NORTH SYDNEY BOYS HIGH SCHOOL

2026 Y12 Extension 2 Task 3

Extension 2 Mathematics

- **General Instructions**

- Working time – 3 hours
  Reading time – 10 minutes
- **LABEL** question paper and **all** writing booklets with your student number and **TICK** teacher
- **USE** the **correct numbered booklet** for each question.
- Write using blue or black pen
- NESA approved calculators may be used
- All necessary working should be shown in every question
- Marks may not be awarded for carelessly or badly arranged work.
- Attempt all questions

TICK your teacher's name:

- ○ Mr. Berry
- ○ Ms. Lee
- ○ Ms. Moss
- ○ Dr. Vranešević

Student Number:_____

Question No	1-10	11	12	13	14	15	16	Total (%)
Mark	___ 10	___ 15	___ 15	___ 15	___ 15	___ 15	___ 15	___ 100

Section I

10 Marks
Attempt Question $1-10$
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions $1-10$

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1. If $z=1+2i$ and $w=3-i$ then $z-\bar{w}$ is:

   (A) $3i-2$

   (B) $4+3i$

   (C) $i-2$

   (D) $4+i$

2. If $2-i$ is a root of the polynomial $x^3-7x^2+17x-15=0$ then the other two roots are:

   (A) $2-i, 3$

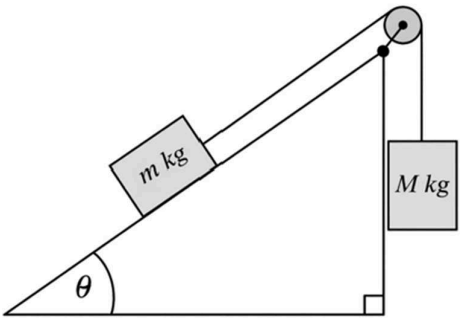
   (B) $2+i, 3$

   (C) $2+i, -3$

   (D) $2-i, -3$

Exam continues overleaf…

3. A smooth plane is inclined at an angle of θ to the horizontal. A mass of m kg, at rest, is supported by a light inextensible rope over a smooth pulley at the top of the plane. The other end of the rope is attached to a mass of M kg, as shown in the diagram below.



Which expression correctly connects m, M and θ?

(A) $M = m \sin\theta$

(B) $M = m \cos\theta$

(C) $\mathrm{m} = M \sin\theta$

(D) $\mathrm{m} = M \cos\theta$

4. Which of the following statements is FALSE?

(A) $\forall\, a, b \in \mathbb{R}$ $\quad a < b \rightarrow a^5 < b^5$

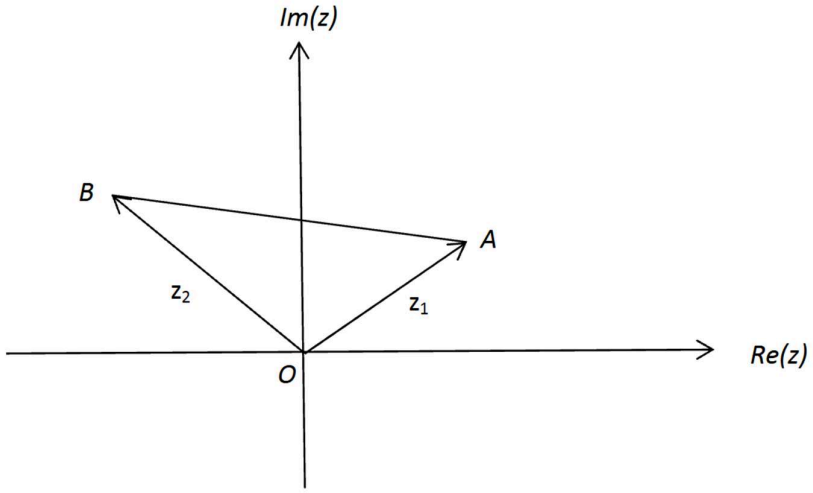
(B) $\forall\, a, b \in (0, \infty)$, $\quad a < b \rightarrow \sqrt{a} < \sqrt{b}$

(C) $\forall\, a, b \in \mathbb{R}$, $\quad a < b \rightarrow \cos a < \cos b$

(D) $\forall\, a, b \in (0, \infty)$, $\quad a < b \rightarrow \frac{1}{a} > \frac{1}{b}$

Exam continues overleaf…

5. In the Argand diagram below, vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$ represent the complex numbers z_1 and z_2 respectively where $|z_2| = 2|z_1|$ and $\angle AOB = \frac{2\pi}{3}$



Vector $\overrightarrow{AB}$ represents:

(A) $\left(-2 + i\sqrt{3}\right)z_1$

(B) $\left(2 - i\sqrt{3}\right)z_1$

(C) $\left(-\sqrt{3} + 2i\right)z_1$

(D) $\left(\sqrt{3} - 2i\right)z_1$

6. Consider the statement:

If I live in Antwerp, then I live in Belgium.

Which of the following are the converse and negation respectively of the statement above

Statement 1: If I don't live in Antwerp then I don't live in Belgium.

Statement 2: If I don't live in Belgium then I don't live in Antwerp.

Statement 3: If I live in Belgium then I live in Antwerp.

Statement 4: I live in Antwerp and I don't live in Belgium.

(A) Statements 1 and 2

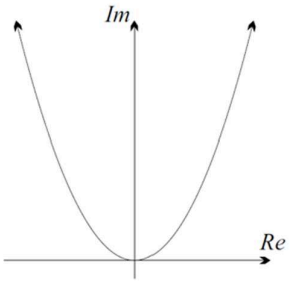
(B) Statements 3 and 1

(C) Statements 2 and 3

(D) Statements 3 and 4

Exam continues overleaf…

7. The diagram shows the locus of points representing the complex number z on the Argand plane, being a parabola with vertex at the origin through the point $(1, \frac{i}{4})$



Which equation in z best matches this locus?

(A) $|z + i| = Re(z) - 1$
(B) $|z + i| = Im(z) - 1$
(C) $|z - i| = Re(z) + 1$
(D) $|z - i| = Im(z) + 1$

8. A particle is moving in a straight line with velocity at any time given by $v = \sin^{-1} x$. Which of the following represents the acceleration of the particle?

A) $\dfrac{\sin^{-1} x}{\sqrt{1 - x^2}}$

B) $\dfrac{-\sin^{-1} x}{\sqrt{1 - x^2}}$

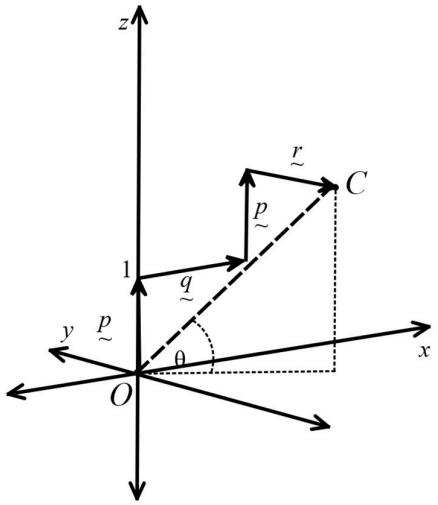
C) $\dfrac{1}{\sqrt{1 - x^2}}$

D) $\dfrac{-\cos x}{\sin^2 x}$

Exam continues overleaf…

9. Given the unit vectors $\underset{\sim}{p} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, $\underset{\sim}{q} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\underset{\sim}{r} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$.

Vector $\overrightarrow{OC} = \underset{\sim}{p} + \underset{\sim}{q} + \underset{\sim}{p} + \underset{\sim}{r}$ as shown below.



What angle (θ) does $\overrightarrow{OC}$ make with the x-y plane?

(A) $22° 12'$

(B) $30° 16'$

(C) $35° 15'$

(D) $54° 44'$

10. Let a, b, c, d be complex numbers such that $a \neq b$ and $c \neq d$.

Which of the following is a necessary and sufficient condition for the line through a and b to be perpendicular to the line through c and d?

