,24)$ lies on l_2. **2**

~ Question 14 continues over the page ~

d) Consider the line l with parametric equations

$$x = \lambda + 1$$
$$y = \lambda$$
$$z = 2\lambda + 3$$

(i) Find the equation of line l, writing your answer in the form $\underset{\sim}{r} = \underset{\sim}{a} + \lambda \underset{\sim}{b}$, where $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are expressed as column vectors. **1**

Consider the sphere S, with centre $C(2, -1, 0)$ and radius $\sqrt{29}$ units.

It is also known that l intersects the surface S at points P and Q.

(ii) Find the coordinates of points P and Q. **3**

(ii) Hence, or otherwise, determine whether PQ is a diameter of S, **2**
Justify your answer.

~ End of Question 14 ~

Question 15 (15 marks) Use the Question 15 Writing Booklet. **MARKS**

a) Find $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$. **1**

b) Find $\int \frac{e^x}{e^{2x}+3e^x+2}\,dx$, giving your answer in the form $\log_e\left(\frac{e^x+1}{e^x+2}\right)+C$. **2**

c) Calculate $\int_0^{\pi} x^2 \sin x\,dx$. **3**

d) Let $I_n = \int_0^1 x^n e^x\,dx$ for integer values of n, where $n \geq 1$.

(i) Show that $I_n = e - n\,I_{n-1}$. **1**

(ii) Hence or otherwise, calculate I_3. **3**

e) (i) Sketch the graph of $y = \frac{\cos x}{1+\cos x}$ for $-\pi < x < \pi$. **2**

State the coordinates of its intersection/s with the x-axis and of the turning point.

(ii) Find the area of the region bounded by the curve and the x-axis. **3**

~ End of Question 15 ~

Question 16 (15 marks) Use the Question 16 Writing Booklet. **MARKS**

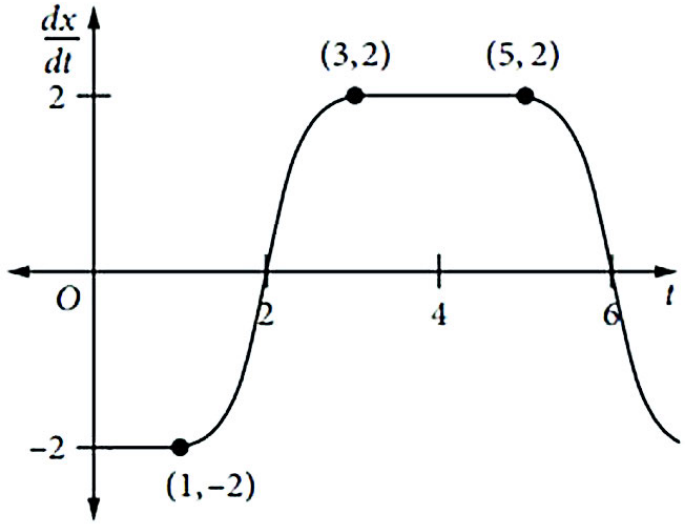
a) A particle is moving in Simple Harmonic Motion. **1**

This motion is described as $x = 5\cos(2t) - 6$,

where x is measured in metres and t in seconds.

Express acceleration as a function of x.

b) The graph shows the velocity $\frac{dx}{dt}$ of a particle as a function of time. Initially, the particle is at the origin.



(i) At what time is the displacement of the particle at a maximum? **1**

(ii) What is the displacement when $t = 4$? **1**

(iii) When does the particle have zero acceleration? **1**

(iv) At what time is the acceleration at a maximum? **1**

(v) Use the trapezoidal rule to estimate the displacement when $t = 6$. **1**

c) A body of mass 4kg is acted upon by a force $F = 48(5 - t)$ newtons for 5 seconds. **2**

If the body starts from rest at $x = 0$ then what is its velocity after 5 seconds?

~ Question 16 continues over the page ~

d) A particle is moving in simple harmonic motion along the x-axis.

Its velocity v, at x, is given by $v^2 = 24 - 8x - 2x^2$.

(i) Find all values of x for which the particle is at rest. **1**

(ii) Find an expression for the acceleration of the particle, in terms of x. **1**

(iii) Find the maximum speed of the particle. **2**

e) An object with mass m kilograms slides down a smooth inclined plane with velocity $\underset{\sim}{v}(t)$,

where t is the time in seconds after the object started sliding down the plane.

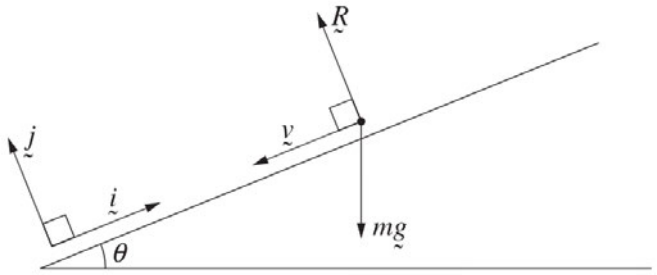
The inclined plane makes an angle θ with the horizontal, as shown in the diagram.

The normal reaction force is $\underset{\sim}{R}$. The acceleration due to gravity is $\underset{\sim}{g}$ and has magnitude g.

No other forces act on the object.

The vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors parallel and perpendicular, respectively, to the plane,

as shown in the diagram.



