Killara High School

Student Number: _____

2026 Higher School Certificate Trial

Mathematics Extension 2

General Instructions

- Reading time - 10 minutes
- Working time - 180 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show mathematical reasoning and/or calculations

Total Marks: 100

Section I – 10 marks (Pages 1–5)

- **Attempt Questions 1–10**
- **Allow about 15 minutes for this section**

Section II – 90 marks (Pages 6–13)

- **Attempt Questions 11–16**
- **Allow about 165 minutes for this section**

Question:	1	2	3	4	5	6	7	8	9
Marks:	1	1	1	1	1	1	1	1	1
Awarded:									

Question:	10	11	12	13	14	15	16		Total
Marks:	1	14	15	16	16	15	14		100
Awarded:									

Do not write on this page

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–10.

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1. How many distinct roots does the polynomial $\left(z^4-1\right)\left(7-6z-z^2\right)$ have, where $z \in \mathbb{C}$ **1**

   A. 3

   B. 4

   C. 5

   D. 6

2. A graph in the complex plane is defined by $|z-(4-3i)|=2$. Which of the following intersects the graph twice? **1**

   A. $|z-4|=1$

   B. $|z+9|=|z-9i|$

   C. $|z+3|=|z+4i|$

   D. $\text{Im}\,(z)=-5$

3. Those who know, know that if a Saiyan survives a near fatal battle, then they become stronger. **1**

   The converse of this statement is given by:

   A. If a Saiyan becomes stronger, then they survived a near fatal battle.

   B. If a Saiyan does not survive a near fatal battle, then they do not become stronger.

   C. If a Saiyan does not become stronger, then they did not survive a near fatal battle.

   D. When a Saiyan becomes stronger it is because they survived a near fatal battle.

4. Two forces P and Q act on a body. P acts in the direction of $\mathbf{i}$ with magnitude one newton and Q acts in the direction of $\mathbf{i} + \sqrt{3}\mathbf{j}$ with magnitude four newtons. **1**

   The magnitude of the total force acting on the body, in newtons, is:

   A. 3

   B. $\sqrt{15}$

   C. $\sqrt{17}$

   D. $\sqrt{21}$

5. Let f be a function with domain $[-\pi, 2\pi]$ where $f(x) = \sin^3(x)$. **1**

Using the substitution $u = \cos(x)$, the area bounded by the graph of f and the x-axis could be found by evaluating

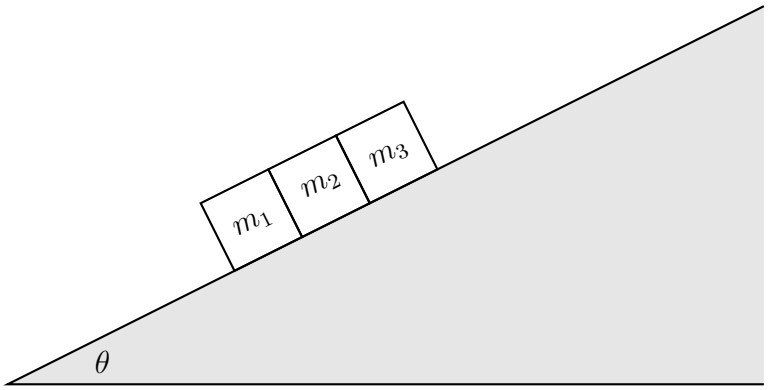
A. $-\int_{\pi}^{2\pi} \left(1 - u^2\right) du$

B. $3\int_{-1}^{1} \left(1 - u^2\right) du$

C. $-3\int_{0}^{\pi} \left(1 - u^2\right) du$

D. $3\int_{1}^{-1} \left(1 - u^2\right) du$

6. Three stationary masses are not connected but are making contact on an inclined plane, as shown. **1**



The surface on which mass m_1 is resting on is rough, with coefficient of friction, μ, and the section of the surface under objects m_2 and m_3 is smooth. The coefficient of friction, μ, is given by

A. $\dfrac{m_2 + m_3}{m_1} \tan(\theta)$

B. $\dfrac{m_1}{m_1 + m_2 + m_3} \tan(\theta)$

C. $\dfrac{m_2 + m_3}{m_1 + m_2 + m_3} \tan(\theta)$

D. $\dfrac{m_1 + m_2 + m_3}{m_1} \tan(\theta)$

7. Consider the statement: **1**

$$\forall \text{ primes } p : px = ab \text{ where } a, b, x \in \mathbb{Z} \implies py = a \cup pz = b \text{ where } y, z \in \mathbb{Z}$$

A. The contrapositive is true and the converse is true.

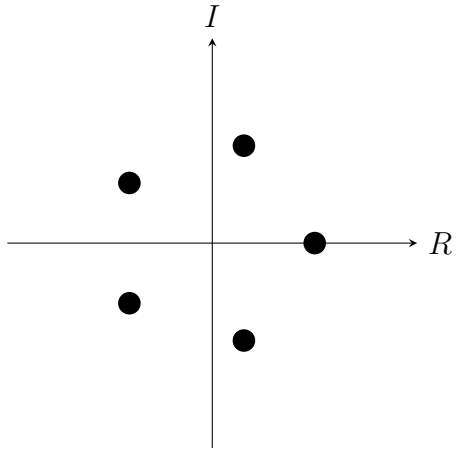
B. The contrapositive is true and the converse is false.

C. The contrapositive is false and the converse is true.

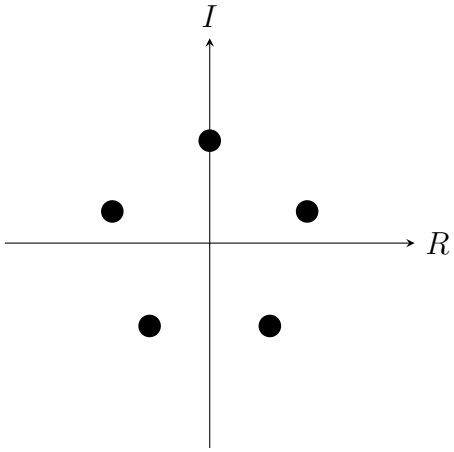
D. The contrapositive is false and the converse is false.

8. Which of the following could be the roots of $P(x) = z^5 - z^2 + z + c$? **1**

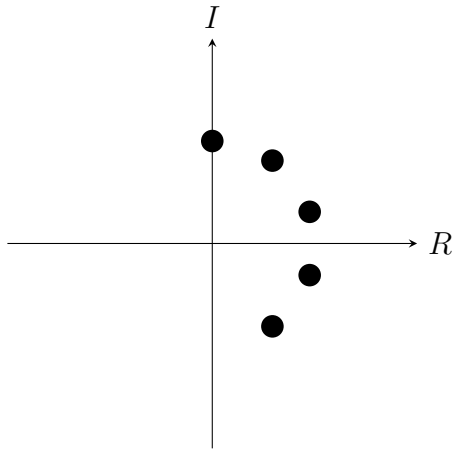
A.



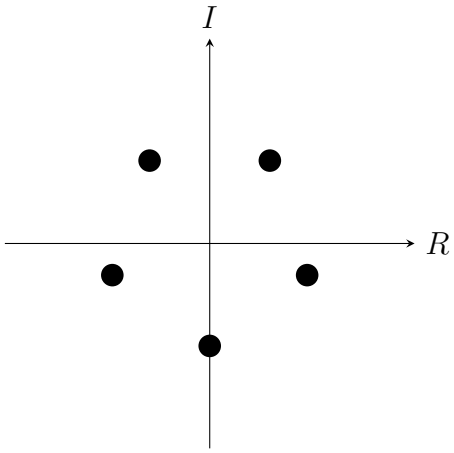
C.