(A) $\dfrac{a-b}{c-d} = -\dfrac{\bar{a}-\bar{b}}{\bar{c}-\bar{d}}\,i$

(B) $\dfrac{a-b}{c-d} = -\dfrac{\bar{a}-\bar{b}}{\bar{c}-\bar{d}}$

(C) $\dfrac{a-b}{c-d} = \dfrac{\bar{a}-\bar{b}}{\bar{c}-\bar{d}}\,i$

(D) $\dfrac{a-b}{c-d} = \dfrac{\bar{a}-\bar{b}}{\bar{c}-\bar{d}}$

Exam continues overleaf...

Section II

90 Marks
Attempt Question 11 – 16
Allow about 2 hours 45 minutes for this section

Answer each question in a separate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include all relevant mathematical reasoning and/or calculations

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Question 11 (15 marks) ** Use **ANSWER BOOKLET 11** for this question **

a) Define $A(2,0,3), B(-1,4,2)$ and $C(0,6,-3)$. Let $\underset{\sim}{u} = \overrightarrow{BA}$ and $\underset{\sim}{v} = \overrightarrow{BC}$

i) Find $proj_{\underset{\sim}{v}}\underset{\sim}{u}$ 2

ii) Find $|\underset{\sim}{u}|$ 1

iii) Hence, find the perpendicular distance of A from a line through BC 1

iv) Find the area of $\triangle ABC$ 1

b) Sketch the region on the Argand diagram whose points satisfy: 2

$$|z - \bar{z}| \leq 2 \text{ and } -\frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{4}$$

c) Prove by contrapositive that if n^2 is even then n is even 2

d) Prove by contradiction that $\sqrt{2} + \sqrt{3} < \sqrt{10}$ 2

e) Find 2

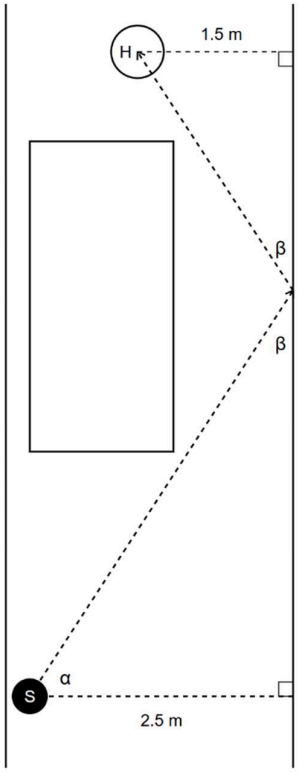
$$\int \frac{3x+5}{x^2+2x+4}\,dx$$

f) Evaluate 2

$$\int_0^1 x^3\sqrt{1-x^2}\,dx$$

Question 12 (15 marks) ** Use **ANSWER BOOKLET 12** for this question **

a) A shot on a mini golf course must bounce off a wall to avoid an obstacle as shown in the diagram to the right. The fairway surface provides a resistance proportional to v^2 giving acceleration as $a = -kv^2$. The ball must reach the hole with a speed of 0.5 ms^{-1} to go into the hole.



A test roll of the ball show that if the ball is hit at 4 ms^{-1} then it slows to 2 ms^{-1} after 2 metres.

Note: The bounce off the wall does not affect the speed of the ball.

i) Find k 2

ii) Given $\alpha = 60°$ find the speed at which the ball should be hit to go into the hole 2

b) Sketch the locus of z if w is purely imaginary and: 3

$$w = \frac{z - i}{z - 2}$$

c) Given 3

$$z = \frac{\sqrt{3} + i}{1 + i}$$

Find the smallest positive integer n such that z^n is real and hence find z^n for this value of n.

d) A particle has acceleration $\ddot{x} = -9x$. Given that when $\dot{x} = 0, x = 4$ find the speed at $x = 2$. 2

e) Given $z = \cos\theta + i\sin\theta$ and by considering $z + z^{-1} = 2\cos\theta$ find the exact solutions to: 3

$$2z^4 - z^3 + 3z^2 - z + 2 = 0$$

Answers should be given in the form $z = a + bi$.

Exam continues overleaf…

Question 13 (15 marks) ** Use **ANSWER BOOKLET 13** for this question **

a) i) Prove: 1

$$\int_a^b f(a+b-x)\,dx = \int_a^b f(x)\,dx$$

ii) Hence, evaluate: 3

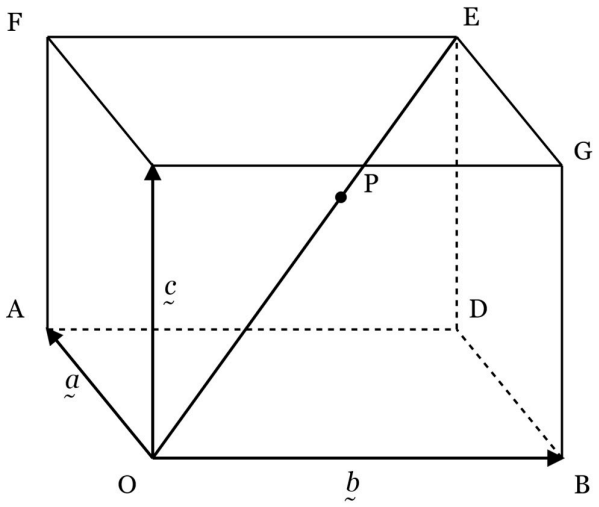
$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{1+\sqrt{\tan x}}\,dx$$

b) Prove there are no integers a and b such that $b^2 = 4a - 2$ 3

c) Find 3

$$\int \frac{x^2+1}{x^4+8x^2+12}\,dx$$

d) In the cuboid shown the point P lies on $\overrightarrow{OE}$ such that $\left|\overrightarrow{OP}\right| : \left|\overrightarrow{PE}\right| = m : n, m > n$



i) Find an expression for $\overrightarrow{OP}$ in terms of $\underset{\sim}{a}, \underset{\sim}{b}, \underset{\sim}{c}$ m and n 1

ii) When BP is extended, it meets FE at a point Y. Using vector methods, find the ratio $FY : YE$ in terms of m and n 4

Question 14 (15 marks) ** Use **ANSWER BOOKLET 14** for this question **

a) Find a vector of length 3 which is perpendicular to $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 4\underset{\sim}{k}$ 3

b) A drag car of mass M crosses the finish line at the end of its run and the driver immediately applies the brakes which provide a constant frictional force of $\frac{1}{5}M$ newtons. Ten seconds after crossing the line, the driver deploys the braking parachute which provides an additional resistive force of $\frac{1}{90}Mv^2$ newtons.

i. Given that the car is travelling at 75 ms^{-1} when it crosses the finish line, determine how far the car has travelled when the parachute is deployed and what the velocity is at that point 3

ii. Find the distance from the finish line when the car comes to rest, answering to the nearest metre. 2

c) Two particles travel according to trajectories defined by: 3

$$\vec{r_1}(t) = (t^2 + t)\underset{\sim}{i} + (3t - 1)\underset{\sim}{j} + (t^2 - t + 2)\underset{\sim}{k}$$

$$\vec{r_2}(t) = (4t - 2)\underset{\sim}{i} + (t^2 + t)\underset{\sim}{j} + (3t - 1)\underset{\sim}{k}$$

Determine if the particles will ever collide, and if so, find the point of collision.

Exam continues overleaf…

d) i) Evaluate 1

$$\lim_{a\to\infty}\int_0^a \frac{1}{1+x^2}\,dx$$

ii) The orbital path of a spaceship can be expressed as 3

$$I=\int_0^{\pi}\frac{d\theta}{1+e\cos\theta}$$

Where $0<e<1$ represents the "eccentricity" of the orbit. By considering $t=\tan\frac{\theta}{2}$ or otherwise, evaluate I.