(i) Show that the resultant force on the object is $F = (-mg\sin\theta)\underset{\sim}{i}$. **2**

(ii) Given that the object is initially at rest,

find its velocity $\underset{\sim}{v}(t)$ in terms of g, θ, t and $\underset{\sim}{i}$. **1**

~ End of Question 16 ~

~ End of Examination ~

2026 Trial Higher School Certificate Examination

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \; r \neq 1$$

$$S = \frac{a}{1 - r}, \; |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

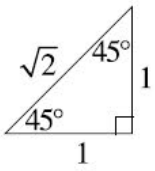
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

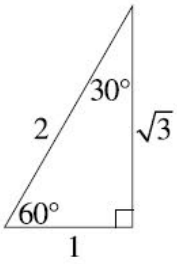
$$A = \frac{1}{2}ab\sin C$$



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$

Trigonometric identities

$$\sec A = \frac{1}{\cos A},\ \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A},\ \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A},\ \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$

$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1+t^2}$

$$\cos A = \frac{1-t^2}{1+t^2}$$

$$\tan A = \frac{2t}{1-t^2}$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

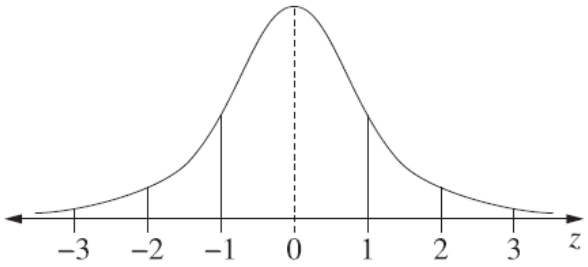
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E\left[(X-\mu)^2\right] = E\left(X^2\right) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x)\,dx$$

$$P(a < X < b) = \int_a^b f(x)\,dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1-p)^{n-r}$$

$$X \sim \text{Bin}(n, p)$$

$$\Rightarrow\ P(X = x) = \binom{n}{x} p^x (1-p)^{n-x},\ x = 0, 1, \ldots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1-p)$$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c \quad \text{where } n \neq -1$$

$$\int f'(x)\sin f(x)\,dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)\,dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)\,dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)\,dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^2+y^2}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_1x_2 + y_1y_2,$$

where $\underset{\sim}{u} = x_1\underset{\sim}{i} + y_1\underset{\sim}{j}$

and $\underset{\sim}{v} = x_2\underset{\sim}{i} + y_2\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$z = a + ib = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$$
$$= r^ne^{in\theta}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^2(x-c)$$

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HURLSTONE AGRICULTURAL HIGH SCHOOL

2026 Trial Higher School Certificate Examination
Mathematics Extension 2

Name _____ Teacher _____

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

Marking Guidelines HAHS 2026 Mathematics Extension 2 Trial Examination

Multiple Choice

Ques 1:

Since the coefficients are real, complex roots occur in conjugate pairs. Hence $P\left(\overline{2i-1}\right)=0$

Option D.

Ques 2:

Multiplying by $-i$ is a clockwise rotation by $\frac{\pi}{2}$ so the argument is $\frac{\pi}{4}$. Option A.

Ques 3:

$$i^i=\left(e^{i\frac{\pi}{2}}\right)^i$$
$$=e^{-\frac{\pi}{2}}$$

Option B

Ques :4

$\overrightarrow{OB}$ is obtained by:
rotating $\overrightarrow{OA}$ clockwise by 90° $\left(\times -i\right)$
and mutiplying by a scale factor of $\frac{2}{3}$

so, $B=3e^{i\frac{\pi}{10}}\times\frac{2}{3}e^{-i\frac{\pi}{2}}$
$$=2e^{-i\frac{2\pi}{5}}$$

Option A

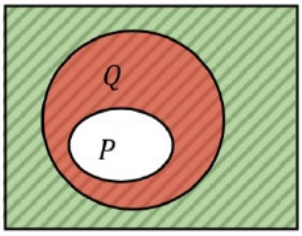
Ques 5:

When negating a statement, the universal quantifier "For all" (∀) flips to the existential quantifier "There exists" (∃), and vice versa. Finally, the conditional operator inside is negated, meaning > 0 flips to ≤ 0.

Option A

Ques 6:

An implication is logically equivalent to its contrapositive. Option C.



$\neg Q$ green shading

$\neg P$ diagonal stripes

Ques 7:

$$\begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \bullet \begin{bmatrix} 4 \\ -6 \\ -3 \end{bmatrix} = 2\times 4 + (-1)\times(-6) + 3\times(-3)$$

$$= 8 + 6 - 9$$

$$= 5$$

Option B

Ques 8:

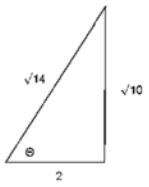
$\left|\overrightarrow{OA}\right| = \sqrt{4+1+9} = \sqrt{14}$

As $\overrightarrow{OA}.\overrightarrow{OB} = \left|\overrightarrow{OA}\right|.\left|\overrightarrow{OB}\right|\cos\theta$ then $6 = \sqrt{14}\times 3\times\cos\theta$

$\therefore \cos\theta = \dfrac{2}{\sqrt{14}}$

Since $\overrightarrow{OA}.\overrightarrow{OB} > 0$ and θ is in a triangle, then θ is acute.

Using the triangle, $\sin\theta = \dfrac{\sqrt{10}}{\sqrt{14}}$



$\therefore Area = \dfrac{1}{2}\times 3\times\sqrt{14}\times\dfrac{\sqrt{10}}{\sqrt{14}}$

$= \dfrac{3\sqrt{10}}{2}$

Option D

Ques 9:

This multiple choice question does not require a fully derived partial fractions decomposition. Consideration of the 4 options is an economical approach, by considering the merits of substituting each option.

Option C.

Ques 10:

The statement shows that $g'(x) = f(x)$, hence a primitive of the expression $f(x)g(x)$ can be gained by reversing the chain rule.

Option C.

