IMPORTANT:
This resource was developed to support a previous version of the syllabus.
It may contain content that differs from the current syllabus.

Mathematics General 2

HSC Specimen Examination Worked Solutions

Section II

Question 26 (a) (i)

Nominal

Question 26 (a) (ii)

How many calls do you make on your mobile phone each day?

Question 26 (a) (iii)

For the sample to be representative by Year group, it would include a fixed percentage of each Year group of the NSW high schools in the sample.

Question 26 (b)

$$\begin{aligned}\text{Area of semicircle} &= \frac{1}{2}\times\pi\times r^2 \\ &= \frac{1}{2}\times\pi\times 9^2 \\ &= 127.23450...\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of trapezium} &= \frac{h}{2}(a+b) \\ &= \frac{3}{2}(6+9) \\ &= 22.5\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of remaining shape} &= 127.23450... - 22.5 \\ &= 104.73450... \\ &= 105\ \text{cm}^2\ \text{(to nearest square cm)}\end{aligned}$$

Question 26 (c)

$$BAC_{\text{female}} = \frac{10N - 7.5H}{5.5M}$$
$$= \frac{10(4.5) - 7.5(3)}{5.5 \times 66}$$
$$= 0.061983471...$$

$$\text{Number of hours for BAC to reach zero} = \frac{BAC}{0.015}$$
$$= \frac{0.061983471...}{0.015}$$
$$= 4.132...$$
$$= 4 \text{ hours and 10 minutes (to nearest 10 minutes)}$$

Question 26 (d)

When it is 8 am on Friday in Perth, it is 8 hours earlier in Greenwich, ie midnight (12 am) at the end of Thursday/beginning of Friday.

When it is midnight at the end of Thursday/beginning of Friday in Greenwich, it is 3 hours earlier in Santiago, ie Thursday at 9 pm.

Question 26 (e)

$$\text{Stamp duty for the car} = 3\% \text{ of } \$45000 + 5\% \text{ of } (\$50000 - \$45000)$$
$$= 0.03 \times \$45000 + 0.05 \times \$5000$$
$$= \$1350 + \$250$$
$$= \$1600$$

$$\text{Total amount to be paid} = \$50000 + \$1995 + \$748 + \$323 + \$920 + \$1600$$
$$= \$55586$$

Question 26 (f)

Cost for 300 two-minute voice calls $= \$(300-250)\times(\$0.40+\$0.45\times 4)$

$= \$110$

Cost for 150 SMS messages is zero (included in plan)

Cost for 1.2 GB of data is zero (included in plan)

Cost for 20 MMS messages $= 20\times\$0.55$

$= \$11$

Cost for 10 five-minute video calls $= 10\times(\$0.35+\$1\times 5)$

$= \$53.50$

Total amount of bill for July $= \$25+\$110+\$11+\$53.50$

$= \$199.50$

Question 27 (a)

$$\text{Perimeter} = \frac{230^\circ}{360^\circ}\times 2\pi r + 2r$$

$$= \frac{230}{360}\times 2\pi \times 13 + 26$$

$$= 78.1853446...$$

$= 78$ cm (to nearest cm)

Question 27 (b)

Rate of 7.2% pa = 0.006 per month

From the table, the present value interest factor is 41.59882

If the monthly repayment is x, then

$$41.59882x = \$10000$$

$$x = \$240.39 \text{ (to nearest cent)}$$

Total repaid $= 48\times\$240.39$

$= \$11538.72$

Interest $= \$11538.72 - \$10000$

$= \$1538.72$

Question 27 (c) (i)

From the data,

$\text{lower quartile } (Q_L) = 12, \text{ upper quartile } (Q_U) = 28$

$$\begin{aligned}\text{Interquartile range} &= Q_U - Q_L \\ &= 28 - 12 \\ &= 16\end{aligned}$$

Question 27 (c) (ii)

Outliers above the median are greater than $Q_U + 1.5 \times IQR$, which is $28 + 1.5 \times 16 = 52$

$\therefore$ the score 51 is not an outlier

Question 27 (d)

Salary is usually stated as a set amount per annum, whereas commission is stated as a percentage of the value of items sold.