Question 15 (15 marks) ** Use **ANSWER BOOKLET 15** for this question **

a) Define for $n \geq 0$

$$I_n = \int_0^1 \frac{x^n}{(x+1)^2}\,dx$$

i) For $n \geq 2$, show that: 3

$$I_n = \frac{1}{2(n-1)} - \frac{n}{n-1} I_{n-1}$$

ii) Hence, find: 2

$$\int_0^1 \frac{x^3}{(x+1)^2}\,dx$$

b) A cyclic quadrilateral (where all 4 vertices lie on a circle) lies in the plane $x = 4$ and has vertices $P(4, 4, 3)$, $Q(4, 3, -4)$, $R(4, -4, 3)$ and $S(4, -3, 4)$.

i) Let $C(x, y, z)$ be the centre of a sphere S which passes through P, Q, R and S. Given the radius of the sphere is $\sqrt{34}$ find all possible positions of C. 3

ii) Hence, write down the vector equation of each possible sphere S_k in the form $\left|\underset{\sim}{r} - \underset{\sim}{c}\right| = r$ 1

iii) The line l, with direction vector $\underset{\sim}{i}$, passes through P. Find the length of the chord that l cuts from each sphere S_k 2

Exam continues overleaf…

c) i) If k is an integer where $k \geq 3$ show that 1

$$\frac{1}{(k-1)k(k+1)} > \frac{1}{k^3}$$

ii) Given that 3

$$S_n = \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \cdots + \frac{1}{n^3}$$

prove that:

$$S_n < \frac{1}{12}$$

Exam continues overleaf…

Question 16 (15 marks) ** Use **ANSWER BOOKLET 16** for this question **

a) i) Prove for $k \in \mathbb{R}^+$ 2

$$\frac{2}{\sqrt{k+1}+\sqrt{k}} < \frac{1}{\sqrt{k}}$$

ii) Hence, prove 3

$$132 < 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{4489}} < 133$$

b) A fairground game requires a player to kick a ball over a horizontal bar. The bar is positioned between two vertical goal posts and is moving vertically in Simple Harmonic Motion with an amplitude of 1m about a centre of 5m off the ground. It has a period of 4 seconds. The bar's movement is mechanically powered and therefore air resistance can be ignored for the bar.

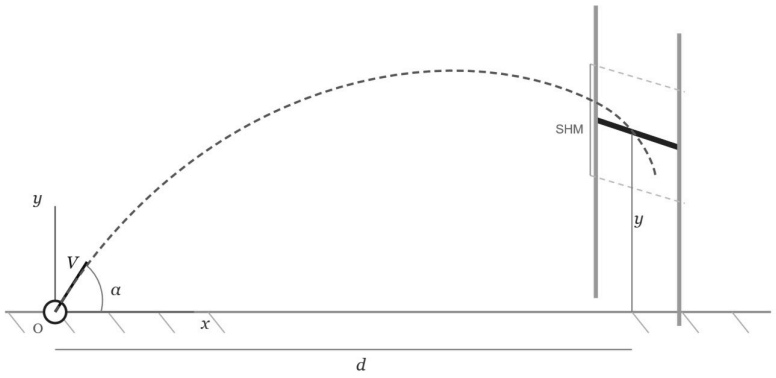
The player is standing on the ground at a horizontal distance $d = 15$ metres from the goalposts in line with the centre of the horizontal bar. He will aim to kick the ball directly over the centre of the bar.

For the motion of the ball, air resistance is proportional to velocity giving equations for displacement:

$$x(t) = \frac{U_x}{k}(1 - e^{-kt}), \qquad y(t) = -\frac{g}{k}t + \frac{1}{k}(U_y + \frac{g}{k})(1 - e^{-kt})$$

U_x and U_y are the initial velocities in the x and y directions respectively. Gravity can be taken as $g = 10\ \text{ms}^{-2}$ and $k = \ln 2$.

The ball is kicked at an initial speed of 24ms^{-1} at an angle of $\alpha = 30°$ to the horizontal.



i) Show that the ball travels 15 metres horizontally, correct to the nearest metre, after 1 second and find the height of the ball at this time, correct to two decimal places. 2

ii) The crossbar is at its centre of motion and moving upwards when $t = 0$. Find the time/s during the first 10 seconds in which the ball should be kicked in order to pass over the crossbar. Answer to two decimal places. 4

c) i) Show that if ω is a non real root of $z^3 = 1$ then $1 + \omega + \omega^2 = 0$ 1

ii) Hence or otherwise, evaluate: 3

$$\binom{2026}{0} + \binom{2026}{3} + \binom{2026}{6} + \cdots + \binom{2026}{2025}$$

END OF EXAMINATION

Solutions

Q1. $z = 1 + 2i \quad \bar{\omega} = 3 + i$

$$z - \bar{\omega} = 1 + 2i - 3 - i$$
$$= -2 + i$$

$\Rightarrow$ (C)

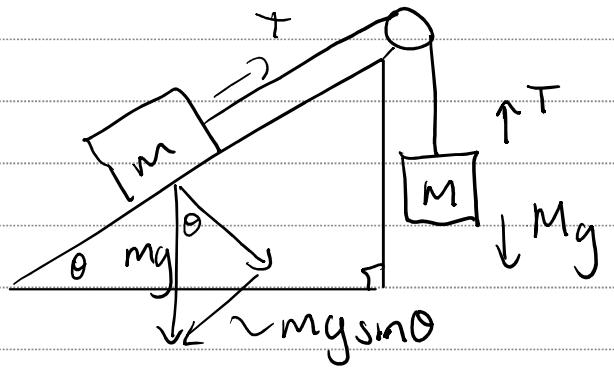
Q2. Since real coefficients, 2nd root is conjugate $2+i$

Also Product of roots $= 15$

$$\therefore (2+i)(2-i)\alpha = 15$$
$$5\alpha = 15$$
$$\alpha = 3$$

$\Rightarrow$ (B)

Q3

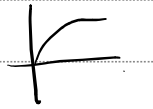


$\therefore Mg = mg\sin\theta$
$M = m\sin\theta$

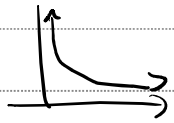
$\Rightarrow$ (A)

Q4

A x^5 is increasing  ∴ True

B $\sqrt{x}$ is increasing for $x>0$  ∴ True

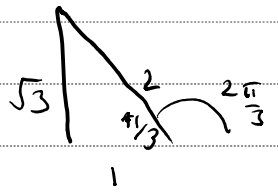
C $\cos x$ increasing / decreasing ∴

D $\frac{1}{x}$ decreasing  ∴ True

$\Rightarrow$ (C)

Q5 $z_2 = 2e^{\frac{2\pi}{3}i} z_1$

$= (-1 + \sqrt{3}i) z_1$



$\overrightarrow{AB} = z_2 - z_1$

$= (-1 + \sqrt{3}i) z_1 - z_1$

$= (-2 + \sqrt{3}i) z_1$

$\Rightarrow$ (A)

Q6 Converse: swap if / then

$\Rightarrow$ If I live in Belgium, then I live in Antwerp S3

Negation: if / then $\Rightarrow$ And not

$\Rightarrow$ I live in Antwerp and I don't live in Belgium S4

$\Rightarrow$ (D)

Q7 Sub $(1, \frac{i}{4})$ into $z = x + yi$

$= 1 + \frac{i}{4}$

A: $\left|1 + \frac{5i}{4}\right| = 1 - 1$ $\Rightarrow$ No

B: $\left|1 + \frac{5i}{4}\right| = \frac{1}{4} - 1$ $\Rightarrow$ No

C: $\left|1 - \frac{3i}{4}\right| = 1 - 1$ $\Rightarrow$ No

$\therefore$ D

Check

D: $\left|1 - \frac{3}{4}i\right| = \sqrt{1 + \frac{9}{16}}$

$= \sqrt{\frac{25}{16}}$

$= \frac{5}{4}$

$= \frac{1}{4} + 1$ $\Rightarrow$ (D)