2026 Yr12 HSC Assessment Task 4

Question 11 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MEX12-1
understands and uses different representations of numbers and functions to model, prove results and find solutions to problems in a variety of contexts

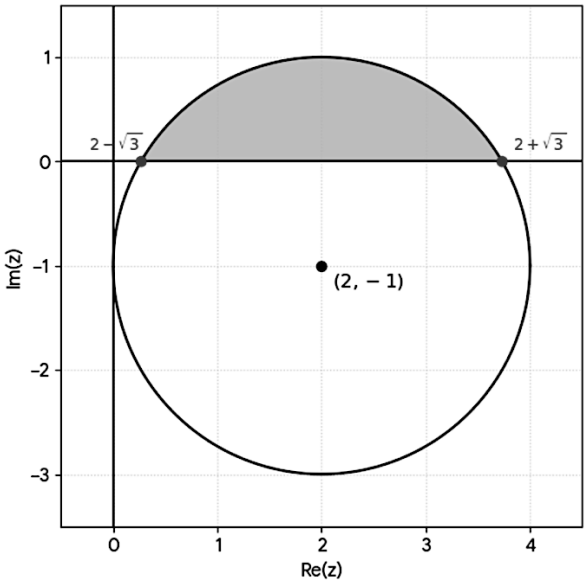
Solutions	Marking Guidelines
a i) $\frac{\left(1+i\sqrt{3}\right)(1-i)}{(1+i)(1-i)}=\frac{\left(1+\sqrt{3}\right)+i\left(\sqrt{3}-1\right)}{2}=\frac{1}{2}\left(1+\sqrt{3}\right)+i\frac{1}{2}\left(\sqrt{3}-1\right)$	1 mark attempts rationalising the denominator 2 marks correct solution
ii) $1+i\sqrt{3}=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ and $1+i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$ $\frac{\left(1+i\sqrt{3}\right)}{(1+i)}=\frac{2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)}{\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)}$ $=\sqrt{2}\left(\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)\right)$ $=\sqrt{2}\left(\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)\right)$	1 mark attempts to find modulus or argument of a complex number 2 marks correctly finds modulus and argument of numerator and denominator 3 marks correct modulus argument form of complex number
iii) $\sqrt{2}\left(\cos\left(\frac{\pi}{12}\right)+i\sin\left(\frac{\pi}{12}\right)\right)=\frac{1}{2}\left(1+\sqrt{3}\right)+i\frac{1}{2}\left(\sqrt{3}-1\right)$ equating the real parts $\sqrt{2}\cos\left(\frac{\pi}{12}\right)=\frac{1}{2}\left(1+\sqrt{3}\right)$ $=\frac{1}{2\sqrt{2}}\left(1+\sqrt{3}\right)$ $=\frac{\sqrt{2}+\sqrt{6}}{4}$	1 mark correct surd

b)

The region $|z-2+i| \le 2$ is a cirlce with the equation $(x-2)^2+(y+1)^2=4$

Setting $y=0$, we have the *x*-intercepts $x=2\pm\sqrt{3}$

Since we need $\text{Im}(z) \ge 0$, then we want the region of the cirlce that is above the real axis.



1 mark
draws circle approximately in correct position on complex plane

2 marks
draws circle with correct diameter and radius

3 marks
correct circle and correct region shaded but missing intercepts

4 marks
correct circle and region with intercepts and extremities of circle labelled

c)

Using De Moivre's Theorem, $z^2=(\cos\theta+i\sin\theta)^2=\cos 2\theta+i\sin 2\theta$

Using the Binomial Theorem, $z^2=(\cos\theta+i\sin\theta)^2=\cos^2\theta+2i\cos\theta\sin\theta-\sin^2\theta$

Equating the two expression for z^2, we have

$\cos 2\theta+i\sin 2\theta=\cos^2\theta+2i\cos\theta\sin\theta-\sin^2\theta$

Matching the reals, we have $\cos 2\theta=\cos^2\theta-\sin^2\theta$

1 mark
applies either De Moirve's Theorem or the Binomial Theorem

2 marks
clearly shows $\cos 2\theta=\cos^2\theta-\sin^2\theta$

d)

The vectors that represent two arms of the triangle are:

$$z_1-\frac{z_1-iRz_2}{1-iR}=\frac{z_1-iRz_1-(z_1-iRz_2)}{1-iR}=\frac{iR(z_1-z_2)}{1-iR}$$

and

$$z_2-\frac{z_1-iRz_2}{1-iR}=\frac{z_2-iRz_2-(z_1-iRz_2)}{1-iR}=\frac{z_1-z_2}{1-iR}$$

What we notice is that one vector is a iR factor of the other.

This means that the angle between them is a right angle, as multiplying by *i* rotates a vector by 90°. Hence, the points for the vertices of a right-angled triangle.

1 mark
Recognises that multiplying by i rotates a vector 90 degrees

2 marks
Finds vectors that represent sides of the triangle

OR

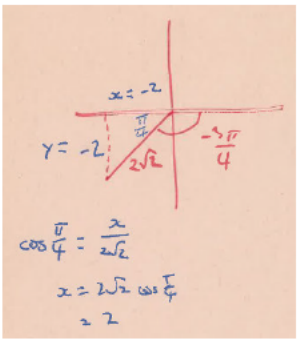
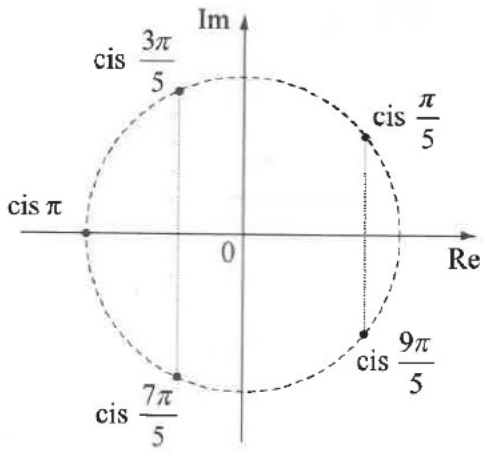
Correctly applies Pythagoras' Theorem

3 marks
Clearly justifies the vectors are a rotation of each other

Year 12	Mathematics Extension 2	Ass Task 4 2026 HSC
Question 12	Solutions and Marking Guidelines	TRIAL HSC

Outcomes Addressed in this Question

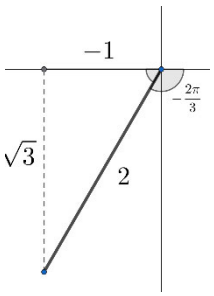
MEX 12-4 uses the relationship between algebraic and geometric representations of complex number techniques to prove results, model and solve problems.