B.



D.



9. A particle is moving along a straight line. At time t, its velocity is v and its displacement from a fixed origin is x. **1**

If $\dfrac{dv}{dx} = \dfrac{1}{2v}$, which of the following best describes the particle's acceleration and velocity?

A. constant acceleration and constant velocity.

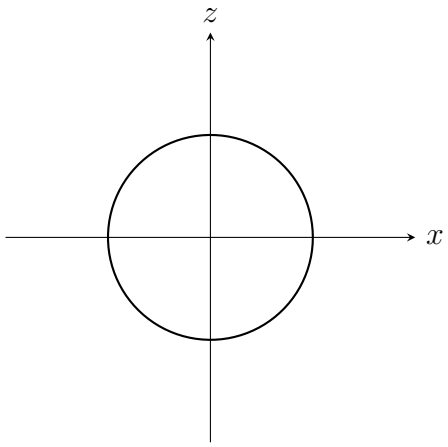
B. constant acceleration and decreasing velocity.

C. constant acceleration and increasing velocity.

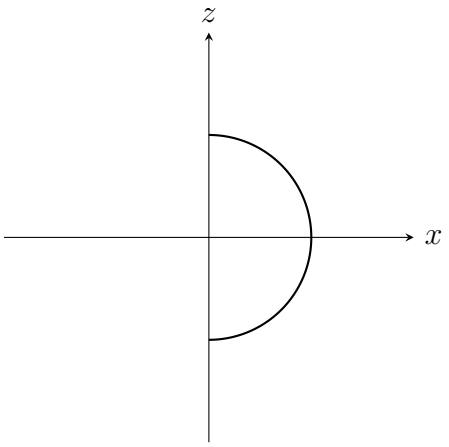
D. increasing acceleration and increasing velocity.

10. The vector $\big(\cos t, \ln(\sin t), \sin t\big)$ makes a continuous curve. Which of the following is the projection onto the x-z plane? **1**

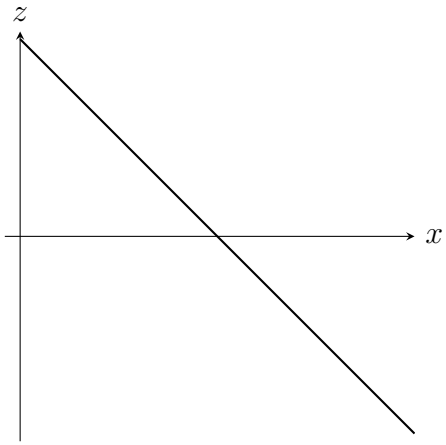
A.



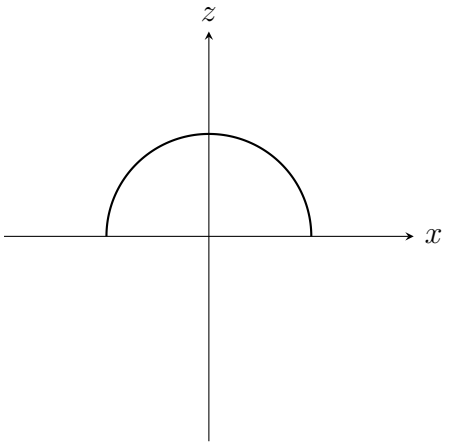
C.



B.



D.



Section II

90 marks
Attempt Questions 11–16
Allow about 165 minutes for this section.
Answer each question in the appropriate writing booklet.
All necessary working should be shown in every question.

Question 11 (14 Marks) Use a SEPARATE writing booklet.

(a) Evaluate $\displaystyle\int_0^1 \frac{x}{\sqrt{2-x}}dx$ **3**

(b) Let $z = a + ib$, where a and b are real numbers and $a \neq 0$.

(i) Express $|z|$ and $\tan(\arg z)$ in terms of a and b. **1**

(ii) Express $\dfrac{z}{3+5i}$ in the form $x + iy$, where x and y are real. **2**

(c) Find the angle between the two vectors $\underset{\sim}{u} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 3 \\ -3 \\ 1 \end{pmatrix}$, giving your answer in radians to 1 decimal place. **2**

(d) (i) Express $z = \sqrt{2} - i\sqrt{2}$ in modulus-argument form. **2**

(ii) Hence write z^{22} in the form $a + ib$, where a and b are real. **1**

(e) Sketch the region of the Argand plane defined by $\text{Re}\left(\dfrac{1}{z}\right) \leqslant \dfrac{1}{2}$ **3**

End of Question 11

Question 12 (15 Marks) Use a SEPARATE writing booklet.

(a) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \frac{dx}{4+2\sin 2x}$ using the substitution $t = \tan(x)$. **3**

(b) Given the free vectors $\underset{\sim}{u} = \begin{pmatrix} 8 \\ 7 \\ p \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}$ are perpendicular to each other, find the value of p. **2**

(c) (i) Prove $\sqrt{30}$ is irrational. **2**

(ii) Hence or otherwise prove that $\sqrt{2}+\sqrt{3}+\sqrt{5}$ is irrational. **2**

(d) On an Argand diagram, sketch the set of points z such that $\arg\left(\dfrac{z-3}{z+1}\right) = \dfrac{\pi}{2}$. **2**

(e) A cable car leaves from point C with a position vector, $\underset{\sim}{c} = \begin{pmatrix} 10 \\ 3 \\ 0 \end{pmatrix}$, it moves in the direction $\begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}$ with a velocity of 4.5 ms^{-1} and arrives at its final destination $\begin{pmatrix} 450 \\ a \\ b \end{pmatrix}$.

(i) Find an equation for the line formed by the cable car's journey, in the form $r = \mathbf{A} + t\mathbf{B}$ including appropriate restrictions for t (time in seconds). **3**

(ii) Find the length of the cable car journey in metres. **1**

End of Question 12

Question 13 (16 Marks) Use a SEPARATE writing booklet.

(a) Given free vector $\mathbf{p} = \begin{pmatrix} -10 \\ 8 \\ 7 \end{pmatrix}$ is orthogonal to both $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$ and $\mathbf{n} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}$ find the shortest distance between $\mathbf{r}$ and $\mathbf{n}$. **2**

(b) The velocity (in ms^{-1}) of a particle moving along the x-axis is given by the equation $v^2 = 14 + 12x - 2x^2$, where x is the displacement in metres from the origin. **3**

Show that this is simple harmonic motion and find the centre of motion, amplitude, period, and greatest velocity.

(c) (i) Express $\sin^2 x \cos 4x$ using complex numbers and leave your answer in Euler form. **2**

(ii) Hence, find $\int \sin^2 x \cos 4x dx$. **3**

(d) Given a, b and c are real numbers:

(i) Show that $a^2 + 9b^2 \geqslant 6ab$. **2**

(ii) Hence or otherwise, show that $a^2 + 5b^2 + 9c^2 \geqslant 3(ab + bc + ca)$. **2**

(iii) Hence if $a > b > c > 0$, show that $a^2 + 5b^2 + 9c^2 > 9bc$. **2**

End of Question 13

Question 14 (16 Marks) Use a SEPARATE writing booklet.