Salary does not change per payment period, whereas the amount of retainer plus commission increases as the number of items sold increases.

Question 27 (e) (i)

$$\begin{aligned}\text{Jason's } z\text{-score} &= \frac{85 - 70}{10} \\ &= 1.5\end{aligned}$$

Question 27 (e) (ii)

Since Mary has a z-score of 0, she has achieved the mean score in the test, ie her mark is 70.

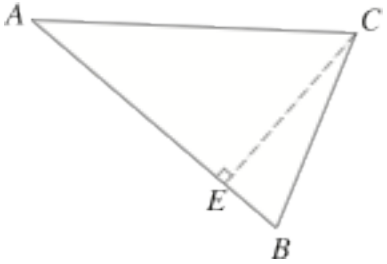
Question 27 (e) (iii)

A mark of 50 is 2 standard deviations below the mean mark (70), ie a mark of 50 has a z-score of -2, while a mark of 80 is 1 standard deviation above the mean (ie it has a z-score of 1).

For marks that are normally distributed, approximately 68% of scores will have z-scores between -1 and 1. A further $\frac{(95 - 68)}{2}$%, or 13.5%, of scores will have z-scores between -2 and -1.

$$\begin{aligned}\therefore \text{ the percentage of marks that lie between 50 and 80 } &= (68 + 13.5)\% \\ &= 81.5\%\end{aligned}$$

Question 27 (f)



Let the vertices of the shaded area (triangular) be *A, B* and *C,* and let *E* be the point at which the perpendicular from *C* meets *AB*.

On the given map, AB is approx 4 cm and CE is approx 2.5 cm

From the scale given, 2 cm $= 60$ km, and so 1 cm $= 30$ km

The actual length of AB is approx $4 \times 30 \text{ km} = 120 \text{ km}$

and of CE is approx $2.5 \times 30 \text{ km} = 75 \text{ km}$

$\therefore$ the actual shaded area is approx $\frac{1}{2} \times 120 \times 75 \text{ km}^2 = 4500 \text{ km}^2$

Question 28 (a)

$4x + y = 13$ (1)

$2x - y = 2$ (2)

Adding (1) and (2)

$$6x = 15$$
$$x = \frac{15}{6}$$
$$x = 2\frac{1}{2}$$

Substituting $2\frac{1}{2}$ for x in (1)

$$4\left(2\frac{1}{2}\right) + y = 13$$
$$10 + y = 13$$
$$y = 3$$

$\therefore x = 2\frac{1}{2},\ y = 3$

Question 28 (b) (i)

$$\text{Number of possible numberplates} = 10 \times 10 \times 26 \times 26 \times 10 \times 10$$
$$= 6\,760\,000$$

Question 28 (b) (ii)

$$\text{Number of possible numberplates with 'JO' in the middle} = 10 \times 10 \times 10 \times 10$$
$$= 10\,000$$

Two of these are the numberplates that Jo would like.

$$\therefore \text{ the probability that she is issued with one of these two numberplates} = \frac{2}{10\,000}$$
$$= \frac{1}{5000}$$

Question 28 (c)

$$\text{Area of land } (A) = 1.5 \text{ ha}$$
$$= 15000 \text{ m}^2$$
$$\text{Depth of water } (h) = 17 \text{ cm}$$
$$= 0.17 \text{ m}$$

$$\text{Using } V = Ah,$$
$$\text{Volume of water} = 15000 \times 0.17$$
$$= 2550 \text{ m}^3$$
$$= 2550 \text{ kilolitres}$$

Question 28 (d)

$$\text{Financial expectation for Tai's game} = \frac{1}{5} \times \$10 + \frac{1}{2} \times \$3 - \frac{3}{10} \times \$8 - \$2$$
$$= -\$0.90$$

ie Tai's financial expectation for this game is a loss of 90 cents

Question 28 (e)

From this point, use a trundle wheel or tape to measure the distances to each of *A, B, C* and *D*. On the paper, draw a line of sight to each of *A, B, C* and *D*. Mark on these lines the distances measured.