Question	Solutions	Marking Guidelines
(a)	$2\sqrt{2}e^{\frac{3\pi}{4}i} = 2\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right)$ $= 2\sqrt{2}\left(-\frac{1}{\sqrt{2}}-i\frac{1}{\sqrt{2}}\right)$ $= -2-2i$ 	**2 marks:** correct solution (ie execution) **1 mark:** rewriting in mod arg form *and* attempting to calculate Note: diagram could work too
(b)	Simplest way is to do this: Start at $z=-1$, and rotate by angles of $\frac{2\pi}{5}$ So roots are $z_1 = \cos\frac{\pi}{5}+i\sin\frac{\pi}{5}$ $z_2 = \cos\frac{3\pi}{5}+i\sin\frac{3\pi}{5}$ $z_1 = \cos\pi+i\sin\pi = 1$ $z_1 = \cos\frac{7\pi}{5}+i\sin\frac{7\pi}{5}$ $= \cos\left(-\frac{3\pi}{5}\right)+i\sin\left(-\frac{3\pi}{5}\right)$ $z_1 = \cos\frac{9\pi}{5}+i\sin\frac{9\pi}{5}$ $= \cos\left(-\frac{\pi}{5}\right)+i\sin\left(-\frac{\pi}{5}\right)$ Let $z=\cos\theta+i\sin\theta$ $z^5 = \cos 5\theta+i\sin 5\theta = -1$ $\therefore \cos 5\theta = -1$ $5\theta = \pi,\ 3\pi,\ 5\pi,\ 7\pi,\ 9\pi$ $\therefore \theta = \frac{\pi}{5},\ \frac{3\pi}{5},\ 1,\ \frac{7\pi}{5},\ \frac{9\pi}{5}$ 	**3 marks:** correct solution Diagram needs indication of argument, either on diagram, or by labelling specific roots. **2 marks:** substantially correct solution. eg correct argument and diagram, but not written in mod-arg form… or equivalent merit. **1 mark:** partial progress towards correct solution Eg unlabelled diagram, or correct angles found only

X2 question 12 continued…

(c)(i)

$$-1-\sqrt{3}i = 2\left(\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right)$$



2 marks: correct modulus *and* argument (**and** written in mod-arg form)

1 mark: correct modulus *or* argument (or equivalent merit)

(c)(ii)

$$\begin{aligned}\left(-1-\sqrt{3}i\right)^{17} &= \left[2\left(\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right)\right]^{17}\\ &= 2^{17}\left(\cos\left(-\frac{34\pi}{3}\right)+i\sin\left(-\frac{34\pi}{3}\right)\right)\\ &= 2^{17}\left(\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)\right)\\ &= 2^{17}\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\\ &= -2^{16}+2^{16}\sqrt{3}i\\ &= -65\,536+65\,536\sqrt{3}i\end{aligned}$$

2 marks: correct solution from answer in (i)

1 mark: obtaining $\frac{2\pi}{3}$ or equivalent merit

X2 question 12 continued…

(d)(i)

RTP: $\left(z+z^{-1}\right)^2+\left(z+z^{-1}\right)-1=0$

$$\begin{aligned}
\text{LHS} &= \left(z+z^{-1}\right)^2+\left(z+z^{-1}\right)-1 \\
&= z^2+2+z^{-2}+z+z^{-1}-1 \\
&= z^{-1}+z^{-2}+z^2+z+1 \\
&= z^{-1}\left(z^5\right)+z^{-2}\left(z^5\right)+z^2+z+1 \qquad \left(\text{noting } z^5=1\right) \\
&= z^4+z^3+z^2+z+1 \qquad \text{(1 for getting to here, or legitim} \\
&= \frac{(z-1)\left(z^4+z^3+z^2+z+1\right)}{z-1} \quad \text{as } z\neq 1 \\
&= \frac{z^5-1}{z-1} \qquad \left(\text{but } z^5=1\right) \\
&= 0 \qquad \text{(1 for these last three lines)}
\end{aligned}$$

Or…

$$z^5=1$$

$$z^5-1=0$$

sum of roots

$$z^4+z^3+z^2+z+1=-\frac{b}{a}$$

$$z^4+z^3+z^2+z+1=0$$

$$z^2+z+1+z^{-1}+z^{-2}=0 \qquad \left(\text{dividing by } z^2\right)$$

$$\left(z^2+z^{-2}\right)+\left(z+z^{-1}\right)+1=0$$

noting $\left(z+z^{-1}\right)^2=z^2+2+z^{-2}$

$$z^2+z^{-2}=\left(z+z^{-1}\right)^2-2$$

$$\left(z^2+z^{-2}\right)+\left(z+z^{-1}\right)+1=0$$

$$\left(z+z^{-1}\right)^2-2+\left(z+z^{-1}\right)+1=0$$

$$\left(z+z^{-1}\right)^2+\left(z+z^{-1}\right)-1=0$$

2 marks: correct solution *(with unambiguous clarity)*

1 mark: progress towards correct solution
Eg showing LHS = $z^4+z^3+z^2+z+1$ in method 1.

Or showing $z^2+z+1+z^{-1}+z^{-2}=0$ in method 2

Or equivalent merit

(d)(ii)

$$z=e^{i\theta}$$

$$\begin{aligned}
z+z^{-1} &= e^{i\theta}+e^{-i\theta} \\
&= \cos\theta+i\sin\theta+\cos(-\theta)+i\sin(-\theta) \\
&= \cos\theta+i\sin\theta+\cos\theta-i\sin\theta
\end{aligned}$$

$$z+z^{-1}=2\cos\theta$$

$$\frac{z+z^{-1}}{2}=\cos\theta$$

1 mark: SHOW!
Correct solution.
Note: $e^{i\theta}+e^{-i\theta}=2\cos\theta$ needs to be shown or have an accompanying statement/explanation

Main thing is to show/use odd/even sin/cos

X2 question 12 continued…

(d)(iii)