(a) Let $\omega = e^{\frac{4\pi i}{7}}$. Evaluate: **3**

$$(2+\omega)\left(2+\omega^2\right)\left(2+\omega^3\right)\left(2+\omega^4\right)\left(2+\omega^5\right)\left(2+\omega^6\right)$$

(b) An object with a mass 100kg is found to experience a resistive force, in Newtons, of one-tenth the square of its velocity in metres per second when it moves through the air.

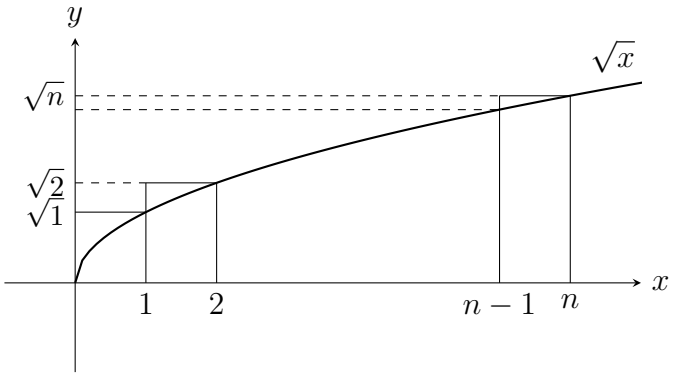
If the object falls from rest under gravity, assuming constant value $g = 9.8\ \text{ms}^{-2}$, calculate:

(i) Its terminal velocity. **1**

(ii) The maximum height, to the nearest metre, of the release point if it reaches 80% of its terminal velocity before striking the ground. **3**

(c) Let a, b, c be real positive numbers. Prove that $\dfrac{bc}{(a^2b+c)} \leqslant \dfrac{b+c}{1+a^2}$. **2**

(d) Consider the function $y = \sqrt{x}$.



(i) Show that the curve is increasing for all $x \geqslant 0$. **1**

(ii) Hence show that $\sqrt{1} + \sqrt{2} + \ldots + \sqrt{x} \geqslant \displaystyle\int_0^n \sqrt{x}dx = \frac{2}{3}n\sqrt{n}$. **2**

(iii) Use mathematical induction to show that $\sqrt{1} + \sqrt{2} + \ldots + \sqrt{n} \leqslant \dfrac{4n+3}{6}\sqrt{n}$, for all integers $n \geqslant 1$. **2**

(iv) Use parts (ii) and (iii) to estimate $\sqrt{1}+\sqrt{2}+\ldots+\sqrt{10000}$ to the nearest hundred. **2**

End of Question 14

Question 15 (15 Marks) Use a SEPARATE writing booklet.

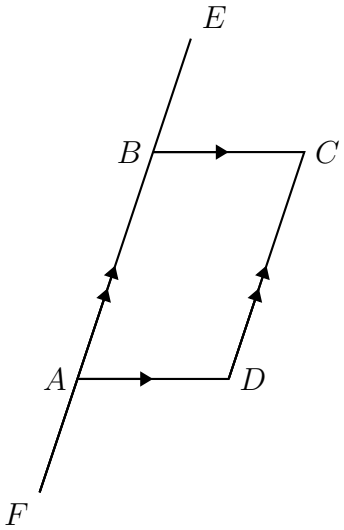
(a) $(x-22)^2+(y+12)^2+(z+22)^2=169$ and $(x-22)^2+(y+12)^2+(z-12)^2=225$ describe two spheres that intersect in a circle. **2**

Find the centre and radius of the circle.

(b) The non-zero complex numbers a, b, and c satisfy $\frac{a}{b}=\frac{b}{c}=\frac{c}{a}$. What are the possible values of $\frac{a+b-c}{a-b+c}$? **3**

(c) Find $\int \frac{2}{x^2+4x\sqrt{x}+4x}dx$ **3**

(d) $ABCD$ is a parallelogram. AB is produced to E and BA is produced to F such that $|\overrightarrow{BE}|=|\overrightarrow{AF}|=|\overrightarrow{BC}|$. Lines EC and FD are produced to meet at X.



(i) Prove that the lines EX and FX meet at right angles. **2**

(ii) If $\overrightarrow{EX}=\lambda\overrightarrow{EC}$ and $\overrightarrow{FX}=\mu\overrightarrow{FD}$ and $|\overrightarrow{AB}|=k|\overrightarrow{BC}|$, find the values of λ and μ. **2**

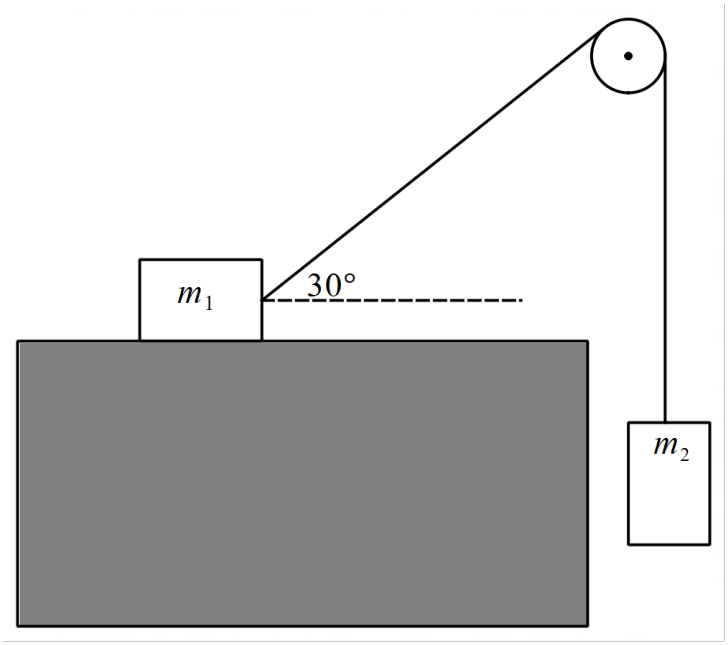
(iii) Find the values of λ and μ if $ABCD$ is a rhombus. **1**

(iv) If $|\overrightarrow{EX}|=|\overrightarrow{FX}|$, prove that $ABCD$ is a rectangle. **2**

End of Question 15

Question 16 (14 Marks) Use a SEPARATE writing booklet.

(a) Masses m_1 kg and m_2 kg are connected by string to a smooth pulley hanging from the ceiling. The end of the string attached to m_1 has an angle of 30° from the horizontal. The coefficient of friction between the surface and mass m_1 is 0.5. **4**



Mass m_1 is moving right and mass m_2 is moving downwards. Determine their acceleration if $m_2 = \sqrt{3}m_1$, and $g = 10\text{ms}^{-2}$.

(b) Let z_1 be a complex number. Let z_2 be the complex number obtained when z_1 is rotated counterclockwise about the origin by $\frac{\pi}{2}$, and let z_3 be the complex number obtained when z_2 is rotated counterclockwise about $4+3i$ by $\frac{\pi}{2}$. **2**

Then z_3 is the same as the point obtained when z_1 is rotated counterclockwise about ω by θ.

Find ω and θ.

(c) Let $I_n = \int x^m (\ln x)^n dx$.

(i) Show that $I_n = \dfrac{x^{m+1}(\ln x)^n}{m+1} - \dfrac{n}{m+1} I_{n-1}$. **2**

(ii) Hence, or otherwise, evaluate $\displaystyle\int_1^e x^3 (\ln x)^3 dx$ **2**

(d) A particle of unit mass is fired at a wall, the wall has a gelatinous cover that is 3m thick and provides a force resisting motion of 0.6mv. If the particle hits the wall with 10N force it will stick to the wall. **4**

Given that the particle is initially launched from the edge of the gelatinous cover at 8ms^{-1} at 30° from the horizontal.