Question 28 (f)

From the similar triangles in the diagram,

$$\frac{4}{3+d}=\frac{1.5}{d}$$

$$\begin{aligned}4d &= 1.5(3+d)\\ 4d &= 4.5+1.5d\\ 2.5d &= 4.5\\ d &= 1.8\end{aligned}$$

$\therefore$ the length of Joe's shadow is 1.8 metres

Question 29 (a)

$$\begin{aligned}\frac{5x}{4}+9 &= 10x-12\\ 5x+36 &= 40x-48\\ 84 &= 35x\\ x &= \frac{84}{35}\\ &= 2\frac{2}{5}(\text{or } 2.4)\end{aligned}$$

Question 29 (b) (i)

Volume of prism = Area of base $\times$ perpendicular height

Volume of tank = [area of rectangle + area of two (equal) semicircles] $\times$ perpendicular height

$$\begin{aligned}&=1.8\times 0.6+2\left(\frac{1}{2}\times\pi\times 0.3^2\right)\times 2.1\\ &= 2.8617611012...\ \text{m}^3\end{aligned}$$

Capacity of tank $= 2.8617611012...\times 1000\ \text{m}^3$

$= 2862$ litres (to nearest litre)

Question 29 (b) (ii)

Using $V = b(0.95)^n$,

$$\text{amount of water remaining in tank} \approx 2862 \times 0.95^{21}$$
$$= 975 \text{ litres}$$

$$\therefore \text{ amount of water lost from tank} = 2862 - 975$$
$$= 1887 \text{ litres}$$

Question 29 (c) (i)

Tax payable on a taxable income of \$21000 = \$3000

Question 29 (c) (ii)

$$\text{Gradient } m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$
$$= \frac{6000}{18000}$$
$$= \frac{1}{3}$$

Question 29 (c) (iii)

$$\text{Amount of each dollar payable in tax } = \frac{1}{3} \text{ of a dollar (using (ii))}$$
$$= 33 \text{ cents (to the nearest cent)}$$

Question 29 (c) (iv)

$$T = \frac{1}{3} I$$

Question 29 (d)

Answers could include:

- More cars are stolen at night (6 pm – 6 am) than during the day (6 am – 6 pm).
- During the day on Sundays is the time when the fewest cars are stolen, and during the night on Saturdays is the time when the most cars are stolen.
- The number of cars stolen during the day is fairly constant during the week, with a smaller number on Sundays.
- The number of cars stolen during the night increases during the week from Monday to Saturday.