From (i) $\left(z+z^{-1}\right)^2+\left(z+z^{-1}\right)-1=0$

$$z+z^{-1}=\frac{-1\pm\sqrt{1^2-4.1.(-1)}}{2(1)}$$

$$z+z^{-1}=\frac{-1\pm\sqrt{5}}{2}$$ [1 mark here]

$$\frac{z+z^{-1}}{2}=\frac{-1\pm\sqrt{5}}{4}$$

$$\text{ie}\quad \cos\theta=\frac{-1\pm\sqrt{5}}{4}$$

Now, $z=\text{cis}\,\frac{2\pi}{5}$ is a 5^{th} root of 1

as $\left(\text{cis}\,\frac{2\pi}{5}\right)^5=\text{cis}\;2\pi=1$

and $\frac{2\pi}{5}$ is in the fist quadrant, $\therefore\cos\frac{2\pi}{5}>0$

so, $\cos\frac{2\pi}{5}=\frac{-1+\sqrt{5}}{4}=\frac{\sqrt{5}-1}{4}$

2 marks: correct solution *including* showing or explaining why $\theta=\frac{2\pi}{5}$ can be used (not just subbing it in)

1 mark: meaningful progress towards correct solution
eg solving the (meaningful) quadratic, r equivalent merit

2026 Yr12 HSC Assessment Task 4

Question 13 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MEX12-2
chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings

Solutions	Marking Guidelines
a) Assume $\log_6 12$ is rational, that is: $\log_6 12 = \frac{a}{b}$ where a and b are coprime positive integers $6^{\frac{a}{b}} = 12$ $6^a = 12^b$ $2^a \times 3^a = \left(2^2\right)^b \times 3^b$ For the above equality to be true, both $a = 2b$ and $a = b$ need to be satisfied. However, this cannot be done for any positive integer. Hence, our assumption that $\log_6 12$ is rational is false. Thus $\log_6 12$ is irrational. Note: A number of students confused when LHS and RHS having common factors meant that the powers had to have a common factor (other than 1). See example: $2^3 = 8^1$.	1 mark sets up statement to contradict 2 marks applies index laws 3 marks validly reasons why there is a contradiction
b i) $(a-b)^2 \geq 0$ $a^2 - 2ab + b^2 \geq 0$ $2ab \leq a^2 + b^2$	1 mark Correct solution
ii) using par (i), we have: $2ab \leq a^2 + b^2 \quad (1)$ $2ac \leq a^2 + c^2 \quad (2)$ $2bc \leq b^2 + c^2 \quad (3)$ (1) + (2) + (3) gives: $2ab + 2ac + 2bc \leq 2a^2 + 2b^2 + 2c^2$ $ab + ac + bc \leq a^2 + b^2 + c^2$ $ab + ac + bc + 2(ab + ac + bc) \leq a^2 + b^2 + c^2 + 2(ab + ac + bc)$ $3(ab + ac + bc) \leq (a + b + c)^2$ Note: $(a+b+c)^2 = (a+b)^2 + 2(a+b)c + c^2$ $= a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$	1 mark uses part i) result 2 marks adds together inequalities 3 marks uses $(a+b+c)^2$ to arrive at inequality

c)

Prove true for base case, $n = 2$

$z_2 = 1 + i$

$|z_2| = \sqrt{1^2 + 1^2} = \sqrt{2}$ There for $|z_n| = \sqrt{n}$ is true for base case

Assume true for $n = k$, $|z_k| = \sqrt{k}$ for $z_k = z_{k-1}\left(1 + \frac{i}{|z_{k-1}|}\right)$

Prove true for $n = k+1$, $|z_k| = \sqrt{k+1}$ for $z_{k+1} = z_k\left(1 + \frac{i}{|z_k|}\right)$

$$z_{k+1} = z_k\left(1 + \frac{i}{|z_k|}\right)$$

$$|z_{k+1}| = \left|z_k\left(1 + \frac{i}{|z_k|}\right)\right|$$

$$|z_{k+1}| = |z_k|\left|\left(1 + \frac{i}{|z_k|}\right)\right|$$

$$= \sqrt{k}\left|1 + \frac{i}{\sqrt{k}}\right|$$

$$= \sqrt{k}\left|\frac{\sqrt{k} + i}{\sqrt{k}}\right|$$

$$= \sqrt{k}\frac{\left|\sqrt{k} + i\right|}{\left|\sqrt{k}\right|}$$

$$= \left|\sqrt{k} + i\right|$$

$$= \sqrt{\sqrt{k}^2 + 1^2}$$

$$= \sqrt{k+1}$$

So the statement is also true for $n = k+1$.

Hence the statement is true for $n \geq 2$ by mathematical induction

1 mark
tests base case

2 marks
uses assumption in inductive step

3 marks
correctly shows LHS = RHS with key steps shown

Year 12	Mathematics Extension 2	Ass Task 4 2026 HSC
Question 14	Solutions and Marking Guidelines	TRIAL HSC