What height will the particle strike the wall and will it stick to the wall?

END OF PAPER

KILLARA HIGH SCHOOL

Extension 2 Mathematics

2026 Higher School Certificate Trial

Student Number: _____

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.
Sample: $2 + 4 =$

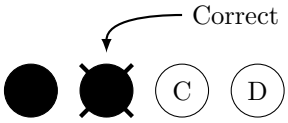
(A) 2 (B) 6 (C) 8 (D) 9



If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



1. A B C D
2. A B C D
3. A B C D
4. A B C D
5. A B C D
6. A B C D
7. A B C D
8. A B C D
9. A B C D
10. A B C D

Multiple Choice

1 C

2 A

3 B

4 D

5 D

6 D

7 A

8 A

9 C

10 D

Question 11

(3) a) $\int_0^1 \frac{x}{\sqrt{2-x}}\,dx = \int_0^1 \frac{x-2+2}{\sqrt{2-x}}\,dx$

① +2 -2 or u sub

$= -\int_0^1 \sqrt{2-x}\,dx + 2\int_0^1 (2-x)^{-1/2}\,dx$

② applied correctly & integrated correctly

$= \left[\frac{2}{3}(2-x)^{3/2}\right]_0^1 * - \left[4\sqrt{2-x}\right]_0^1$

③ with correct answer

$= \frac{2}{3} - \frac{4\sqrt{2}}{3} - (4 - 4\sqrt{2})$

$= -\frac{10}{3} + \frac{8}{3}\sqrt{2}$

(1) b) i) $|z| = \sqrt{9^2+5^2}$ $\tan(\text{Arg}(z)) = \tan(\tan^{-1}(\frac{5}{9}))$

① for both answers correct. $= \frac{5}{9}$.

(2) ii) $\frac{z}{3+5i} = \frac{(a+ib)(3-5i)}{(3+5i)(3-5i)}$

① multiply conjugate $= \frac{3a+3ib-5ai+5b}{34}$

② final answer in form. $= \frac{3a+5b}{34} - \frac{(5a-3b)}{34}i$.

(2) c) $\underset{\sim}{u} \cdot \underset{\sim}{v} = 2\times 3 + (-3)\times 2 + (-2)\times 1 = -2$

$|\underset{\sim}{u}| = \sqrt{2^2+2^2+(-2)^2} = 2\sqrt{3}$

$|\underset{\sim}{v}| = \sqrt{3^2+(-3)^2+1^2} = \sqrt{19}$

$\underset{\sim}{u} \cdot \underset{\sim}{v} = |\underset{\sim}{u}||\underset{\sim}{v}| \cos\theta.$

① for $\underset{\sim}{u} \cdot \underset{\sim}{v}$ or $|\underset{\sim}{u}|$ & $|\underset{\sim}{v}|$

$\cos\theta = \dfrac{-2}{2\sqrt{3}.\sqrt{19}}$

$\theta = \cos^{-1}\left(\dfrac{-2}{2\sqrt{3}.\sqrt{19}}\right)$

$= 1.7$ radians.

② for final equivalent answer with radians.

(2) d) i) $r = \sqrt{(\sqrt{2})^2+(-\sqrt{2})^2} = 2.$

① for r

$\tan\theta = -\dfrac{\sqrt{2}}{\sqrt{2}} = -1$

① for θ.

$\theta = -\pi/4$

So $z = 2e^{-\pi/4 i}$

① ii) $z^{22} = \left(2e^{-\pi/4 i}\right)^{22}$

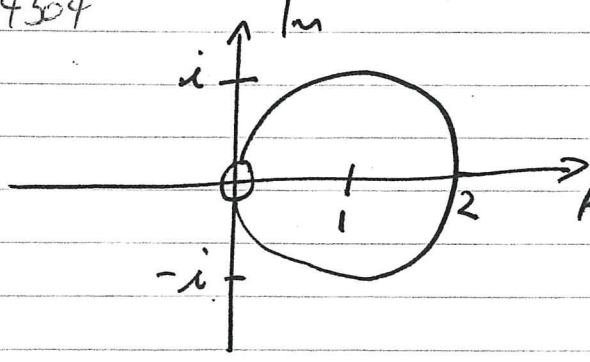
$= 2^{22} e^{-\frac{11}{2}\pi i}$

$= 2^{22}\cos\left(-\frac{11\pi}{2}\right) + 2^{22} i \sin\left(\frac{11\pi}{2}\right)$

$= 2^{22} i$ 1 for final answer.

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(3) e)



① working with x & y

① circle

① open at origin

12a) let $t = \tan(x)$

$$\frac{dt}{dx} = \sec^2 x$$

$$dt = 1 + t^2\, dx$$

$$dx = \frac{1}{1+t^2}\, dt$$

$x: 0 \sim \frac{\pi}{4}$

$t: 0 \sim \tan^{-1}\frac{\pi}{4}$

$$\int_0^{\tan\frac{\pi}{4}} \frac{1}{4 + 2\left(\frac{2t}{1+t^2}\right)} \times \frac{1}{t^2+1}\, dt$$

$$\int_0^{\tan\frac{\pi}{4}} \frac{1}{4 + 4t^2 + 4t}\, dt$$

$$\frac{1}{4}\int_0^{\tan\frac{\pi}{4}} \frac{1}{t^2 + t + 1}\, dt$$

$$\frac{1}{4}\int_0^{\tan\frac{\pi}{4}} \frac{1}{\left(t+\frac{1}{2}\right)^2 + \frac{3}{4}}\, dt.$$

$$= \frac{1}{4}\left[\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2\left(t+\frac{1}{2}\right)}{\sqrt{3}}\right)\right]_0^{\tan\frac{\pi}{4}}$$

$$= \frac{1}{2\sqrt{3}}\left[\tan^{-1}\left(\frac{\left(2\tan\frac{\pi}{4}\right) + \frac{1}{2}\right)}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$$

$$= \frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{2\tan\left(\frac{\pi}{4}\right)+1}{\sqrt{3}}\right) - \frac{\pi}{12\sqrt{3}}$$