Q8 $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1 - x^2}}$

$a = v\frac{dv}{dx}$ $\therefore$ $\frac{\sin^{-1} x}{\sqrt{1 - x^2}}$

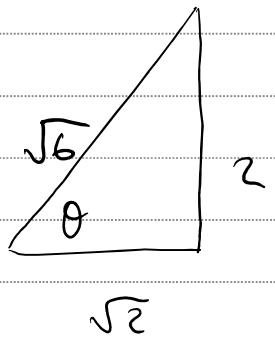
$\Rightarrow$ (A)

Q9 $\overrightarrow{OC} = 2\underset{\sim}{p} + \underset{\sim}{q} + \underset{\sim}{r}$

$$= 2\begin{pmatrix}0\\0\\1\end{pmatrix} + \begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\-1\\0\end{pmatrix}$$

$$= \begin{pmatrix}1\\-1\\2\end{pmatrix}$$

xy plane $|i + j| = \sqrt{2}$



$$\cos\theta = \frac{\sqrt{2}}{\sqrt{6}}$$

$$= \frac{1}{\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\approx 54°44'$$

$\Rightarrow$ (D)

Q10 If $\underline{\quad}$ then $\omega + \bar{\omega} = 0$

$$\omega = -\bar{\omega}$$

Let $\omega = \dfrac{a-b}{c-d}$

$$\bar{\omega} = \frac{\bar{a}-\bar{b}}{\bar{c}-\bar{d}}$$

$\Rightarrow$ (B)

Q11

i) $\underset{\sim}{u} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix}$ $\underset{\sim}{v} = \begin{pmatrix} 0 \\ 6 \\ -8 \end{pmatrix} - \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$ $= \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$

$$\text{proj}_{\underset{\sim}{v}} \underset{\sim}{u} = \frac{\underset{\sim}{u} \cdot \underset{\sim}{v}}{|v|^2} \underset{\sim}{v}$$

$$= \frac{\begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}}{1 + 4 + 25} \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

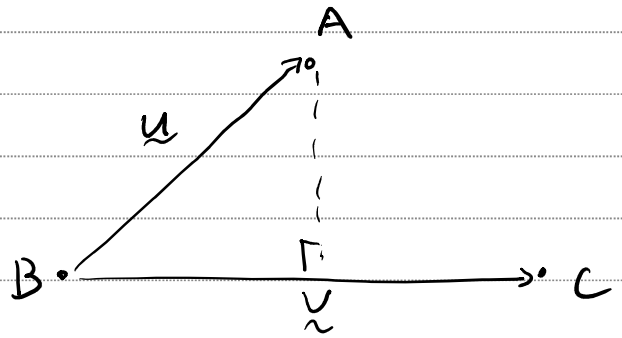
$$= \frac{3 - 8 - 5}{30}$$

$$= -\frac{1}{3} \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

ii $|\underset{\sim}{u}| = \sqrt{3^2 + 4^2 + 1^2}$

$= \sqrt{26}$

iii)



$$d^2 = |\underset{\sim}{u}|^2 - \left|\begin{pmatrix} -1/3 \\ -2/3 \\ 5/3 \end{pmatrix}\right|^2$$

$$= 26 - \frac{10}{3}$$

$$= \frac{68}{3}$$

$$\text{Distance} = \frac{2\sqrt{17}}{3} = \sqrt{\frac{68}{3}} = \frac{2\sqrt{51}}{3}$$

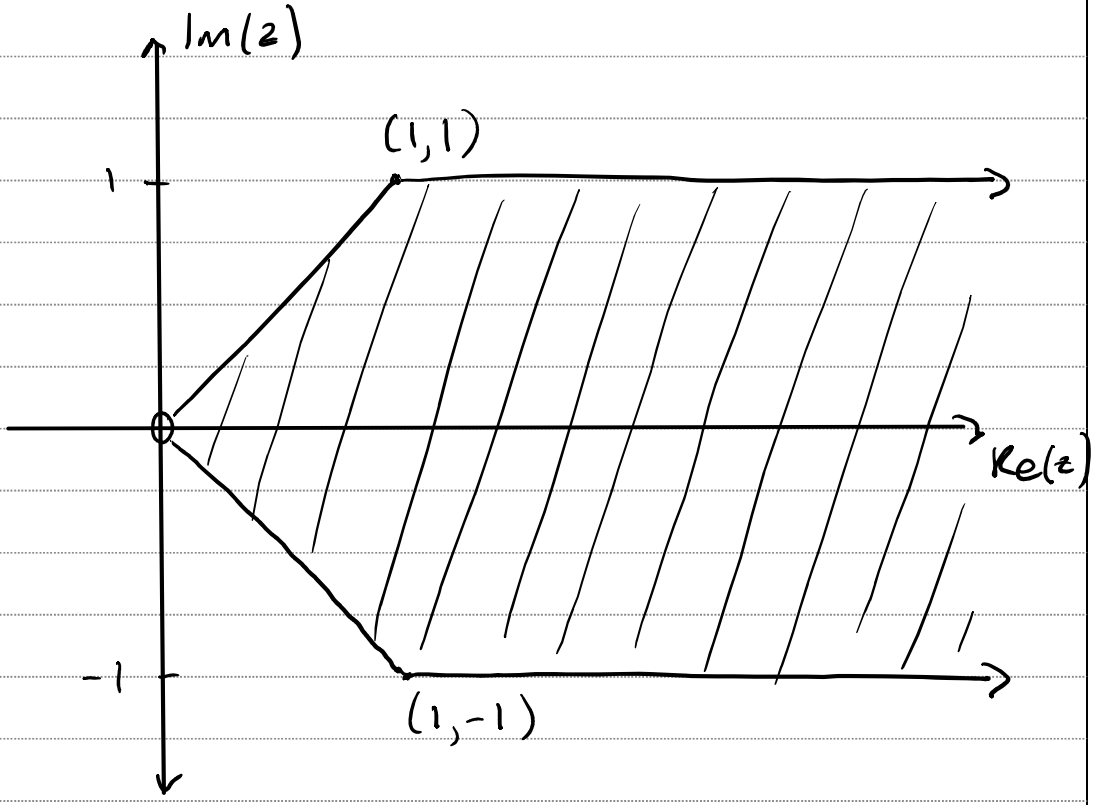
iv) $$\text{Area} = \frac{1}{2} \times \sqrt{30} \times \sqrt{\frac{68}{3}}$$

$$= \frac{1}{2}\sqrt{680}$$

$$= \sqrt{170}$$

Q11

b) $|z - \bar{z}| \leq 2$ $\quad -\frac{\pi}{4} \leq \text{Arg}(z) \leq \frac{\pi}{4}$



c) Contrapositive:

If n is odd then n^2 is odd.

Let $n = 2k + 1$, $k \in \mathbb{Z}$

$$\begin{aligned} n^2 &= (2k+1)^2 \\ &= 4k^2 + 4k + 1 \\ &= 2(2k^2 + 2k) + 1 \\ &= 2m + 1 \quad m \in \mathbb{Z} \text{ which is odd.} \end{aligned}$$

$\therefore$ If n even, n^2 is even is true by contrapositive

Q11

d) For sake of contradiction

Let $\sqrt{2} + \sqrt{3} \geq \sqrt{10}$

then $(\sqrt{2} + \sqrt{3})^2 \geq 10$

$2 + 2\sqrt{6} + 3 \geq 10$

$2\sqrt{6} \geq 5$

Squaring again $4 \times 6 \geq 25$

$24 \geq 25$ which is clearly false.