Outcomes Addressed in this Question

MEX12-3 uses vectors to model and solve problems in two and three dimensions

Question	Solutions	Marking Guidelines
(a)	$\overrightarrow{RT} = 3\begin{bmatrix}-4\\-1\\2\end{bmatrix} = \begin{bmatrix}-12\\-3\\6\end{bmatrix}$ now, $\overrightarrow{RT} = \overrightarrow{OT} - \overrightarrow{OR}$ so, $\overrightarrow{OT} = \overrightarrow{RT} + \overrightarrow{OR}$ $= \begin{bmatrix}-12\\-3\\6\end{bmatrix} + \begin{bmatrix}2\\1\\-1\end{bmatrix}$ $= \begin{bmatrix}-10\\-2\\5\end{bmatrix}$ $\therefore$ the coordinates of T are $(-10,-2,5)$	**2 marks:** correct solution (ie execution) **1 mark:** correct expression for $\overrightarrow{RT}$ (or equivalent merit)
(b)	*Method 1* $\left\lvert \underset{\sim}{m} - \underset{\sim}{p} \right\rvert^2 = \left(\underset{\sim}{m} - \underset{\sim}{p}\right)\cdot\left(\underset{\sim}{m} - \underset{\sim}{p}\right)$ $= \underset{\sim}{m}\cdot\underset{\sim}{m} - 2\underset{\sim}{m}\cdot\underset{\sim}{p} + \underset{\sim}{p}\cdot\underset{\sim}{p}$ $= \lvert\underset{\sim}{m}\rvert^2 - 2\underset{\sim}{m}\cdot\underset{\sim}{p} + \lvert\underset{\sim}{p}\rvert^2$ $= 5^2 - 2(9) + 3^2$ $= 16$ $\therefore \quad \left\lvert \underset{\sim}{m} - \underset{\sim}{p} \right\rvert = 4$ *Method 2* $\underset{\sim}{m}\cdot\underset{\sim}{p} = \lvert\underset{\sim}{m}\rvert\,\lvert\underset{\sim}{p}\rvert\cos\theta$ $9 = 5\times 3\times\cos\theta$ $\cos\theta = \frac{3}{5}$ $\left\lvert \underset{\sim}{m} - \underset{\sim}{p} \right\rvert^2 = \lvert\underset{\sim}{m}\rvert^2 + \lvert\underset{\sim}{p}\rvert^2 - 2\lvert\underset{\sim}{m}\rvert\lvert\underset{\sim}{p}\rvert\cos\theta$ $\left\lvert \underset{\sim}{m} - \underset{\sim}{p} \right\rvert^2 = 5^2 + 3^2 - 2(5)(3)\left(\frac{3}{5}\right)$ $\left\lvert \underset{\sim}{m} - \underset{\sim}{p} \right\rvert^2 = 16$ $\left\lvert \underset{\sim}{m} - \underset{\sim}{p} \right\rvert = 4$	**2 marks:** correct solution (ie execution) **1 mark:** Substantial progress towards correct solution. ie correctly applies the relationship $\lvert\underset{\sim}{v}^2\rvert = \underset{\sim}{v}\cdot\underset{\sim}{v}$ *OR* finds $\cos\theta$, or equivalent merit.

Question 14 continued…

(c)(i)

From lines l_1 and l_2

$$x = 7 + \lambda = -1 + 2\mu \qquad \langle 1 \rangle$$
$$y = -1 + 3\lambda = 2 - 3\mu \qquad \langle 2 \rangle$$
$$z = 2 - 5\lambda = -6 + 6\mu \qquad \langle 3 \rangle$$

from $\langle 1 \rangle$ $\quad \lambda - 2\mu = -8$

from $\langle 2 \rangle$ $\quad \lambda + \mu = 1$

and $\langle 1 \rangle - \langle 2 \rangle$ $\quad -3\mu = -9$

$$\mu = 3$$
$$\lambda = -2$$

check $\langle 3 \rangle$ $\quad 2 - 5\lambda = -6 + 6\mu$

$$2 - 5(-2) = -6 + 6(3)$$
$$12 = 12$$

$\therefore$ lines intersect when $\lambda = -2,\ \mu = 3$.

sub $\lambda = -2$ into l_1

$$P = \begin{pmatrix} 7 + (-2) \\ -1 + 3(-2) \\ 2 - 5(-2) \end{pmatrix} = \begin{pmatrix} 5 \\ -7 \\ 12 \end{pmatrix}$$

3 marks: correct solution
Including verification in the third equation (confirming the lines actually meet, and are not skew) **and** correct point of intersection stated

2 marks: substantially correct solution
ie correct values of λ and μ found, and subbed in to find P… or equivalent merit

1 mark: progress toward correct solution
Eg correct system of three equations set up, or equivalent merit

(c)(i) was a good discriminator, since a correct answer *without* the consistency check in the third equation (i.e. a student who just solves two equations and asserts intersection) is a system that could easily describe skew lines — the verification step is where the real justification lives.

Note: many students used a combination of all three equations to find λ and μ in the first place, effectively verifying the lines were not skew and did, in fact intersect.

(c)(ii)

$$l_2 : \begin{bmatrix} -1 \\ 2 \\ -6 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix} \quad \text{and} \quad B \begin{bmatrix} 9 \\ k \\ 24 \end{bmatrix}$$

comparing x gives $\quad -1 + 2\mu = 9$

$$\mu = 5$$

comparing y gives $\quad 2 - 3\mu = k$

$$2 - 15 = k$$
$$k = -13$$

$\therefore \mu = 5$ and $k = -13$

2 marks: correct μ *and* k

1 mark: correct μ *or* k

(d)(i)

Question 14 continued…

$$\underset{\sim}{r} = \begin{bmatrix} \lambda+1 \\ \lambda \\ 2\lambda+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

1 mark: both correct position vector $\underset{\sim}{a}$ and direction vector $\underset{\sim}{b}$

(d)(ii)

$$\left| \underset{\sim}{r} - \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \right| = \sqrt{29}$$

$$\left| \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \right| = \sqrt{29}$$

$$\left| \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} \right| = \sqrt{29}$$

$$(\lambda-1)^2 + (\lambda+1)^2 + (2\lambda+3)^2 = 29$$

$$6\lambda^2 + 12\lambda - 18 = 0$$

$$\lambda^2 + 2\lambda - 3 = 0$$

$$(\lambda+3)(\lambda-1) = 0$$

$$\lambda = -3,\ 1$$

$$\lambda = -3: \quad P = \begin{pmatrix} 1-3 \\ 0-3 \\ 3-6 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ -3 \end{pmatrix}$$

$$\lambda = 1: \quad Q = \begin{pmatrix} 1+1 \\ 0+1 \\ 3+2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

3 marks: correct solution

2 marks: substantially correct solution
ie correctly simplified quadratic in λ, solved for $\lambda = 1, -3$

1 mark: progress towards correct solutions
Eg correct substitution of the parametrised point into the sphere equation
$|\text{(point)} - C|^2 = 29$

(d)(iii)

$$PQ = \sqrt{(-2-2)^2 + (-3-1)^2 + (-3-5)^2}$$

$$= \sqrt{16+16+64}$$

$$= \sqrt{96}$$

$$= 2\sqrt{24}$$

$$\neq 2\sqrt{29}$$

where $\sqrt{29}$ is the radius

$\therefore PQ$ is not the diameter of the sphere

2 marks: correct conclusion (not a diameter) **with clear justification from the comparison**

1 mark: valid method — either midpoint of PQ compared to C, or $|PQ|$ compared to $2\sqrt{29}$

Question 15

Outcomes Addressed in this Question

MEX12-5 applies techniques of integration to structured and unstructured problems.