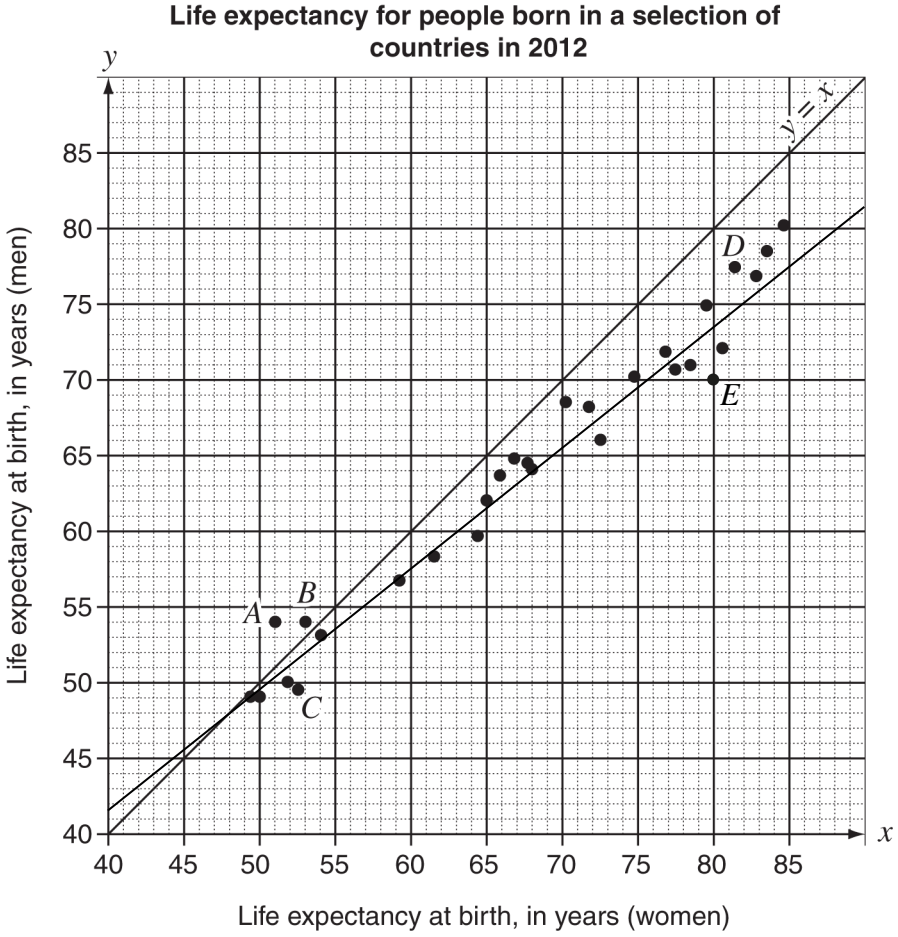
Question 30 (a) (i)

E is the point (80, 70). (The plot of the point and the labelling of the point are shown on the graph in answer to Q30 (a) (ii) (2).)

Question 30 (a) (ii) (1)

$$y = 0.8(60) + 9.4$$
$$= 48 + 9.4$$
$$= 57.4$$

Question 30 (a) (ii) (2) (also shows answer to Q30 (a) (i))



Question 30 (a) (iii)

Line represents equal life expectancy at birth for women and men.

Question 30 (a) (iv)

For most of the countries represented in the data, the life expectancy at birth for women is higher than that for men.

Question 30 (a) (v)

The life expectancy at birth for women is higher for Country *B* than for Country *A*, while the life expectancy at birth for men is the same for the two countries.

Question 30 (a) (vi)

Answers could include two of the following, regarding Country *D* compared to Country *C*:

- higher income per person
- lower child mortality rate
- higher health spending as a percentage of gross domestic product (GDP)
- higher number of medical doctors per 100 people
- higher percentage of population in urban areas
- higher food supply per person per day
- higher availability of energy, eg electricity.

Question 30 (b) (i)

$$\begin{aligned} 30\% \text{ of Xiang's monthly gross salary} &= 0.3 \times \$7000 \\ &= \$2100 \end{aligned}$$

From rows 11 and 12 of the spreadsheet, in October 2012 the maximum amount that Xiang's bank would have approved for her to borrow would have been an amount lying between \$300 000 and \$350 000, since \$1942 < \$2100 < \$2266.

Using rows 5 and 11 of the spreadsheet,

$$\begin{aligned} \text{maximum amount bank would have approved} &= \$300\,000 + \frac{\$2100 - \$1942}{\$6.47} \times 1000 \\ &= \$300\,000 + \$24\,420.40185... \\ &= \$324\,420 \text{ (to nearest dollar)} \end{aligned}$$

Question 30 (b) (ii)

Possible answers include:

- The graph of Jack's loan drops more steeply than Xiang's graph over the first 12 years. During that time, he may have had a lower interest rate, be making higher repayments, or be making more frequent repayments (or some combination of the three).
- After 12 years, Jack's graph changes to be less steep than Xiang's, and extends over a longer period (to 30 years). He may be making the same or lower repayments over the longer loan period, or the loan may have been moved to a higher interest rate but he is making the same repayments.