c i) Assume $\sqrt{30} = \frac{a}{b}$

$30 = \frac{a^2}{b^2}$ $\quad a, b \in \mathbb{Z}$ and are coprime

$30b^2 = a^2$

$\therefore$ which means a^2 must be even

$\therefore a$ must be even

let $a = 2k$

$\sqrt{30} = \frac{2k}{b}$

$30b^2 = 4k^2$

$15b^2 = 2k^2$

$\therefore b^2$ must be even

$\therefore b$ must be even

$\therefore$ Contradiction as a & b should be coprime aswell

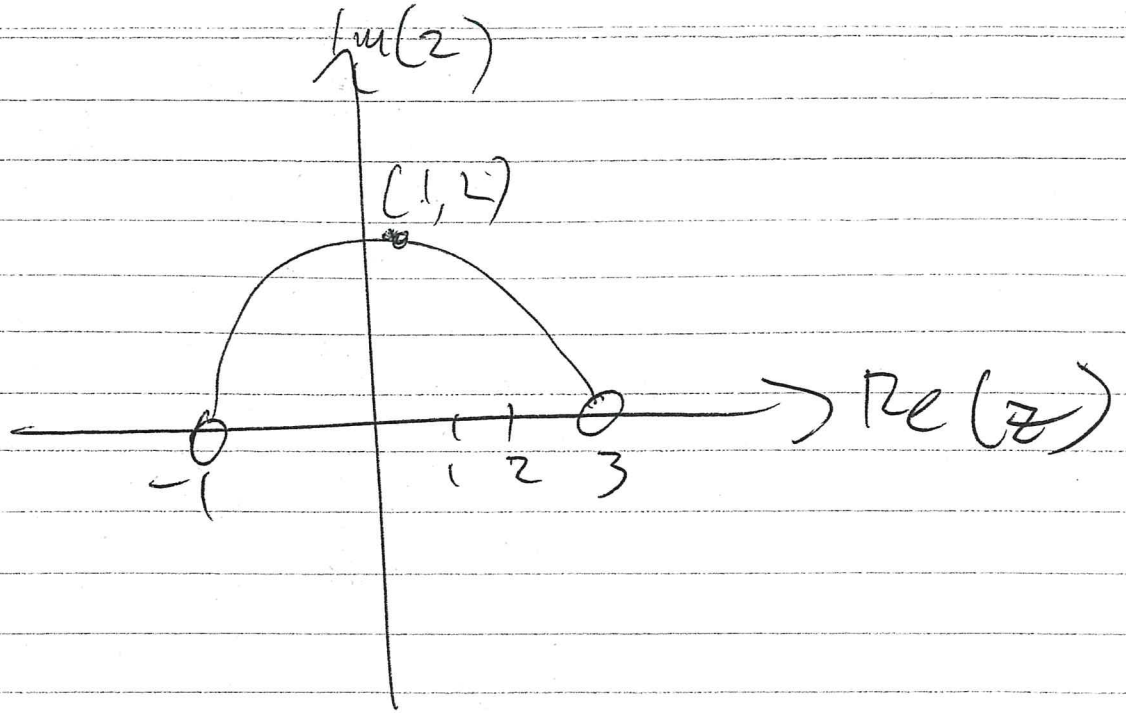
$\therefore \sqrt{30}$ is irrational

iv) $\sqrt{30} = \sqrt{2} \times \sqrt{3} \times \sqrt{5}$

$\therefore$ At least one of $\sqrt{2}, \sqrt{3}$ or $\sqrt{5}$ must be irrational as the product of rationals is rational.

$\therefore \sqrt{2} + \sqrt{3} + \sqrt{5}$ must be irrational

d)



(i) $\vec{r} = \begin{pmatrix} 10 \\ 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 3 \\ -3 \\ 1.5 \end{pmatrix} t$

$0 \leq t \leq \frac{440}{3}$

(ii) $\begin{pmatrix} 3 \\ -3 \\ 1.5 \end{pmatrix} \times \frac{440}{3} = \begin{pmatrix} 440 \\ -440 \\ 220 \end{pmatrix}$

$\left| \begin{pmatrix} 440 \\ -440 \\ 220 \end{pmatrix} \right| = \sqrt{440^2 + (-440)^2 + 220^2}$

$= 660 \text{m}.$

Question 13:

a) let vector $\underset{\sim}{q} = \begin{pmatrix} 5-1 \\ 0-2 \\ 1-3 \end{pmatrix}$ be a vector connecting $\underset{\sim}{r}$ & $\underset{\sim}{r}$

$= \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix}$ $\underset{\sim}{l} = \begin{pmatrix} -10 \\ 8 \\ 7 \end{pmatrix}$

~~Vector q~~

(1) use proj. $\text{proj}_{\underset{\sim}{l}} \underset{\sim}{q} = \left| \frac{\underset{\sim}{l} \cdot \underset{\sim}{q}}{|\underset{\sim}{l}|} \underset{\sim}{l} \right|$

(1) find length

$= \left| \frac{4x-10 + -2x8 + -2x7}{(\sqrt{10^2+8^2+7^2})^2} \cdot \underset{\sim}{l} \right|$

$= \left| \frac{-70}{\sqrt{213}} \begin{pmatrix} -10 \\ 8 \\ 7 \end{pmatrix} \right|$

$= \left| \begin{pmatrix} \frac{700}{\sqrt{213}} \\ \frac{-560}{\sqrt{213}} \\ \frac{-490}{\sqrt{213}} \end{pmatrix} \right| = \sqrt{\frac{490000}{\sqrt{213}}}$

$= \frac{70}{\sqrt{213}}$? $\frac{70}{\sqrt{213}}$ (2)

b) $v^2 = -2(x^2 - 6x - 7)$

$= -2(x^2 - 6x + 9 - 9 - 7)$

$= -2((x-3)^2 - 16)$

(1) → $= -2((x-3)^2 - 4^2)$

(1) → So motion about $x=3$ with Amplitude 4

Period $\frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$

greatest velocity at $x = 3 + 4 = 7$

$v^2 = -2((3-3)^2 - 4^2)$

$v^2 = 32$

(1) → $v = 4\sqrt{2}\ ms^{-1}$

(1) c) i) $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$ $\cos x = \frac{e^{ix} + e^{-ix}}{2}$.

$\sin^2 x = -\frac{1}{4}\left(e^{i2x} - 2 + e^{-i2x}\right)$ $\cos 4x = \frac{e^{i4x} + e^{-i4x}}{2}$

so $\sin^2 x \cos 4x = -\frac{1}{8}\left(e^{2xi} + e^{-2ix} - 2\right)\left(e^{i4x} + e^{-i4x}\right)$.

(2) - final answer. $= -\frac{1}{8}\left(e^{6xi} + e^{-2xi} + e^{i2x} + e^{-6xi} - 2e^{i4x} - 2e^{-4xi}\right)$

ii) $\int \sin^2 x \cos 4x\,dx = -\frac{1}{8}\int e^{6xi} + e^{-6xi} + e^{2xi} + e^{-2xi} - 2e^{4xi} - 2e^{-4xi}\,dx$

$= -\frac{1}{8}\left(\frac{e^{6xi}}{6i} - \frac{e^{-6xi}}{6i} + \frac{e^{2xi}}{2i} - \frac{e^{-2xi}}{2i}\right.$

(1) $= \frac{-1}{8}\int 2\cos 6x + 2\cos 2x - 4\cos 4x\,dx \Big/ \left(\frac{e^{4xi}}{2i} + \frac{e^{-4xi}}{2i}\right)$

(2) $= -\frac{1}{8}\left[\frac{2\sin 6x}{6} + \frac{2\sin 2x}{2} - \frac{4\sin 4x}{4}\right] + C$.

(3) $= \frac{\sin 4x}{8} - \frac{\sin 2x}{8} - \frac{\sin 6x}{24} + C$

-1 for sign error or integration error or missing $+C$

① d) i) $(a-3b)^2 \geq 0$ $\quad a, b \in \mathbb{R}$.