Question	Solutions	Marking Guidelines
(a)	(a) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\int \frac{1}{2\sqrt{x}} \times \cos\sqrt{x}\,dx$ $= 2\sin\sqrt{x} + C$ Note: This amount of working is sufficient due to the reverse chain rule (reference sheet).	**a) 1 mark**: Correct answer.
(b)	(b) Let $u = e^x \rightarrow du = e^x dx$ Then: $\int \frac{e^x}{e^{2x}+3e^x+2}\,dx = \int \frac{du}{u^2+3u+2}$ $= \int \frac{du}{(u+1)(u+2)}$ Partial fractions separate this to become: $\int \frac{1}{u+1} - \frac{1}{u+2}\,du = \ln\lvert u+1\rvert - \ln\lvert u+2\rvert + C$ $= \ln\left\lvert\frac{u+1}{u+2}\right\rvert + C$ $= \ln\left(\frac{e^x+1}{e^x+2}\right) + C$ Which is true because $e^x > 0$. (Hence the absolute value signs can be discarded.) Notes: • Marks were awarded regardless of justification being given that the absolute value symbols could be discarded. This justification should be part of your best possible response, and could be part of a future marking scheme.	**b) 2 marks:** Correct solution. **1 mark:** Successfully forming the partial fraction
(c)	(c) Integration by parts. $\int_0^{\pi} x^2 \sin x\,dx = \left[-x^2\cos x\right]_0^{\pi} + \int_0^{\pi} 2x\cos x\,dx$ $= -\left(\pi^2\cos\pi - 0\right) + 2\left[x\sin x\right]_0^{\pi} - 2\int_0^{\pi}\sin x\,dx$ $= \pi^2 + 2(0-0) - 2\left[-\cos x\right]_0^{\pi}$ $= \pi^2 + 2(-1-1)$ $= \pi^2 - 4$ Common reasons for not gaining marks: Choosing the incorrect order for IBP; carelessly integrating sin or cos; carelessly evaluating cos π or cos 0.	**c) 3 marks:** Correct solution would include: -IBP twice, each one providing progress towards the solution; -Evaluation of final solution. **2 marks:** One component incorrect. **1 mark:** Relevant progress towards result.

(d)

Recurrence relation.

(i)

$$I_n = \int_0^1 x^n e^x \, dx = \left[x^n e^x \right]_0^1 - \int_0^1 n x^{n-1} e^x \, dx$$

$$= \left(e^1 - 0 \right) - n \int_0^1 x^{n-1} e^x \, dx$$

$$= e - I_{n-1}$$

(ii)

$$I_3 = e - 3I_2; \quad I_2 = e - 2I_1; \quad I_1 = e - I_0$$

$$I_0 = \int_0^1 e^x \, dx = \left[e^x \right]_0^1 = e - 1$$

$$\therefore I_1 = 1; \quad I_2 = e - 2; \quad I_3 = 6 - 2e$$

Notes:

- The method shown (Going "up" from I_0 to I_3) seemed to be the least likely to feature errors because it coped with the negative signs the best.
- The placement of the "$n \geq 1$" in the question led some responses to disregard $n=0$, which was accepted, but the aim of recurrence relations is to make the integration simple, and calculating I_1 required another IBP.

(d) (i) 1 mark: Correct solution including working to justify result.

(ii) 3 marks: Correct solution with justifications.
2 marks: Correct reasoning with one calculation error.
1 mark: Correct evaluation of I_0 or I_1.
1 mark: Correct expressions for each stage of the recurrence, either in steps or one expression.

(e)(i)

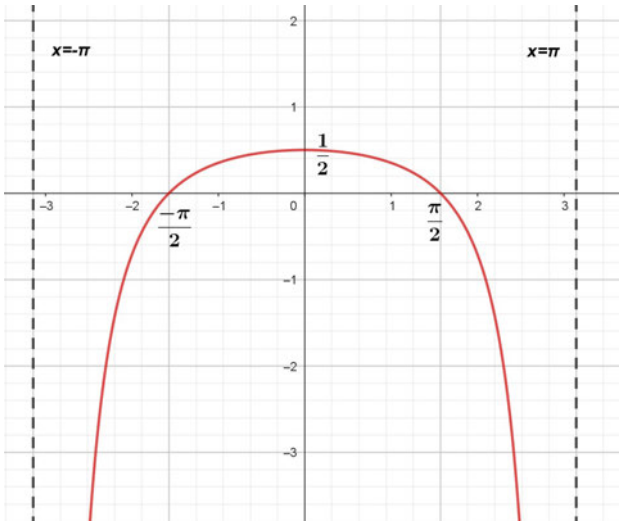
(e)(i)

The function is even, so we can just consider behaviour when $x \in [0, \pi)$.

The only stationary point occurs when $x = 0$.

A zero occurs when $x = \frac{\pi}{2}$.

The limit as $x \to \pi = -\infty$



(e) (i) 2 marks: Correct graph with maximum, intercepts and asymptotes labelled.
1 mark: Significant progress of shape and position of curve.