$\therefore \sqrt{2} + \sqrt{3} < \sqrt{10}$ by contradiction

e) $\int \frac{3x+5}{x^2+2x+4}\,dx$

$= \frac{3}{2}\int \frac{2x+2}{x^2+2x+4}\,dx + \int \frac{2}{x^2+2x+4}\,dx$

$= \frac{3}{2}\ln|x^2+2x+4| + \int \frac{2}{(x+1)^2+3}\,dx$

$= \frac{3}{2}\ln|x^2+2x+4| + \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right) + C$

Q 11

f) $\int_0^1 x^3 \sqrt{1-x^2}\, dx$

Let $u = 1 - x^2$ $\quad du = -2x\, dx$

$x^2 = 1 - u$ $\quad dx = \frac{-du}{2x}$

$x = 1 \quad u = 0$

$x = 0 \quad u = 1$

$$\int_1^0 (1-u)\, x \sqrt{u} \left(\frac{-du}{2x}\right)$$

$$= \frac{1}{2}\int_0^1 (1-u)\, u^{1/2}\, du$$

$$= \frac{1}{2}\int_0^1 u^{1/2} - u^{3/2}\, du$$

$$= \frac{1}{2}\left[\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right]_0^1$$

$$= \frac{1}{2}\left[\frac{2}{3} - \frac{2}{5}\right]$$

$$= \frac{2}{15}$$

Q 12

a) i) $v\frac{dv}{dx} = -kv^2$

$\frac{dv}{dx} = -kv$

$\int_{v_0}^{v} \frac{dv}{v} = \int_{0}^{x} -k\,dx$

$\ln \frac{v}{v_0} = -kx$

For $x = 2$, $v = 2$, $v_0 = 4$

$\ln\left(\frac{1}{2}\right) = -2k$

$-\ln 2 = -2k$

$k = \frac{\ln 2}{2}$.

ii) $\alpha = 60$ means the ball travels $5 + 3 = 8$ m

∴ Find v_0 when $v = 0.5$, $x = 8$

$\ln \frac{0.5}{v_0} = -\frac{\ln 2}{2} \times 8$

$= \ln 2^{-4}$

$\frac{0.5}{v_0} = \frac{1}{16}$

$v_0 = 8$ m/s

Q12

b) $\omega = \frac{z-i}{z-2}$ is imaginary when the angle between $z-i$ and $z-2$ is $\pi/2$ i.e. they are perpendicular

Let $z = x+yi$

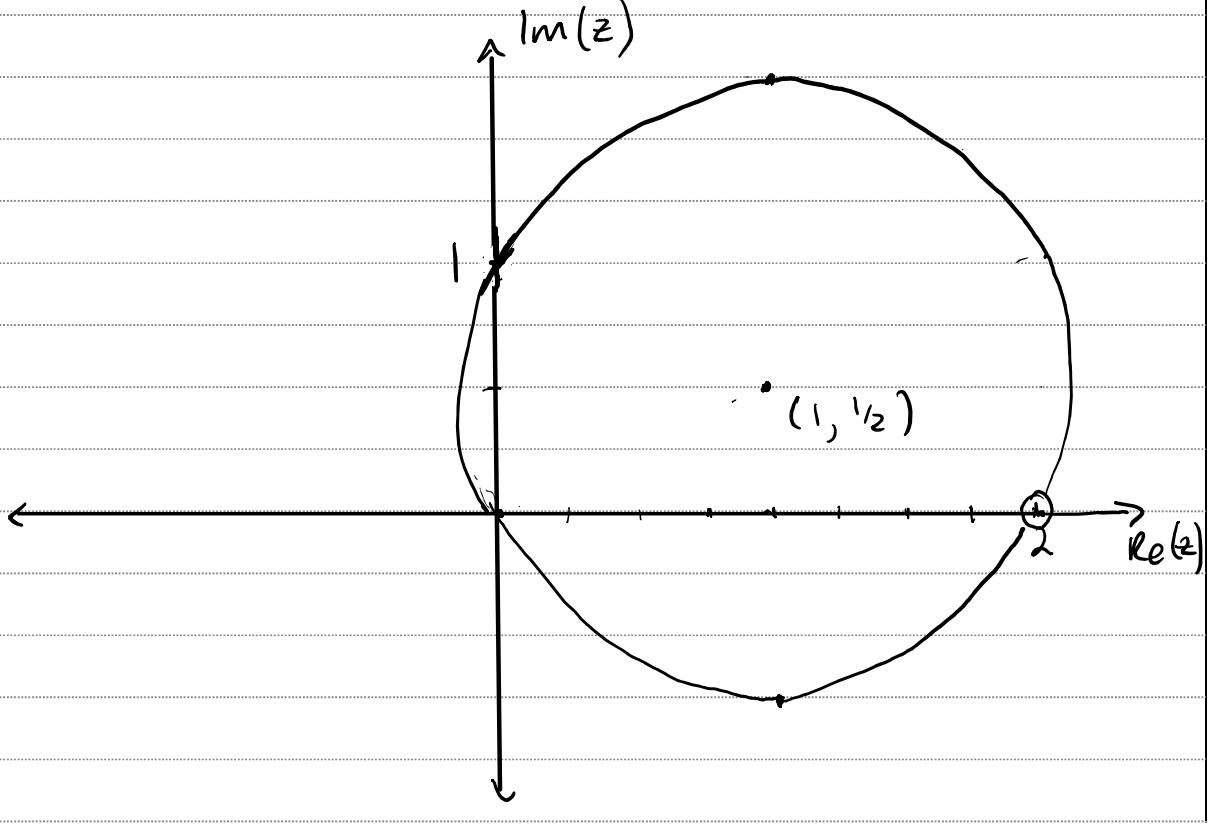
$$(x+(y-1)i)\cdot((x-2)+yi) = 0$$

$$x(x-2) + y(y-1) = 0$$

$$x^2 - 2x + 1 + y^2 - y + 1/4 = 5/4$$

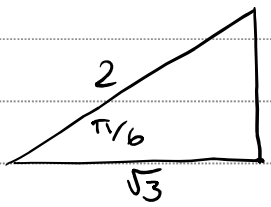
$$(x-1)^2 + (y-1/2)^2 = 5$$

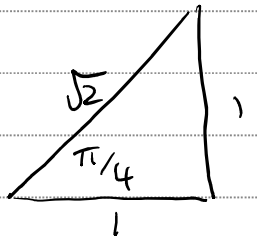
Exclude $z = 2$ since ω must exist



Q11

c) $z = \dfrac{\sqrt{3}+i}{1+i}$





$= \dfrac{2e^{\pi/6 i}}{\sqrt{2}e^{\pi/4 i}}$

$= \sqrt{2}e^{-\pi/12 i}$

Smallest n for real is $n = 12$

$z^{12} = (\sqrt{2})^{12} e^{-\pi i}$

$= -64$

d) $\ddot{x} = -9x \Rightarrow n^2 = 9, \quad c = 0$

$\dot{x} = 0$ at $x = 4 \therefore a = 4$

$v\dfrac{dv}{dx} = -9x$

$\int v\,dv = \int -9x\,dx$

$\dfrac{v^2}{2} = \dfrac{-9x^2}{2} + C$

At $x = 4, v = 0 \quad 0 = \dfrac{-9(4)^2}{2} + C$

$C = 72$

Now at $x = 2 \quad \dfrac{v^2}{2} = \dfrac{-9(2)^2}{2} + 72$

$v^2 = 108$

$|v| = 6\sqrt{3}$

Q 12

e) $z = \cos\theta + i\sin\theta$ $\quad z + z^{-1} = 2\cos\theta$

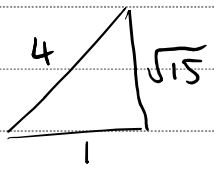
$z^2 = \cos 2\theta + i\sin 2\theta$ $\quad z^2 + z^{-2} = 2\cos 2\theta$

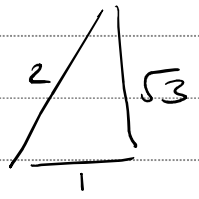
$$2z^4 - z^3 + 3z^2 - z + 2 = 0$$

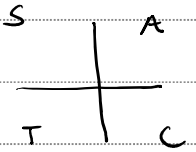
$\div z^2$ $\quad 2z^2 - z + 3 - z^{-1} + 2z^{-2} = 0$

$$2(z^2 + z^{-2}) - (z + z^{-1}) + 3 = 0$$
$$2(2\cos 2\theta) - 2\cos\theta + 3 = 0$$
$$4(2\cos^2\theta - 1) - 2\cos\theta + 3 = 0$$
$$8\cos^2\theta - 4 - 2\cos\theta + 3 = 0$$
$$8\cos^2\theta - 2\cos\theta - 1 = 0$$
$$8\cos^2\theta - 4\cos\theta + 2\cos\theta - 1 = 0$$
$$(4\cos\theta + 1)(2\cos\theta - 1) = 0$$