$a^2 - 6ab + 9b^2 \geq 0$

② $\therefore a^2 + 9b^2 \geq 6ab$. as required

ii) Similarly to (i), $b^2 + 9c^2 \geq 6bc$

① $a^2 + 9c^2 \geq 6ac$

adding all 3 ~~eq~~ inequalities together:

$a^2 + 9b^2 + b^2 + 9c^2 + a^2 + 9c^2 \geq 6ab + 6bc + 6ac$

$2a^2 + 10b^2 + 18c^2 \geq 6(ab + bc + ac)$

② $a^2 + 5b^2 + 9c^2 \geq 3(ab + bc + ac)$

as required.

iii) as $a > b > c > 0$

① $a > 6c$ $\quad a > b$

$ab > bc$ $\quad ac > bc$

hence $a^2 + 5b^2 + 9c^2 \geq 3(ab + bc + ac)$

$\geq 3(bc + bc + bc)$

② $> 3(3bc)$

$a^2 + 5b^2 + 9c^2 > 9bc$ as required

very poor notation & setting out

(a) $w = e^{\frac{4\pi i}{7}}$

w is a 7th root of unity

let $P(z) = z^7 - 1 = (z-1)(z-w)(z-w^2)\ldots(z-w^6)$

$P(-2) = -2^7 - 1 = (-2-1)(-2-w)(-2-w^2)\ldots(-2-w^6)$

$-129 = -3(-1)^6(2+w)(2+w^2)\ldots(2+w^6)$

$43 = (2+w)(2+w^2)(2+w^3)(2+w^4)(2+w^5)(2+w^6)$

b) i) $\ddot{x} = 100 \times 9.8 - \frac{v^2}{10}$

$100a = 100 \times 9.8 - \frac{v^2}{10}$

let $a = 0$

$9800 = v^2$

$v = \sqrt{9800}$

$= 70\sqrt{2}\ m\,s^{-1}$

ii) $\ddot{x} = \frac{9800 - v^2}{1000}$

$v\frac{dv}{dx} = \frac{9800 - v^2}{1000}$

$\int \frac{-2v}{9800 - v^2} dv = \int -\frac{1}{500}\, dx$

$$= -\frac{1}{2}\ln|1000g - v^2| = -\frac{x}{500} + C$$

$x = 0 \quad v = 0$

$$\ln|1000g| = C$$

$$\ln|1000g - v^2| = \frac{x}{500} + \ln(1000g)$$

$$x = -500 \ln\left|\frac{1000g - v^2}{1000g}\right|.$$

let $v = 56\sqrt{2}$

$$x = -500 \ln\left|\frac{1000 \times 9.8 - (56\sqrt{2})^2}{1000 \times 9.8}\right|$$

$$\approx 511 \text{ m}.$$

c) RHS - LHS

$$= \frac{b+c}{1+a^2} - \frac{bc}{a^2b+c}$$

$$= \frac{(b+c)(a^2b+c) - bc(1+a^2)}{(1+a^2)(a^2b+c)}$$

$$= \frac{a^2b^2 + bc + a^2bc + c^2 - bc - a^2bc}{(1+a^2)(a^2b+c)}$$

$$= \frac{a^2b^2 + c^2}{(1+a^2)(a^2b+c)} \geq 0$$

as a, b & c are $\in \mathbb{Z}^+$.

$$\therefore \frac{b+c}{1+a^2} \geq \frac{bc}{a^2b+c}$$

i) $f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f'(x) \geq 0$ as $x \in \mathbb{R}$,

$\therefore$

ii) Sum of Area of rectangles $\geq$ Area under curve
as $f(x)$ is an increasing function

$$\int_0^n \sqrt{x} = \frac{2}{3} x^{\frac{3}{2}} \Big|_0^n$$

$$= \frac{2n\sqrt{n}}{3}$$

iii) Consider $n = 1$

$$LHS = \sqrt{1} < \frac{7}{6}\sqrt{1} = RHS$$

$\therefore$ True for $n = 1$

Assume true for $n = k$

$$\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots \sqrt{k} \leq \frac{4k+3}{6}\sqrt{k+1}$$

prove true for $n = k+1$

$$LHS = \frac{4k+3}{6}\sqrt{k} + \sqrt{k+1} \quad \text{(by assumption)}$$

$$\leq \frac{4k+3}{6}\sqrt{k+1} + \frac{6\sqrt{k+1}}{6} = \frac{4k+9}{6}\sqrt{k+1}$$

Question 15

a) a typo means the two spheres do not intersect, so evidence to that effect referencing the sum of the radii and the distance between the centres will earn 2 marks. (2)

b) recognise $a = r_a \operatorname{cis}\theta_a$, $b = r_b \operatorname{cis}\theta_b$, $c = r_c \operatorname{cis}\theta_c$ (2)

$$\frac{r_a \operatorname{cis}\theta_a}{r_b \operatorname{cis}\theta_b} = \frac{r_b \operatorname{cis}\theta_b}{r_c \operatorname{cis}\theta_c} = \frac{r_c \operatorname{cis}\theta_c}{r_a \operatorname{cis}\theta_a}$$

(1) gives $\frac{r_a}{r_b} = \frac{r_b}{r_c} = \frac{r_c}{r_a}$ & $\theta_a - \theta_b = \theta_b - \theta_c = \theta_c - \theta_a$

(2) for roots of unity so r must be 1 & θ's evenly distributed forming 3rd roots of unity

(3). giving solutions $\boxed{1,\ \operatorname{cis}\frac{2\pi}{3},\ \operatorname{cis}\frac{4\pi}{3}}$

as $\frac{qb+c-2c}{qb+c-2b} = \frac{c}{b}$ as $qb+c=0$ for roots of unity

(3)

c) $\int \frac{2}{x^2+4\sqrt{x}+4x}\,dx = 2\int \frac{u}{u^4+4u^3+4u^2}\,du$

let $u=\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx.$

$= 2\int \frac{1}{u(u+2)^2}\,du.$

① substitution

$= 2\int -\frac{1}{4(u+2)} - \frac{1}{2(u+2)^2} + \frac{1}{4u}\,du.$

① partial fractions

$= -\frac{1}{2}\int \frac{1}{u+2}\,du - \int \frac{1}{(u+2)^2}\,du + \frac{1}{2}\int \frac{1}{u}\,du.$

$v=u+2.$

$= -\frac{1}{2}\ln(u+2) + \frac{1}{u+2} + \frac{1}{4}\ln u$

$u=\sqrt{x}$

$= \frac{\ln(\sqrt{x})}{4} + \frac{1}{\sqrt{x}+2} - \frac{\ln(\sqrt{x}+2)}{2} + C$

$\ln(\sqrt{x}) = \frac{\ln x}{2}$

① $= \frac{\ln x}{2} + \frac{2}{\sqrt{x}+2} - \ln(\sqrt{x}+2) + C$

correct integration with logs.

(3)

d) i) $\vec{EC} = \underset{\sim}{q} - \underset{\sim}{b}$ & $\vec{FD} = \underset{\sim}{q} + \underset{\sim}{b}$

① $\underset{\sim}{q}$ & $\underset{\sim}{b}$

$$\vec{EC} \cdot \vec{FD} = (\underset{\sim}{q} - \underset{\sim}{b}) \cdot (\underset{\sim}{q} + \underset{\sim}{b})$$

$$= |\underset{\sim}{q}|^2 - |\underset{\sim}{b}|^2$$

② conclusion

$$\vec{EC} \perp \vec{FD} \Rightarrow \vec{EX} \perp \vec{FX} \quad (=0)$$ hence

(2)

ii) $\vec{EX} = \lambda (\underset{\sim}{q} - \underset{\sim}{b})$

$\vec{EX} = \vec{EF} + \vec{FX}$

(2)