	(ii) $\text{Area} = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+\cos x}\,dx = 2\int_{0}^{\frac{\pi}{2}} \frac{1+\cos x}{1+\cos x} - \frac{1}{1+\cos x}\,dx$ $= 2\int_{0}^{\frac{\pi}{2}} 1\,dx - 2\int_{0}^{\frac{\pi}{2}} \frac{1}{1+\cos x}\,dx$ $= 2\left[\frac{\pi}{2} - 0\right] - 2\int_{0}^{1} \frac{2}{1+t^2+1-t^2}\,dt$ (t result simplification) $= \pi - 2\int_{0}^{1} 1\,dt$ $= \pi - 2(1-0) = \pi - 2 \text{ units}^2$ Alternate method that uses *t* results before splitting the fraction into 2 integrals: $\text{Area} = 2\int_{0}^{1} \frac{1-t^2}{1+t^2}\,dt$ (t result simplification) $= 2\left[\tan^{-1} t\right]_0^1 - 2\left[t - \tan^{-1} t\right]_0^1$ $= \frac{\pi}{2} - 2 + \frac{\pi}{2} = \pi - 2 \text{ units}^2$ Note: The alternate method above was the more popular, but the 1st method shown is more elegant, involving much easier simplifications and integrals.	(ii)**3 marks:** A correct solution would include: -Splitting the integrand into easier to handle parts and simplifying. -Determining an integral in terms of the *t*-results. -Evaluating the correct primitive. **2 marks:** 2 components from above correct. **1 mark:** 1 correct component. **Alternate: 3 marks:** -Simplifying the t result in the integrand. -Evaluating the primitive involving $\tan^{-1}$t terms. -Evaluating the primitive. **2 marks:** As above **1 mark:** As above.

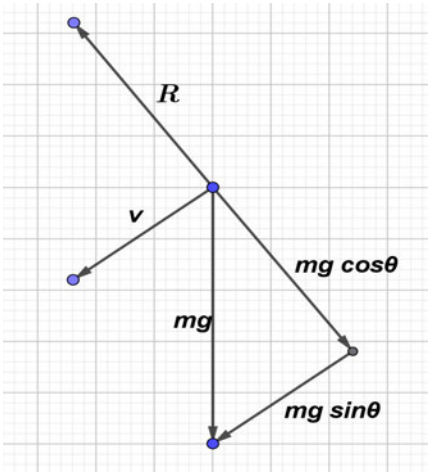
Solutions and Marking Guidelines

Question 16

Outcomes Addressed in this Question

MEX12-6 uses mechanics to model and solve practical problems.

Question	Solutions	Marking Guidelines
(a)	(a) $x = 5\cos(2t) - 6$ $\dot{x} = -10\sin 2t$ $\ddot{x} = -20\cos 2t$ $= -4(5\cos 2t)$ $= -4(x+6)$	**a) 1 mark**: Correct answer.
(b)	(b) (i) $t = 6$ only. The change in sign comes after a period of positive velocity. (ii) The symmetry of the integral from x=0 to x=4 around x=2 means that movement is equal in both directions. Hence displacement over the first 4 seconds is 0. (iii) $t \in (0,1)$ and when $t \in (3,5)$ (iv) $t = 2$ only. The inflection point is on an increasing curve, representing positive acceleration. (v) From part (ii), displacement is zero after the first 4 sec. Hence calculation is only needed from t=4 to t=6. There are 3 co-ordinate points that will form 2 applications of the trapezoidal rule. $\int_4^6 v\,dt \approx \frac{1}{2}(2+2(2)+0) = 3$ Hence the displacement is 3 units	**b)** **(i) 1 mark:** Correct answer. **(ii) 1 mark:** Correct answer. **(iii) 1 mark:** Correct answer. **(iv) 1 mark:** Correct answer. **(v) 1 mark:** Correct answer.
(c)	(c) F=ma gives us: $a = 12(5-t) = \frac{dv}{dt}$ $\int_0^v dv = 12\int_0^5 5-t\,dt$ $v = 12\left[5t - \frac{1}{2}t^2\right]_0^5$ $= 12\left(25 - 12\frac{1}{2}\right) = 150\text{ms}^{-1}$	**c) 2 marks:** Correct solution. **1 mark:** Correct expression for acceleration and choosing $a = \frac{dv}{dt}$.

(d)	(d) (i) $v^2 = 24 - 8x - 2x^2$ $0 = 2\left(12 - 4x - x^2\right)$ $0 = 2(6+x)(2-x)$ Hence v=0 at x=2, -6. (ii) $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{d}{dx}\left(12 - 4x - x^2\right)$ $= -4 - 2x$ (iii) We have been told this is SHM, so we know that maximum speed is where a=0 (sub into the answer to part (ii)), which is also the midpoint of the two extreme points of motion found in part (i). Both methods give x=-2. Sub into the equation in (i), $v^2 = 32$. Hence maximum speed = $\sqrt{32}$ or $4\sqrt{2}$ units.	**(d) (i) 1 mark:** Correct solutions. **(ii) 1 mark:** Correct solution. (iii) **2 marks:** Correct solution. **1 mark:** Correct x value.
(e)(i)	(e) (i) Forces diagram.  Resolving the forces: $\underset{\sim}{R} = \underset{\sim}{j}\left(-mg\cos\theta\right)$ $\therefore F_{\underset{\sim}{j}} = mg\cos\theta - mg\cos\theta = 0$ $\underset{\sim}{v} = -\underset{\sim}{i}\left(mg\sin\theta\right)$ $\therefore F_{\underset{\sim}{i}} = -mg\sin\theta$ $F = 0\underset{\sim}{j} - mg\sin\theta\underset{\sim}{i} = \left(-mg\sin\theta\right)\underset{\sim}{i}$	**(e) (i) 2 marks:** Solution that includes acknowledgement of both the $\underset{\sim}{i}$ and $\underset{\sim}{j}$ components. **1 mark:** Only considers $\underset{\sim}{i}$ direction without justification. Note: The "show that" given in the question means that responses that didn't show why $mg\cos\theta$ is not included in the final expression.
(ii)	(ii) F=ma gives us: $a = -g\sin\theta = \frac{dv}{dt}$ $\int_0^v dv = \int_0^t -gt\sin\theta\, dt$ Hence $\underset{\sim}{v}(t) = -gt\sin\theta\left(\underset{\sim}{i}\right)$	(ii) **1 mark:** Expressing a as $\frac{dv}{dt}$ and the integral dt.