$\therefore \cos\theta = -\frac{1}{4}$ $\quad \cos\theta = \frac{1}{2}$







$\sin\theta = \frac{\sqrt{15}}{4}$ $\quad \sin\theta = \frac{\sqrt{3}}{2}$

$z = \frac{-1 \pm \sqrt{15}i}{4}$ $\quad z = \frac{1 \pm \sqrt{3}i}{2}$

Q13

a) i)

$$LHS = \int_a^b f(a+b-x)\,dx$$

$$= -\int_a^b -f(a+b-x)\,dx$$

$$= -\left[F(a+b-x)\right]_a^b \quad \text{where } F(x) \text{ is the primitive of } f(x)$$

$$= -\left[F(a) - F(b)\right]$$

$$= F(b) - F(a)$$

$$= RHS$$

Let $I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}}\,dx$ * $f(x) \to f(a+b-x) \to f(\pi/2 - x)$

$\therefore \tan x \to \cot x$

$$= \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\cot x}}\,dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\tan x}}{\sqrt{\tan x}+1}\,dx \quad **$$

Now $2I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\tan x}} + \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\,dx$

$$= \int_{\pi/6}^{\pi/3} 1\,dx$$

$$= \pi/3 - \pi/6$$

$$I = \frac{\pi}{12}$$

Q 13

b) For sake of contradiction

Assume a, b are integers such that $b^2 = 4a - 2$.

Then $b^2 = 2(2a-1)$

which means b^2 is even

$\therefore$ b is even (from 11c)

Let $b = 2k$, $k \in \mathbb{Z}$

Now $4k^2 = 2(2a-1)$

$2k^2 = 2a - 1$

But LHS is even while RHS is odd

$\Rightarrow$ contradiction.

$\therefore$ By contradiction there are no integers a, b such that $b^2 = 4a-2$

Q 13

c) $\int \frac{x^2+1}{(x^2+6)(x^2+2)}\,dx$

Consider $\frac{A}{x^2+6} + \frac{B}{x^2+2} = \frac{x^2+1}{(x^2+6)(x^2+2)}$

$Ax^2 + 2A + Bx^2 + 6B = x^2 + 1$

$\therefore \quad A + B = 1 \qquad 2A + 6B = 1$

$A = 1 - B$

$2(1-B) + 6B = 1$

$4B = -1$

$B = -\frac{1}{4}$

$A = \frac{5}{4}$

Now $\int \frac{x^2+1}{x^4+8x^2+12}\,dx = \frac{5}{4}\int \frac{1}{x^2+6}\,dx - \frac{1}{4}\int \frac{1}{x^2+2}\,dx$

$= \frac{5}{4\sqrt{6}} \tan^{-1}\left(\frac{x}{\sqrt{6}}\right) - \frac{1}{4\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$

Q13

d)

i) $\overrightarrow{OP} = \frac{m}{m+n}\overrightarrow{OE}$

$= \frac{m}{m+n}(\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c})$

ii) Consider $\overrightarrow{OY}$ in two ways

Let $\overrightarrow{FY} = \lambda \overrightarrow{FE}$ $\quad 0 < \lambda < 1$

$\overrightarrow{BY} = k\overrightarrow{PP}$ $\quad k > 1$

Now $\overrightarrow{OY} = \underset{\sim}{a} + \underset{\sim}{c} + \lambda \underset{\sim}{b}$

and $\overrightarrow{OY} = \underset{\sim}{b} + k\overrightarrow{BP}$

$= \underset{\sim}{b} + k(\overrightarrow{OP} - \overrightarrow{OB})$

$= \underset{\sim}{b} + k\left(\frac{m}{m+n}(\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}) - \underset{\sim}{b}\right)$

$= \frac{km}{m+n}\underset{\sim}{a} + \left(\frac{km}{m+n} + 1 - k\right)\underset{\sim}{b} + \frac{km}{m+n}\underset{\sim}{c}$

Since $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}$ are linearly independent

$\frac{km}{m+n} = 1 \qquad \frac{km}{m+n} + 1 - k = \lambda$

$$k = \frac{m+n}{m}$$

$$x = 1 + 1 - k$$

$$= 2 - k$$

$$= 2 - \frac{m+n}{m}$$

$$= \frac{2m - m + n}{m}$$

$$= \frac{m-n}{m}$$

$\therefore$ FY : FE is $(m-n) : m$

$$FY = \frac{m-n}{m} FE$$

$$YE = \left(1 - \frac{m-n}{m}\right) FE$$

$$= \frac{m - m + n}{m}$$

$$= \frac{n}{m}$$

$$\therefore \frac{FY}{YE} = \frac{m-n}{m} \times \frac{m}{n}$$

$$= \frac{m-n}{n}$$

FY : YE is $m-n : n$

Q14

a) Let $\underset{\sim}{v} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k}$

$\sqrt{x^2 + y^2 + z^2} = 3$

$x^2 + y^2 + z^2 = 9$

And $2x + y + 2z = 0$ — ①

$2x + 3y + 4z = 0$ — ②

② − ①: $2y + 2z = 0$

$y = -z$ — ③

③ into ① $2x + y - 2y = 0$

$2x = y$

$x = \frac{y}{2}$

Let $x = 1$, $y = 2$, $z = -2$

Length $x^2 + y^2 + z^2 = 1 + 4 + 4$

$= 9$

$\therefore \underset{\sim}{v} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

b) i) $F = ma \Rightarrow \frac{dv}{dt} M = -\frac{1}{5} M$

$$\frac{dv}{dt} = -\frac{1}{5}$$

$$\int_{75}^{v} dv = -\int_{0}^{t} \frac{1}{5}\, dt$$

$$v - 75 = -\frac{t}{5}$$

$$v = -\frac{t}{5} + 75$$

At $t = 10$ $\quad v = -\frac{10}{5} + 75$

$$= 73 \text{ m/s}$$

$$\frac{dx}{dt} = -\frac{t}{5} + 75$$

$$\int_{0}^{x} dx = \int_{0}^{10} -\frac{t}{5} + 75\, dt$$

$$x = \left[-\frac{t^2}{10} + 75t \right]_0^{10}$$

$$= -\frac{100}{10} + 750$$

$$= 740 \text{ m}.$$

∴ The car has travelled 740m and its speed is 73m/s at that point

ii) Now add in parachute

$$Ma = -\left(\frac{1}{5}M + \frac{Mv^2}{90}\right)$$

$$v\frac{dv}{dx} = -\left(\frac{18+v^2}{90}\right)$$

$$\int_{73}^{0} \frac{v}{18+v^2}\,dv = \int_{0}^{x} \frac{-1}{90}\,dx \quad \left(\begin{array}{l}\text{finding distance}\\ \text{from } 740\text{m}\end{array}\right)$$

$$-45\int_{73}^{0} \frac{2v}{18+v^2}\,dv = \int_{0}^{x} dx$$

$$x = -45\ln|18+v^2| - 45\ln|98|$$

$$= 256.226$$

$$\approx 256\text{m}$$

$\therefore$ Total distance from finish line is 996m (nearest metre)

Q 14

c)

For collision there needs to be a consistent value of t which gives matching $\underset{\sim}{i}$, $\underset{\sim}{j}$, $\underset{\sim}{k}$ components.