① $\lambda \underset{\sim}{q} - \lambda \underset{\sim}{b} = -\left(2 + \frac{k|\underset{\sim}{q}|}{|\underset{\sim}{b}|}\right)\underset{\sim}{b} + \mu (\underset{\sim}{q} + \underset{\sim}{b})$

$$= \left(\mu - 2 - \frac{k|\underset{\sim}{q}|}{|\underset{\sim}{b}|}\right)\underset{\sim}{b} + \mu \underset{\sim}{q}$$

equating vectors

$\lambda = \mu$

$$-\lambda = \mu - 2 - \frac{k|\underset{\sim}{q}|}{|\underset{\sim}{b}|}$$

$$2\mu = 2 + \frac{k|\underset{\sim}{q}|}{|\underset{\sim}{b}|}$$

(2)

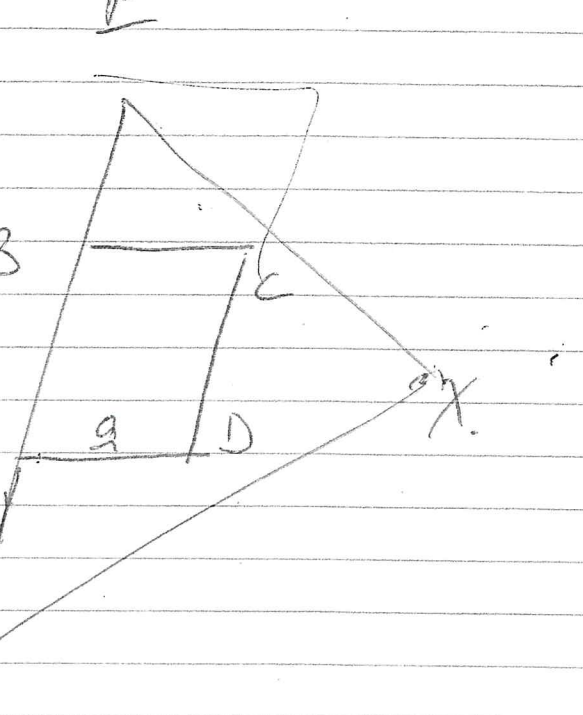
$$\mu = 1 + \frac{k|\underset{\sim}{q}|}{|\underset{\sim}{b}|}$$

$$\lambda = 1 + \frac{k|\underset{\sim}{q}|}{|\underset{\sim}{b}|}$$

iii) for ABCD rhombus, $k=1$

① $\therefore \mu = \lambda = 1 + \frac{|\underset{\sim}{a}|}{|\underset{\sim}{b}|}$ (1)

iv) (2)



$|\vec{EX}| = |\vec{FX}|$

then $|\lambda(\underset{\sim}{a}-\underset{\sim}{b})| = \mu|\underset{\sim}{a}+\underset{\sim}{b}|$

$|\underset{\sim}{a}-\underset{\sim}{b}|^2 = |\underset{\sim}{a}+\underset{\sim}{b}|^2$

$(\underset{\sim}{a}-\underset{\sim}{b})\cdot(\underset{\sim}{a}-\underset{\sim}{b}) = (\underset{\sim}{a}+\underset{\sim}{b})\cdot(\underset{\sim}{a}+\underset{\sim}{b})$

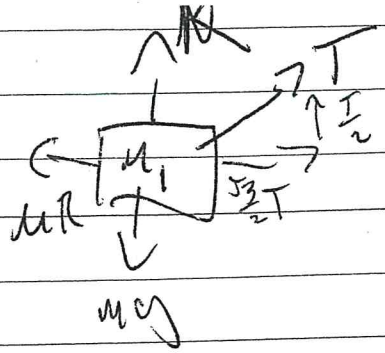
$|\underset{\sim}{a}|^2+|\underset{\sim}{b}|^2-2\underset{\sim}{a}\underset{\sim}{b} = |\underset{\sim}{a}|^2+|\underset{\sim}{b}|^2+2\underset{\sim}{a}\underset{\sim}{b}$

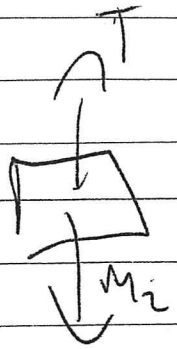
$4\underset{\sim}{a}\underset{\sim}{b} = 0$

so $\underset{\sim}{a} \perp \underset{\sim}{b}$

$\therefore$ ABCD is a rectangle

16a)





$m_2 a_2 = m_2 g - T$

Horizontal

$m_1 a_1 = \frac{\sqrt{3}}{2} T \cos - \mu (mg - \frac{1}{2} T)$ 0

$a_1 = a_2$

given $m_2 = \sqrt{3} m_1$

$\sqrt{3} m_1 a = 10\sqrt{3} m_1 - T$

$T = 10\sqrt{3} m_1 - \sqrt{3} m_1 a$

$= \sqrt{3} m_1 (10 - a)$

sub into Horizontal of m_1

$m_1 a = \left(\frac{\sqrt{3}}{2} + \frac{1}{4}\right) T - 5 m_1$

$m_1 a = \left(\frac{\sqrt{3}}{2} + \frac{1}{4}\right)\left(\sqrt{3} m_1 (10 - a)\right) - 5 m_1$

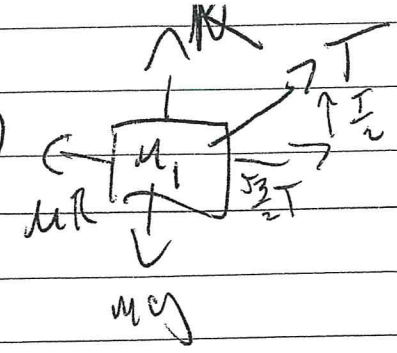
$= \frac{3}{2} m_1 (10 - a) + \frac{\sqrt{3}}{4} m_1 (10 - a) - 5 m_1$

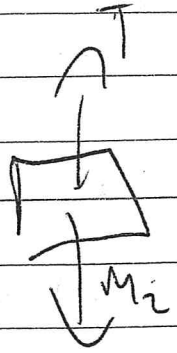
$= 15 m_1 - \frac{3}{2} a + 5\sqrt{3} m_1 - \frac{\sqrt{3}}{4} a - 5$

$a\left(1 + \frac{3}{2} + \frac{\sqrt{3}}{4}\right) = \frac{30 + 5\sqrt{3}}{2}$

$a\left(\frac{4 + 6 + \sqrt{3}}{4}\right) = \frac{5\sqrt{3} + 20}{2}$

16a)





$m_2 a_2 = m_2 g - T$

Horizontal

$m_1 a_1 = \frac{\sqrt{3}}{2} T \cos - \mu (mg - \frac{1}{2} T)$ ①

②

given $m_2 = \sqrt{3} m_1$

$\sqrt{3} m_1 a = 10\sqrt{3} m_1 - T$

$T = 10\sqrt{3} m_1 - \sqrt{3} m_1 a$

$= \sqrt{3} m_1 (10 - a)$

sub into Horizontal of m_1

$m_1 a = \left(\frac{\sqrt{3}}{2} + \frac{1}{4}\right) T - 5 m_1$

$m_1 a = \left(\frac{\sqrt{3}}{2} + \frac{1}{4}\right)\left(\sqrt{3} m_1 (10 - a)\right) - 5 m_1$