$\therefore$ For $\underset{\sim}{i}$

$$t^2 + t = 4t - 2$$
$$t^2 - 3t + 2 = 0$$
$$(t-2)(t-1) = 0$$

$$\therefore t = 2 \text{ or } t = 1$$

For $\underset{\sim}{j}$

$$3t - 1 = t^2 + t$$
$$0 = t^2 - 2t + 1$$
$$= (t-1)^2$$

$$\therefore t = 1$$

Check $\underset{\sim}{k}$ at $t = 1$

$$t^2 - t + 2 = 2$$
$$3t - 1 = 2 \quad \therefore t = 1 \text{ works.}$$

Collision when $t = 1$ at $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$

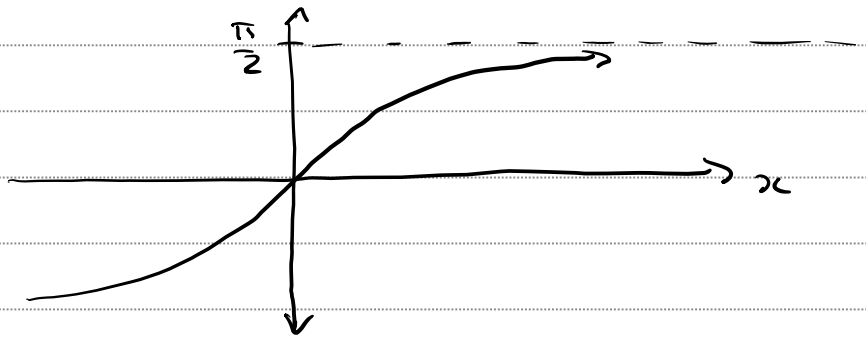
Q14

d) i) $\lim_{a \to \infty} \int_0^a \frac{1}{1+x^2}\, dx$

$= \lim_{a \to \infty} \left[\tan^{-1} x \right]_0^a$

$= \lim_{a \to \infty} \tan^{-1} a$

As $a \to \infty$ $\tan^{-1} a \Rightarrow \pi/2$



ii) Let $t = \tan \frac{\theta}{2}$ $\quad \frac{dt}{d\theta} = \frac{1}{2} \sec^2 \frac{\theta}{2}$

$= \frac{1}{2}\left(1 + \tan^2 \frac{\theta}{2}\right)$

$\cos\theta = \frac{1-t^2}{1+t^2}$ $\quad = \frac{1+t^2}{2}$

$\therefore d\theta = \frac{2\,dt}{1+t^2}$

When $t = 0$ $\tan \frac{\theta}{2} = 0$

$t \approx \pi$ $\tan \frac{\pi}{2} \to \infty$

Now $I = \int_0^\infty \frac{1}{1 + e\left(\frac{1-t^2}{1+t^2}\right)} \frac{2\,dt}{1+t^2}$

$$I = \int_0^\infty \frac{2\,dt}{1 + t^2 + e(1-t^2)}$$

$$= \int_0^\infty \frac{2\,dt}{(1-e)t^2 + (1+e)}$$

$$= \frac{2}{1-e} \int_0^\infty \frac{dt}{t^2 + \frac{1+e}{1-e}}$$

$$= \frac{2}{1-e} \left[\sqrt{\frac{1-e}{1+e}} \tan^{-1} t \left(\frac{\sqrt{1-e}}{\sqrt{1+e}}\right) \right]_0^\infty$$

$$= \frac{2\sqrt{1-e}}{(1-e)\sqrt{1+e}} \lim_{t \to \infty} \tan^{-1} t$$

$$= \frac{2}{\sqrt{1-e^2}} \cdot \frac{\pi}{2} \quad \text{(from i)}$$

$$= \frac{\pi}{\sqrt{1-e^2}}$$

Q15

a) $I_n = \int_0^1 \frac{x^n}{(x+1)^2}\,dx$

i) $u = x^n \qquad dv = \frac{1}{(x+1)^2}\,dx$

$du = nx^{n-1}\,dx \qquad v = \frac{-1}{(x+1)}$

$$I_n = \left[\frac{-x^n}{x+1}\right]_0^1 - \int_0^1 \frac{-nx^{n-1}}{(x+1)}\,dx$$

$$= -\frac{1}{2} + \int_0^1 \frac{nx^{n-1}}{x+1}\,dx$$

$$= -\frac{1}{2} + n\int_0^1 \frac{x^{n-1}(x+1)}{(x+1)^2}\,dx$$

$$I_n = -\frac{1}{2} + n\int_0^1 \frac{x^n}{(x+1)^2} + \frac{x^{n-1}}{(x+1)^2}\,dx.$$

$$I_n = -\frac{1}{2} + n I_n + n I_{n-1}$$

$$(1-n) I_n = -\frac{1}{2} + n I_{n-1}$$

$$= \frac{2n I_{n-1} - 1}{2}$$

$$I_n = \frac{2n I_{n-1} - 1}{2(1-n)}$$

$$= \frac{1}{2(n-1)} - \frac{n}{n-1} I_{n-1}$$

Q15

a) ii) $\int_0^1 \frac{x^3}{(x+1)^2}\,dx = I_3$

$$I_1 = \int_0^1 \frac{x}{(x+1)^2}\,dx$$

$$= \int_0^1 \frac{x+1-1}{(x+1)^2}\,dx$$

$$= \int_0^1 \frac{1}{x+1} - \frac{1}{(x+1)^2}\,dx$$

$$= \left[\ln|x+1| + \frac{1}{x+1}\right]_0^1$$

$$= \ln 2 + \frac{1}{2} - 1$$

$$= \ln 2 - \frac{1}{2}$$

$$I_2 = \frac{1}{2} - 2\left[\ln 2 - \frac{1}{2}\right]$$

$$= \frac{1}{2} - 2\ln 2 + 1$$

$$= \frac{3}{2} - 2\ln 2$$

$$I_3 = \frac{1}{4} - \frac{3}{2}\left[\frac{5}{2} - 2\ln 2\right]$$

$$= \frac{1}{4} - \frac{9}{4} + 3\ln 2$$

$$= -2 + 3\ln 2$$

$$= 3\ln 2 - 2$$

Q15

b) $P(4,4,3)$ $Q(4,3,-4)$ $R(4,-4,3)$

$S(1,-3,4)$

$|\vec{PC}| = |\vec{QC}| = |\vec{RC}| = |\vec{SC}| =$

From $\vec{PC}$ and $\vec{RC}$

$$(x-4)^2 + (y-4)^2 + (z-3)^2 = (x-4)^2 + (y+4)^2 + (z-3)^2$$

$$(y-4)^2 = (y+4)^2$$

$$\therefore \quad y = 0$$

From $\vec{PC}$ and $\vec{SC}$

$$(x-4)^2 + 4)^2 + (z-3)^2 = (x-4)^2 + (-3)^2 + (z-4)^2$$

$$16 \quad z^2 - 6z + 9 = 9 + z^2 - 8z + 16$$

$$-6z = -8z$$

$$\therefore \quad z = 0$$

Now from $\overrightarrow{PC}$

$$(x-4)^2 + (-4)^2 + (3)^2 = 34$$
$$(x-4)^2 + 25 = 34$$
$$(x-4)^2 = 9$$
$$x - 4 = \pm 3$$
$$x = 1 \quad \text{or} \quad x = 7$$

$\therefore$ C $(1, 0, 0)$ or $(7, 0, 0)$

ii) $\left| \underset{\sim}{r} - \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right| = \sqrt{34}$ or $\left| \underset{\sim}{r} - \begin{pmatrix} 7 \\ 0 \\ 0 \end{pmatrix} \right| = \sqrt{34}$

iii) $l = \begin{pmatrix} 4 \\ 4 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

Sub $(4+\lambda, 4, 3)$ into each sphere

$$(4+\lambda-1)^2 + (4)^2 + (3)^2 = 34$$
$$(3+\lambda)^2 + 16 + 9 = 34$$
$$(3+\lambda)^2 = 9$$
$$\lambda = 0 \text{ or } 6$$

$\therefore$ since in direction i only, chord length is 6.

Due to line having direction parallel with line through sphere centres, the chords will be the same length

$\Rightarrow$ 6 units through each sphere.

c)i) Since $k \geq 3$ we can take the reciprocal

RTP becomes $(k-1)k(k+1) < k_3$

$$\begin{aligned} \text{LHS} - \text{RHS} &= (k-1)k(k+1) - k^3 \\ &= k(k^2-1) - k^3 \\ &= k^3 - k - k^3 \\ &= -k \\ &< 0 \end{aligned}$$

$\therefore$ LHS < RHS

$\therefore (k-1)k(k+1) < k^3$

$\therefore \dfrac{1}{(k-1)k(k+1)} > \dfrac{1}{k^3}$

ii) Consider $\dfrac{1}{(k-1)k(k+1)} = \dfrac{A}{k(k-1)} + \dfrac{B}{k(k+1)}$

$$= \frac{Ak + A + Bk - B}{(k-1)k(k+1)}$$

$A - B = 1 \quad A + B = 0 \quad \therefore A = \frac{1}{2}, \; B = -\frac{1}{2}$

Now

$$\left(\frac{1}{k(k-1)} - \frac{1}{k(k+1)}\right) > \frac{1}{k^3}$$

So

$$S_n = \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \ldots + \frac{1}{n^3}$$

$$< \frac{1}{2}\left(\frac{1}{3\times 2} - \frac{1}{3\times 4} + \frac{1}{4\times 3} + \frac{1}{4\times 5} + \ldots + \frac{1}{n(n-1)} - \frac{1}{n(n+1)}\right)$$

$$= \frac{1}{2}\left(\frac{1}{6} - \frac{1}{n(n+1)}\right)$$

$$< \frac{1}{2}\left(\frac{1}{6}\right) \quad \text{since } n > 0$$

$$= \frac{1}{12} \quad \text{as required}$$

Q16

a) i) $\dfrac{2}{\sqrt{k+1}+\sqrt{k}} < \dfrac{1}{\sqrt{k}}$

$$\text{LHS} = \frac{2\sqrt{k+1}-2\sqrt{k}}{k+1-k}$$

$$= 2\sqrt{k+1}-2\sqrt{k}$$

RTP $\quad 2\sqrt{k+1}-2\sqrt{k} < \dfrac{1}{\sqrt{k}}$

Consider FSOC

$$2\sqrt{k+1}-2\sqrt{k} > \frac{1}{\sqrt{k}}$$

$\times\sqrt{k}$ (+ve) $\quad 2\sqrt{k(k+1)}-2k > 1$

$$2\sqrt{k(k+1)} > 1+2k$$

Square (both side +ve) $\quad 4(k(k+1) > (1+2k)^2$

$$4k^2+4k > 1+4k+4k^2$$

$$0 > 1$$

Which is a contradiction

$\therefore \dfrac{2}{\sqrt{k+1}+\sqrt{k}} < \dfrac{1}{\sqrt{k}}$

Q16

a) ii) Consider $\frac{2}{\sqrt{k+1}+\sqrt{k}} < \frac{1}{\sqrt{k}}$

$LHS = 2\sqrt{k+1} - 2\sqrt{k}$ (from i)

$\therefore 2\sqrt{k+1} - 2\sqrt{k} < \frac{1}{\sqrt{k}}$

$2(\sqrt{k+1} - \sqrt{k}) < \frac{1}{\sqrt{k}}$

Now

$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{4489}} > 2(\sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \ldots + \sqrt{4490} - \sqrt{4489})$

$= -2 + 2(67.00\ldots)$

> 132

Now

Consider $2\sqrt{k} > \sqrt{k} + \sqrt{k-1}$ for $k \geq 2$

$\therefore \frac{1}{\sqrt{k}} < \frac{2}{\sqrt{k} + \sqrt{k-1}}$

$= \frac{2\sqrt{k} - 2\sqrt{k-1}}{k - k + 1}$

$= 2(\sqrt{k} - \sqrt{k-1})$

Now $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{4489}} < 2(\sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \ldots + \sqrt{4489} - \sqrt{4488})$

$= -2 + 2(67)$

$= 132$ — (1)

Add 1 to both sides

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{4489}} < 133$$

$$\therefore 132 < 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{4489}} < 133.$$

Q16

b) $u_x = 24\cos 30 = 12\sqrt{3}$ $\qquad u_y = 24\sin 30 = 12$

$k = \ln 2 \qquad g = 10 \qquad t = 1$

$$x(1) = \frac{12\sqrt{3}}{\ln 2}\left(1 - e^{-\ln 2}\right)$$

$$= \frac{12\sqrt{3}}{\ln 2}\left(1 - \frac{1}{2}\right)$$

$$= \frac{6\sqrt{3}}{\ln 2}$$

≈ 15 m (nearest metre)

$$y(1) = \frac{-10}{\ln 2} + \frac{1}{\ln 2}\left(12 + \frac{10}{\ln 2}\right)\left(1 - \tfrac{1}{2}\right)$$

$= 4.636\ldots$

$= 4.64$ m (2 dp).

ii) $a = 1 \quad c = 5 \quad 4 = \frac{2\pi}{n}$

$$n = \frac{\pi}{2}$$

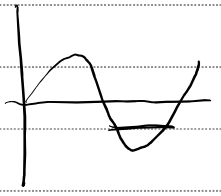
∴ Take SHM of crossbar as

$$h = \sin\left(\frac{\pi}{2}t\right) + 5$$

We need height at $t+1$ s to be less than 4.64m

$$4.64 > \sin\left(\frac{\pi}{2}(t+1)\right) + 5$$

$$-0.36 > \sin\left(\frac{\pi}{2}(t+1)\right)$$



$$\pi + \sin^{-1}(0.36) < \frac{\pi}{2}(t+1) < 2\pi - \sin^{-1}(0.36)$$

$$3.5098 < \frac{\pi}{2}(t+1) < 5.9149$$

$$1.23 < t < 2.77$$

Consider periods

$$5.23 < t < 6.77$$

$$9.23 < t \leq 10$$

Note: 10s should be included.

Q16

c) i) If ω is a root then

$$\omega^3 = 1$$
$$\omega^3 - 1 = 0$$
$$0 = (\omega - 1)(\omega^2 + \omega + 1)$$

Since $\omega \neq 1$ then $\omega^2 + \omega + 1 = 0$

ii) Consider

① : $(1+\omega)^{2026}$

$$= \binom{2026}{0} + \binom{2026}{1}\omega + \binom{2026}{2}\omega^2 + \ldots + \binom{2026}{2026}\omega^{2026}$$

② $(1+\omega^2)^{2026}$

$$= \binom{2026}{0} + \binom{2026}{1}\omega^2 + \binom{2026}{2}\omega^4 + \ldots + \binom{2026}{2026}\omega^{4052}$$

③ $(1+1)^{2026}$

$$= \binom{2026}{0} + \binom{2026}{1} + \binom{2026}{2} + \ldots + \binom{2026}{2026}$$

① + ② + ③ gives

$$3\binom{2026}{0} + (1+\omega+\omega^2)\binom{2026}{1} + (1+\omega^2+\omega^4)\binom{2026}{2} + (1+\omega^3+\omega^6)\binom{2026}{3} + \ldots + (1+\omega^{2026}+\omega^{4052})\binom{2026}{2026} \quad - \text{④}$$

Consider $1 + \omega + \omega^2 = 0$ (from i)

Also $\omega^{3k+1} = \omega$

$\omega^{3k+2} = \omega^2$

$\omega^{3k} = 1$

$\therefore$ (4) gives

$$3\binom{2026}{0} + 3\binom{2026}{3} + 3\binom{2026}{6} + \ldots + 3\binom{2026}{2025}$$

$$= (1+1)^{2026} + (1+\omega)^{2026} + (1+\omega^2)^{2026}$$

$$= 2^{2026} + (-\omega^2)^{2026} + (-\omega)^{2026}$$

$$= 2^{2026} + \omega^{4052} + \omega^{2026}$$

$$= 2^{2026} + \omega^2 + \omega$$

$$= 2^{2026} - 1$$

$\therefore$

$$\binom{2026}{0} + \binom{2026}{3} + \binom{2026}{6} + \ldots + \binom{2026}{2025} = \frac{2^{2026} - 1}{3}$$