$= \frac{3}{2} m_1 (10 - a) + \frac{\sqrt{3}}{4} m_1 (10 - a) - 5 m_1$

$= 15 m_1 - \frac{3}{2} a + 5\sqrt{3} m_1 - \frac{\sqrt{3}}{4} a - 5$

$a\left(1 + \frac{3}{2} + \frac{\sqrt{3}}{4}\right) = \frac{30 + 5\sqrt{3}}{2}$

$a\left(\frac{4 + 6 + \sqrt{3}}{4}\right) = \frac{5\sqrt{3} + 20}{2}$

$$a = \frac{5\sqrt{3}+30}{2}\cdot\frac{4^2}{4+6+\sqrt{3}}$$

$$= \frac{30+5\sqrt{3}}{10+\sqrt{3}}$$

$$\approx 3.29526821\ldots$$

$$\approx 3.3\ m^2$$

b) we have $z_2 = e^{i\frac{\pi}{2}} z_1 = iz_1$ and

$$z_3 = e^{i\frac{\pi}{2}}(z_2 - 4 - 3i) + 4 + 3i$$

$$= i(iz_1 - 4 - 3i) + 4 + 3i$$

$$= -z_1 - 4i + 3 + 4 + 3i$$

$$= -z_1 + 7 - i$$

if z_3 is obtained by rotating z_1 about w by θ

$$z_3 = e^{i\theta}(z_1 - w) + w$$

$$= e^{i\theta} z_1 - e^{i\theta} w + w$$

since $z_3 = -z_1 + 7 - i$

then $e^{i\theta} = -1$ & $\theta = \pi$

and $-e^{i\pi} w + w = 7 - i$

$$2w = 7 - i$$

$$w = \frac{7}{2} - \frac{1}{2}i$$

(i) $I_n = \int x^m (\ln x)^n \, dx$

$$= \frac{x^{m+1}(\ln x)^n}{m+1} - \int \frac{x^{m+1}}{m+1} \times \frac{n}{x} \times (\ln x)^{n-1} \, dx$$

$$= \frac{x^{m+1}(\ln x)^n}{m+1} - \frac{n}{m+1} \int x^m (\ln x)^{n-1} \, dx$$

$$= \frac{x^{m+1}(\ln x)^n}{m+1} - \frac{n}{m+1} I_{n-1}$$

(ii) let $m=3$ & $n=3$

$$I_3 = \frac{x^4(\ln x)^3}{4} \Big|_1^e - \frac{3}{4} I_2$$

$$I_3 = \frac{e^4}{4} - \frac{3}{4}\left[\frac{x^4(\ln x)^2}{4}\Big|_1^e - \frac{2}{4} I_1\right]$$

$$= \frac{e^4}{4} - \frac{3}{4}\left[\frac{e^4}{4} - \frac{1}{2}\left(\frac{x^4 \ln x}{4}\Big|_1^e - \frac{1}{4}\int x^3 dx\right)\right]$$

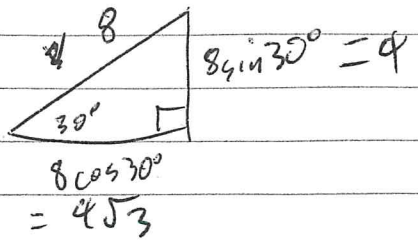
$$= \frac{e^4}{4} - \frac{3e^4}{16} + \frac{3}{8}\left(\frac{e^4}{4} - \frac{x^4}{16}\Big|_1^e\right)$$

$$= \frac{e^4}{4} - \frac{3e^4}{16} + \frac{3e^4}{32} - \frac{3e^4}{128} + \frac{3}{128}$$

$$= \frac{17e^4}{128} + \frac{3}{128}$$

d) $a_x = -0.6 V_x$

$v_x \frac{dv_x}{dx} = -0.6 V_x$



$\int_{4\sqrt{3}}^{V_x} dv_x = \int_0^x -0.6\, dx$

$V_x \Big|_{4\sqrt{3}}^{V_x} = -0.6x$

$V_x - 4\sqrt{3} = -0.6x$

$V_x = -0.6x + 4\sqrt{3}$

$x = 3$

$V_x = -1.8 + 4\sqrt{3}$

$a_x = -0.6 V_x$

$\frac{dv_x}{dt} = -0.6 V_x$

$\int_{4\sqrt{3}}^{V_x} \frac{1}{V_x} dv_x = \int_0^t -0.6\, dt$

$\ln V_x \Big|_{4\sqrt{3}}^{V_x} = -0.6t \Big|_0^t$

$\ln V_x - \ln 4\sqrt{3} = -0.6t$

$\ln \frac{V_x}{4\sqrt{3}} = -0.6t$

$\frac{V_x}{4\sqrt{3}} = e^{-0.6t}$

$V_x = 4\sqrt{3}\, e^{-0.6t}$

$$v_x = 4\sqrt{3}e^{-0.6t}$$

let $v_x = -1.8 + 4\sqrt{3}$

$$-1.8 + 4\sqrt{3} = 4\sqrt{3}e^{-0.6t}$$

$$\frac{-1.8 + 4\sqrt{3}}{4\sqrt{3}} = e^{-0.6t}$$

$$\ln\left(\frac{4\sqrt{3} - 1.8}{4\sqrt{3}}\right) = -0.6t$$

$$t = \frac{10 \ln\left(\frac{4\sqrt{3}}{4\sqrt{3} - 1.8}\right)}{6}$$

$$\approx 0.5$$

$$a_y = g - 0.6v_y$$

$$\frac{dv_y}{dt} = g - 0.6v_y$$

$$\int_4^{v_y} \frac{-0.6}{g - 0.6v_y} = \int_0^t -0.6\,dt$$

$$\ln|g - 0.6v_y| \Big|_4^{v_y} = -0.6\Big|_0^t$$

$$\ln|g - 0.6v_y| - \ln|g - 2.4| = -0.6t$$

$$\ln\left|\frac{g - 0.6v_y}{g - 2.4}\right| = -0.6t$$

$$\frac{g - 0.6v_y}{g - 2.4} = e^{-0.6t}$$

$$g - 0.6v_y = (g - 2.4)e^{-0.6t}$$

$$-0.6v_y = (g - 2.4)e^{-0.6t} - g$$

$$v_y = \frac{2.4 - g}{0.6} e^{-0.6t} + \frac{g}{0.6}$$

$$v_y\left(\frac{10\ln\left(\frac{4\sqrt{3}}{4\sqrt{3} - 1.8}\right)}{6}\right) = \frac{2.4 - 9.8}{0.6} e^{-0.6\left(\frac{10\ln\left(\frac{4\sqrt{3}}{4\sqrt{3} - 1.8}\right)}{6}\right)} + \frac{9.8}{0.6}$$

$$\approx 0.67$$

$$a_y = g - 0.6v_y$$

$$= 9.8 - 0.6 \times 0.67\ldots \times$$

$$= 9.397\ldots$$

$$a^2 = a_x^2 + a_y^2$$

$$= (-1.8 + 4\sqrt{3})^2 + (9.397\ldots)^2$$

$$= 114.61$$

$$a = \sqrt{114.61}$$

$$= 10.7$$

$F = a$

So the particle will stick to the wall as $F > 0$

$$\int v_y dt = \int \frac{2.4 - g}{0.6} e^{-0.6t} + \frac{g}{0.6} dt$$

$$y = \frac{g - 2.4}{0.36} e^{-0.6t} + \frac{gt}{0.6}$$

let $t = 10 \ln\left(\frac{4\sqrt{3}}{4\sqrt{3} - 1.8}\right) = k$

$$y = \frac{9.8 - 2.4}{0.36} e^{-0.6k} + \frac{9.8k}{0.6}$$

$$= 25.4722\ldots$$

$$\approx 25 \text{ m}$$

∴ The particle will stick to the wall at 25 m.